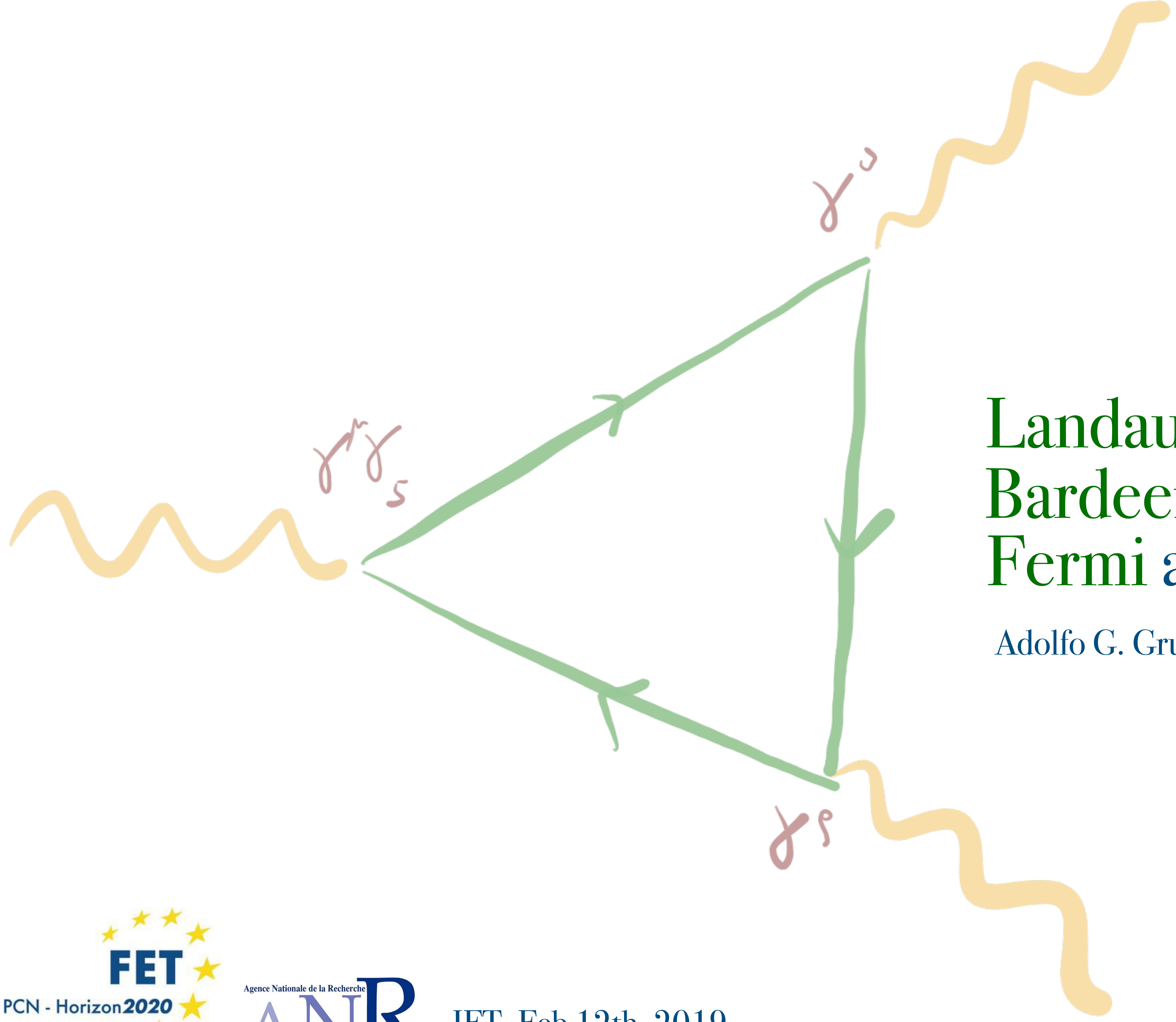


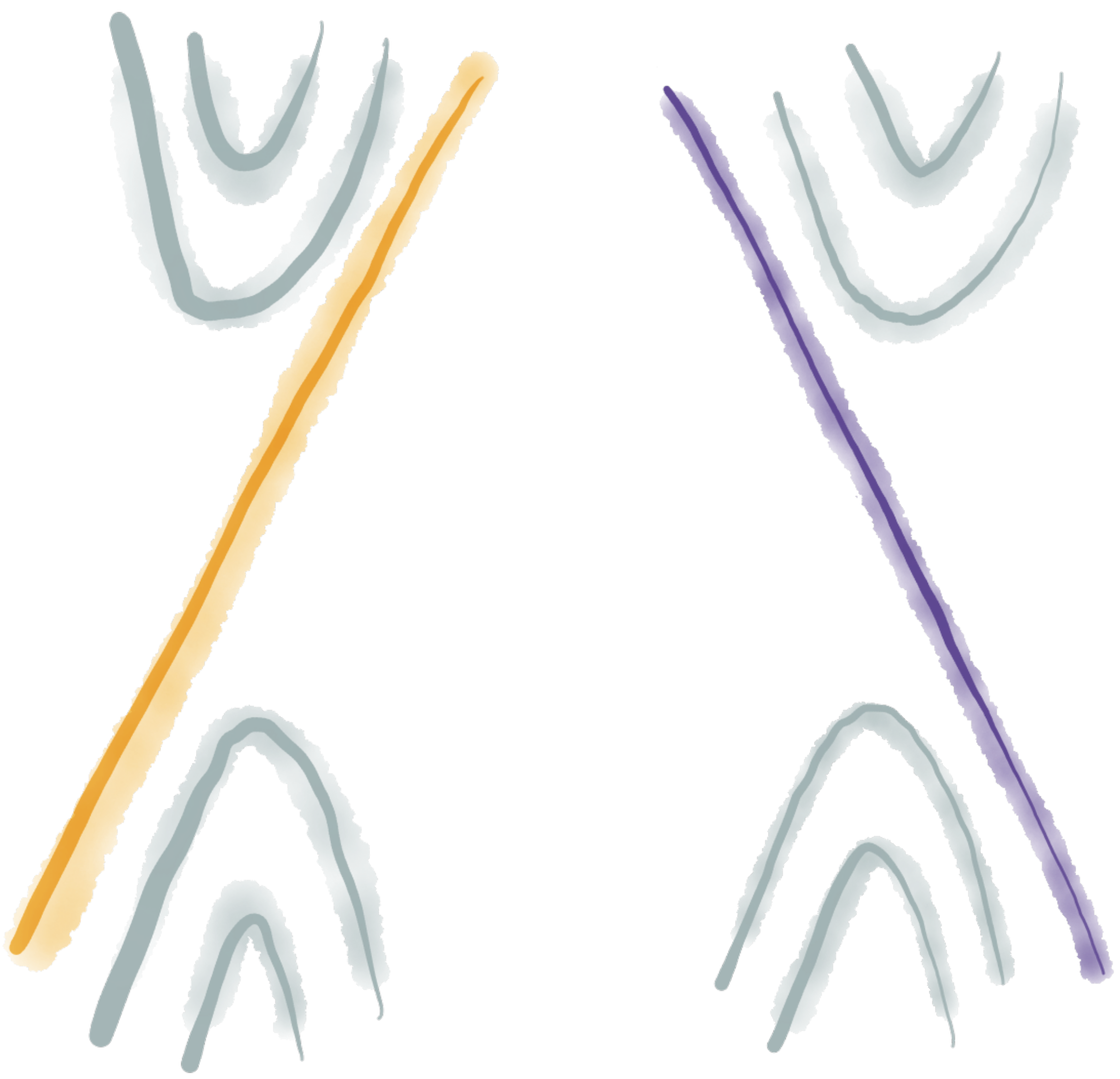
Landau levels Bardeen polynomials and Fermi arcs

Adolfo G. Grushin, Néel Institute, CNRS

IFT, Feb 12th, 2019

J. Behrends, S. Roy, M. Kolodrubetz, J. H. Bardarson, AGG 1807.06615
S. Roy, M. Kolodrubetz, N. Goldman, AGG 2D Mat. (2018)
AGG, J. Venderbos, A. Vishwanath, R. Ilan PRX (2016)

Landau levels



Strain

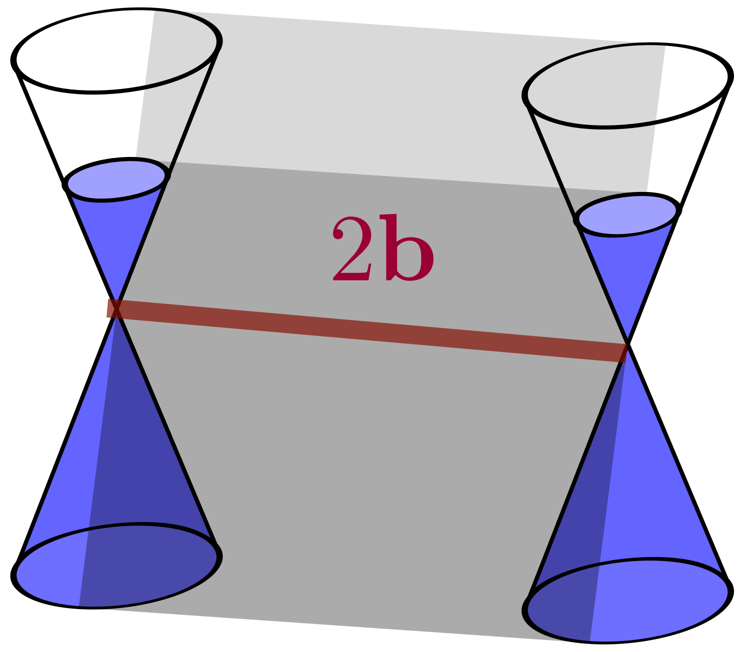
u_{ij} + Dirac/Weyl = pseudo-magnetic fields

Magnetic field

Dirac/Weyl

Pseudo magnetic field

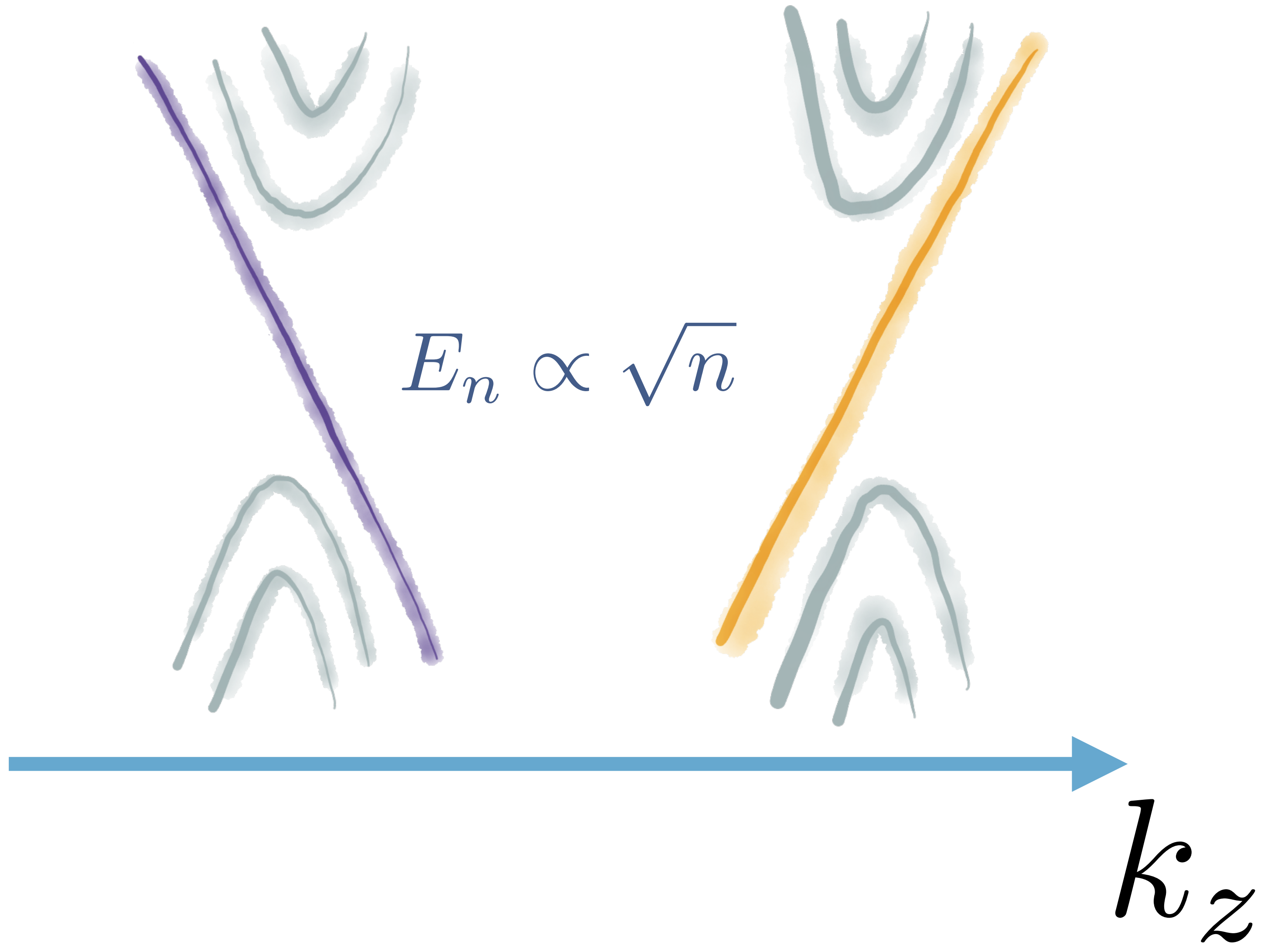
$$k_j \to k_j + A_j$$



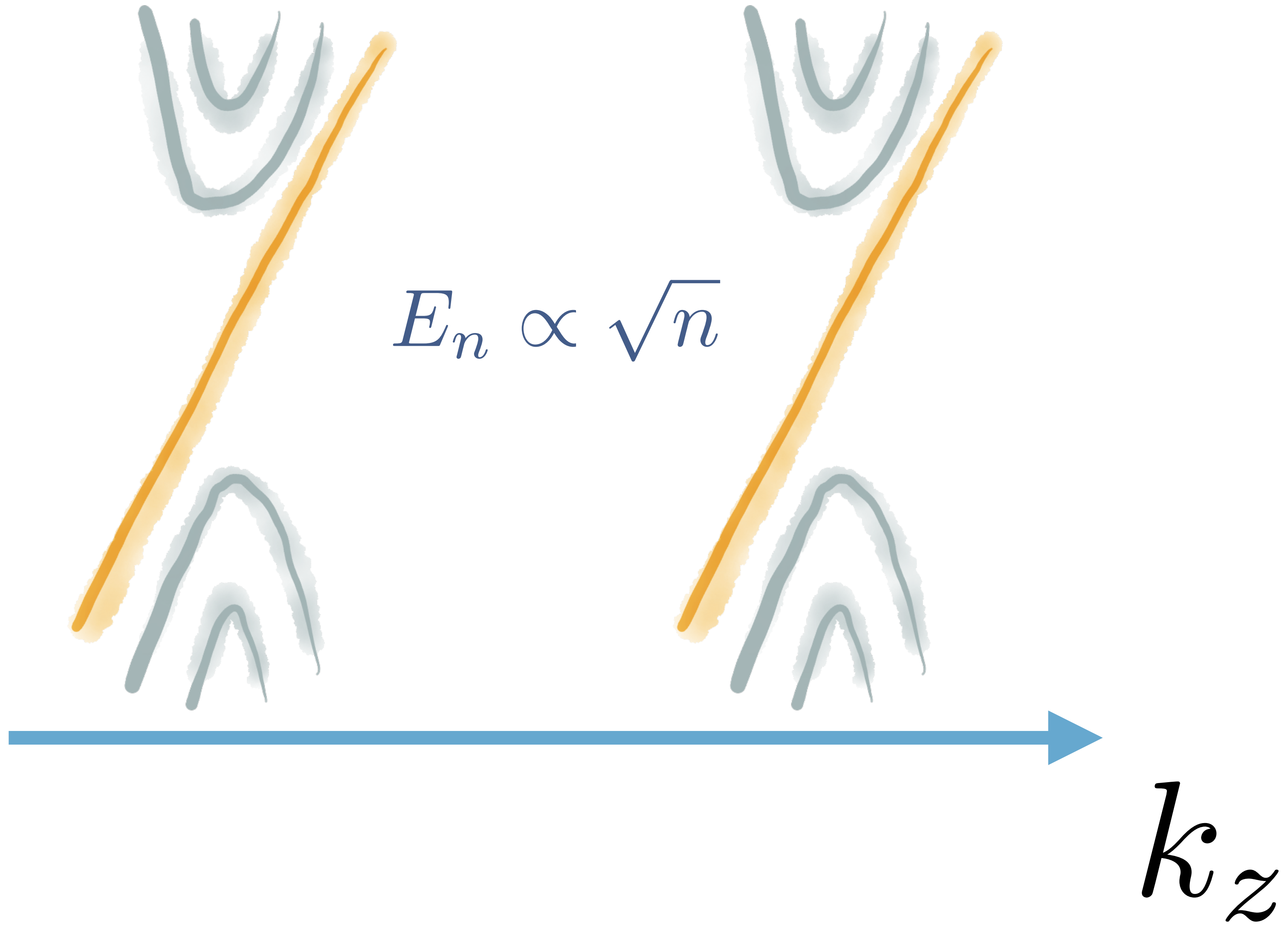
$$\mathbf{k} \pm \mathbf{b}$$

$$\delta b_i(x) \propto u_{ij}(x)\sigma_j$$

Landau levels



Pseudo-Landau levels



Position momentum locking

$$A_x = B_z y$$

$$\langle y \rangle \propto k_x$$

$$b_x = B_z^5 y$$

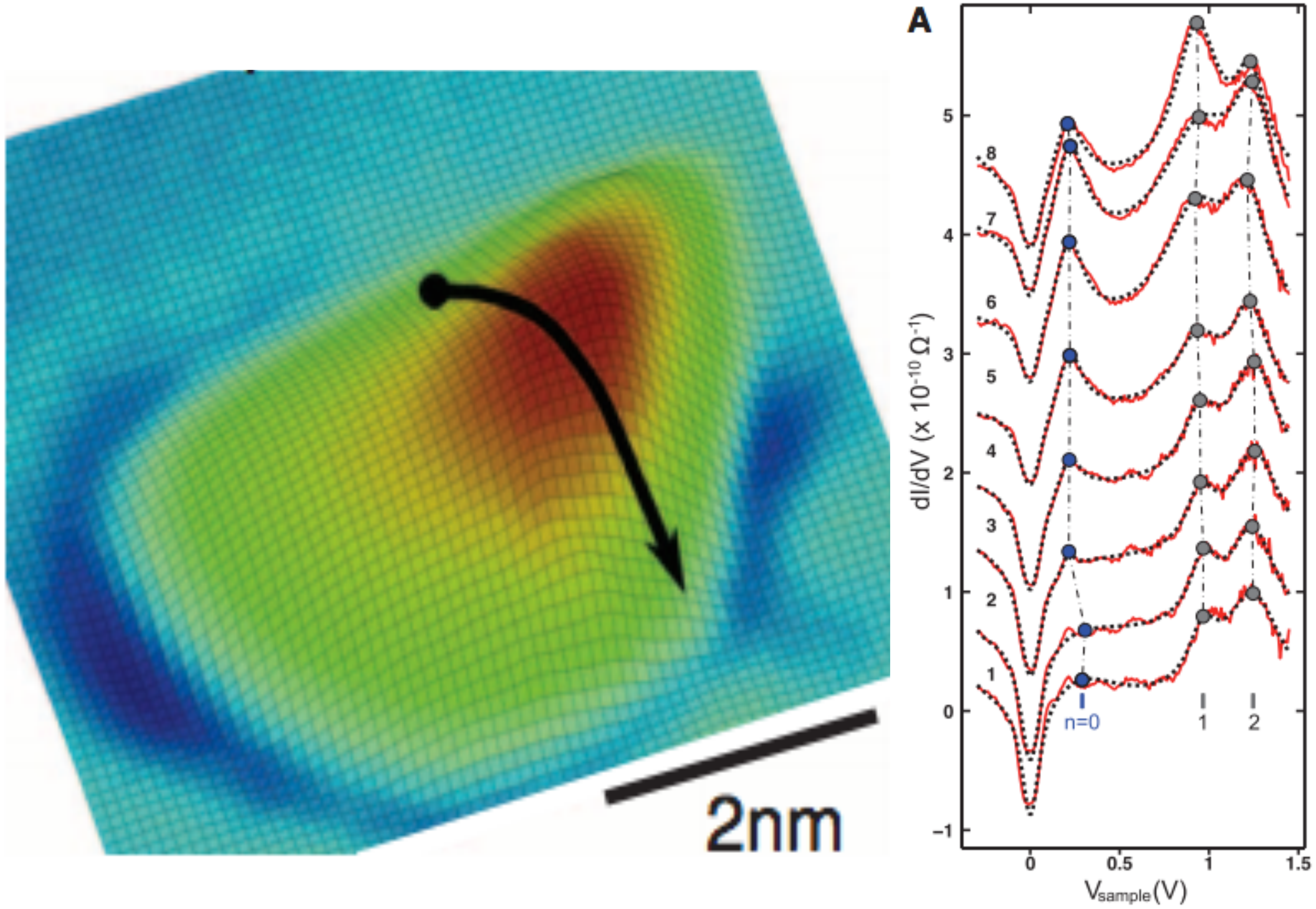
$$k_x \to k_x + B_z y$$

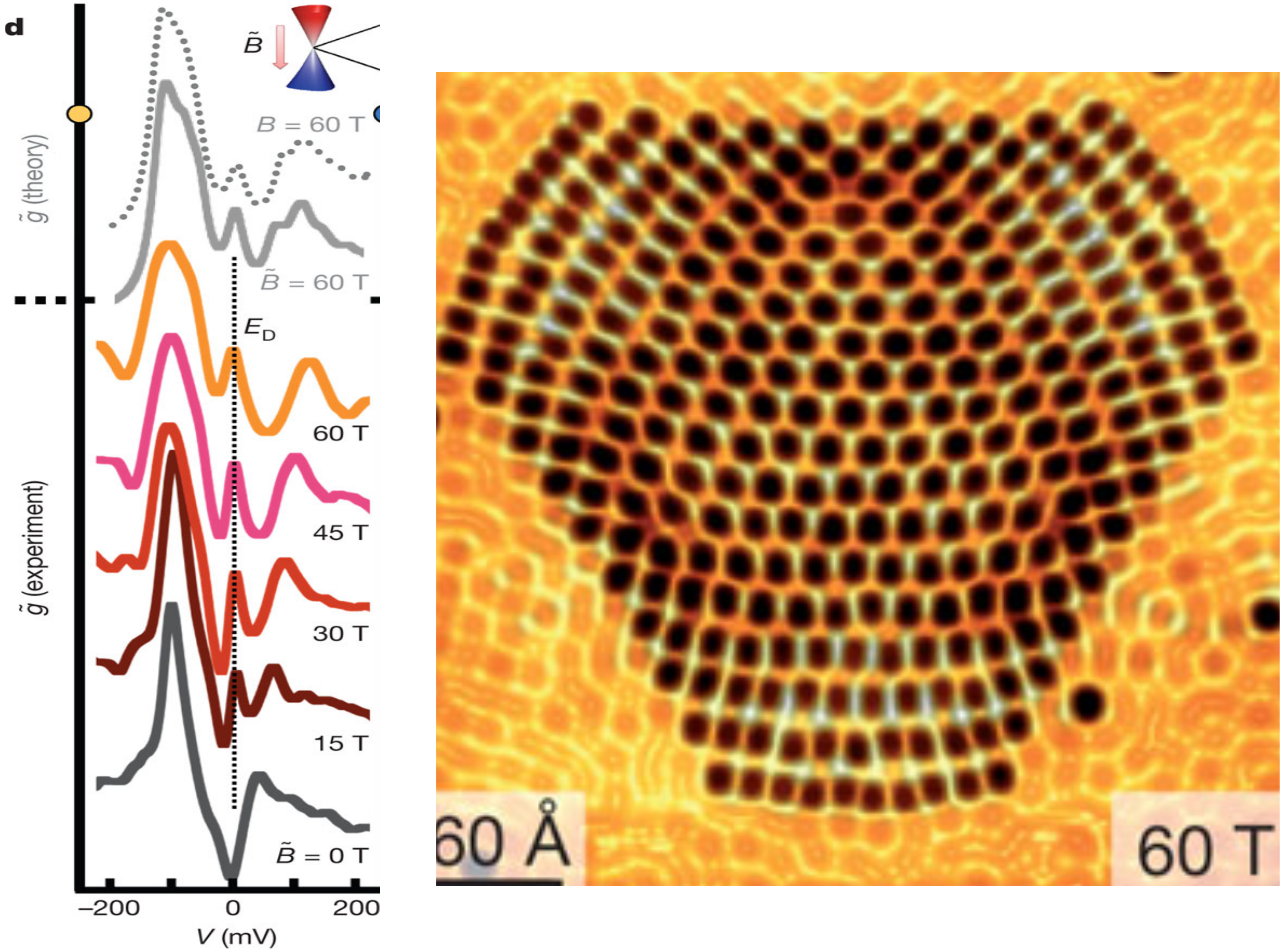
C. X. Liu, P. Ye, X. L.Qi PRB (2013)

A. Cortijo, Y. Ferreiros, K. Landsteiner, M. A. H. Vozmediano PRL (2016)

Stephan Rachel, Ilja Göthel, Daniel P. Arovas, and Matthias Vojta PRL (2016)

Graphene





Levy et al Science (2010) (Crommie Group, Berkeley)

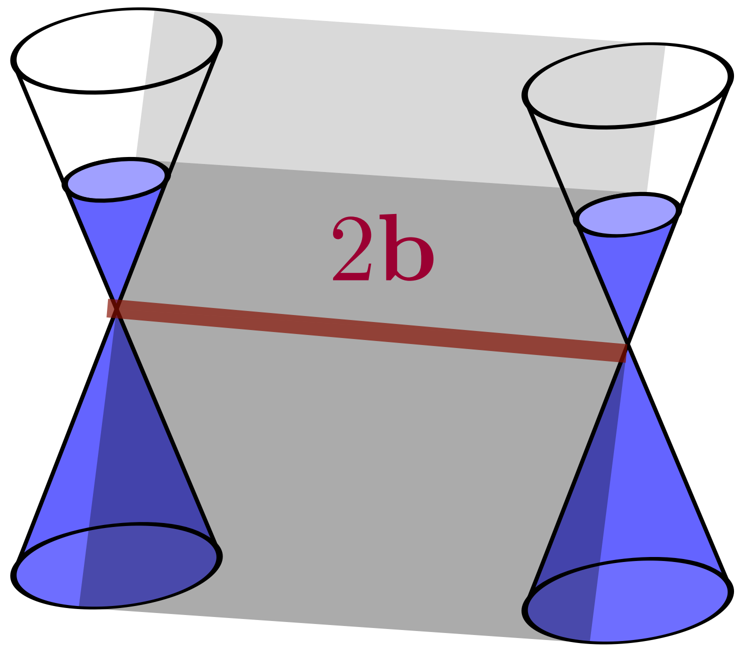
Gomes et al Nature (2012) (Manoharan Group, Stanford)

What is specific to 3D?

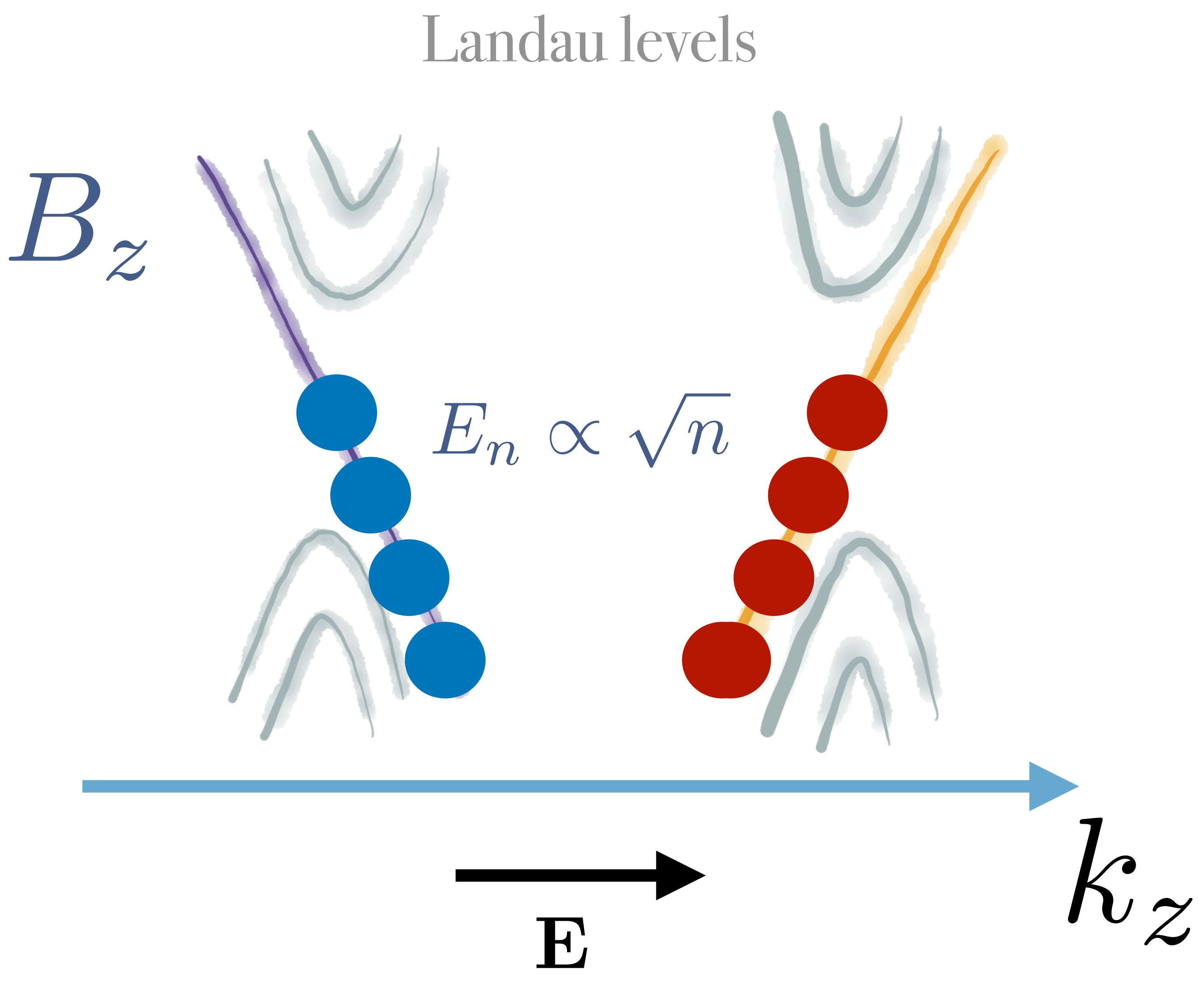
Dirac/Weyl

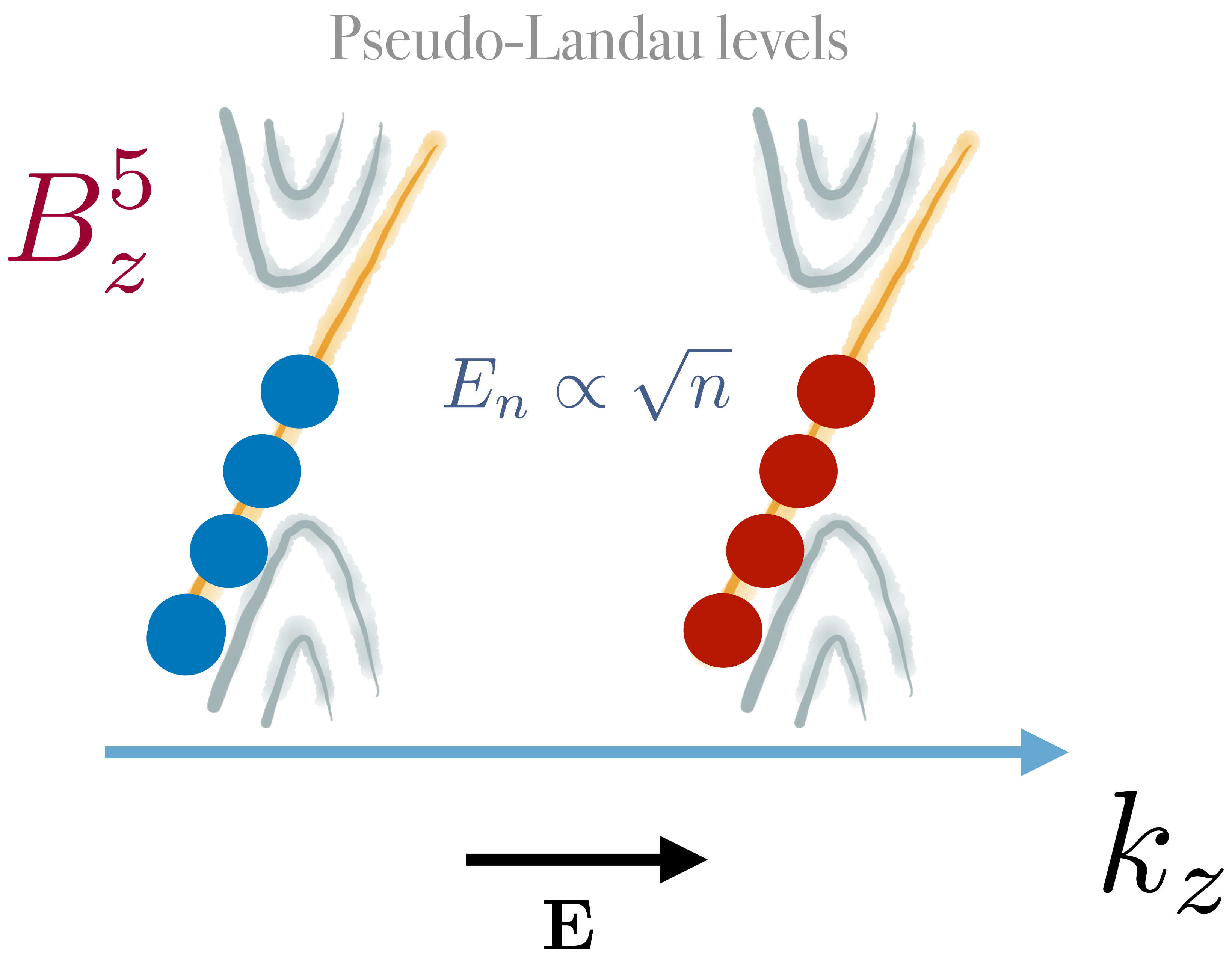
Magnetic field

Pseudo magnetic field

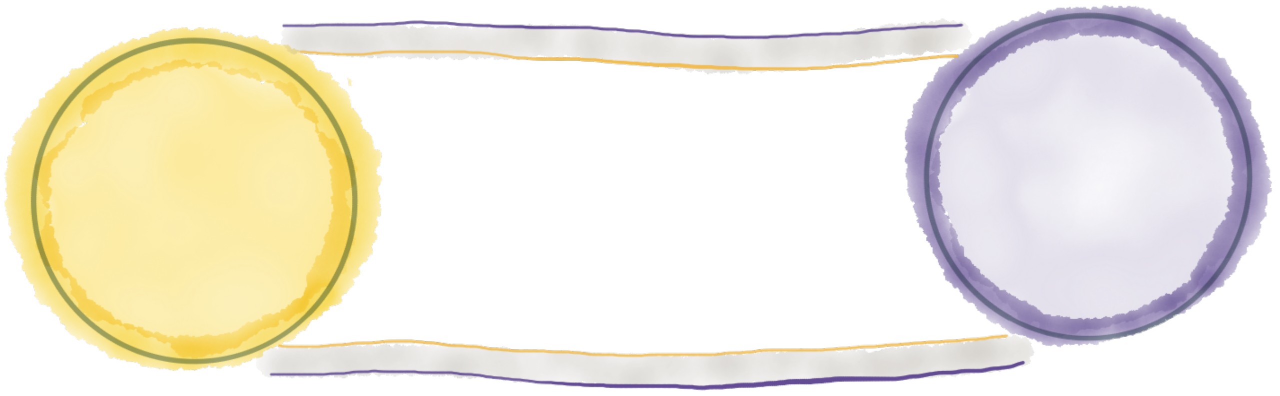


$\mathbf{k} \pm \mathbf{b}$

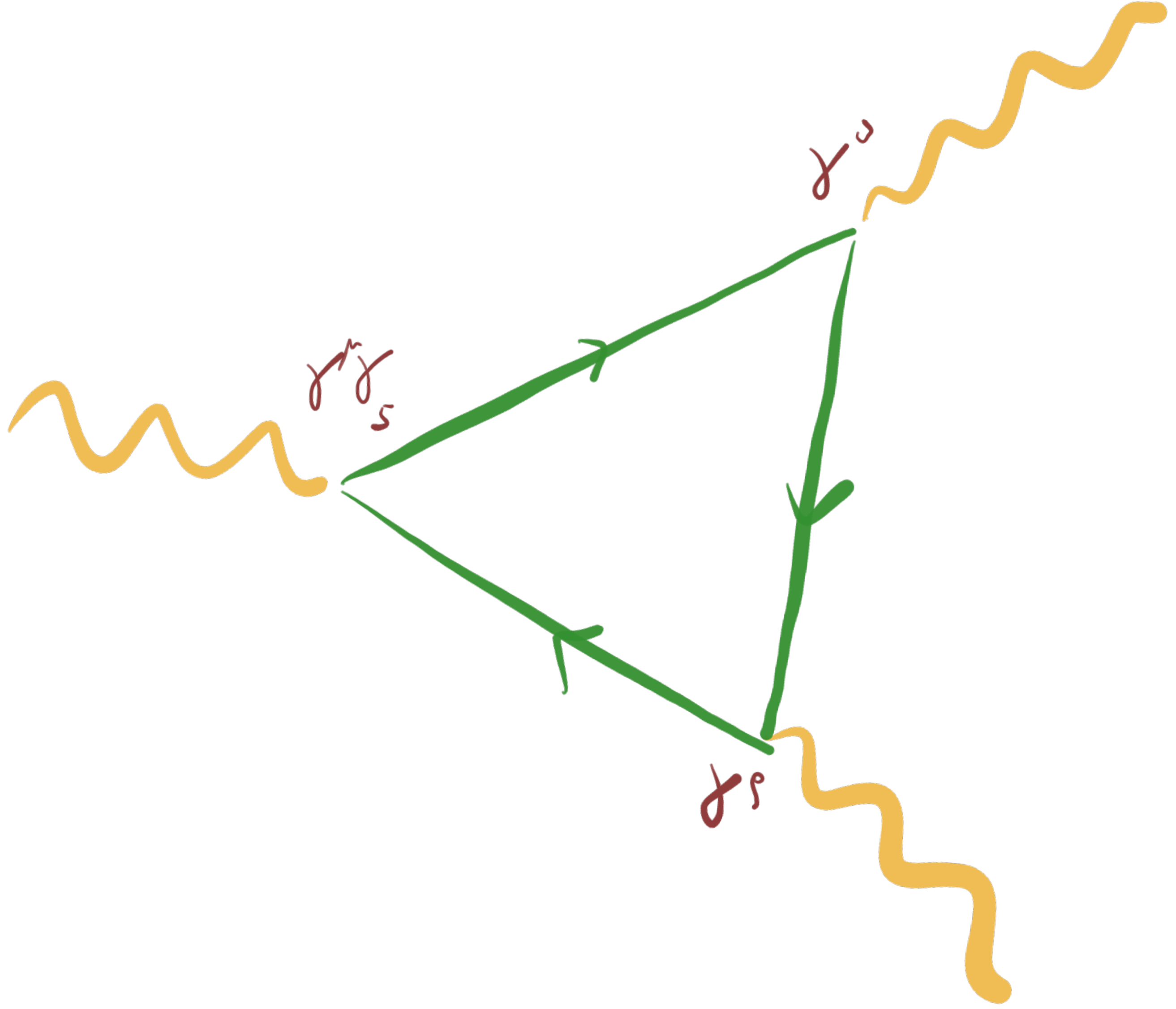




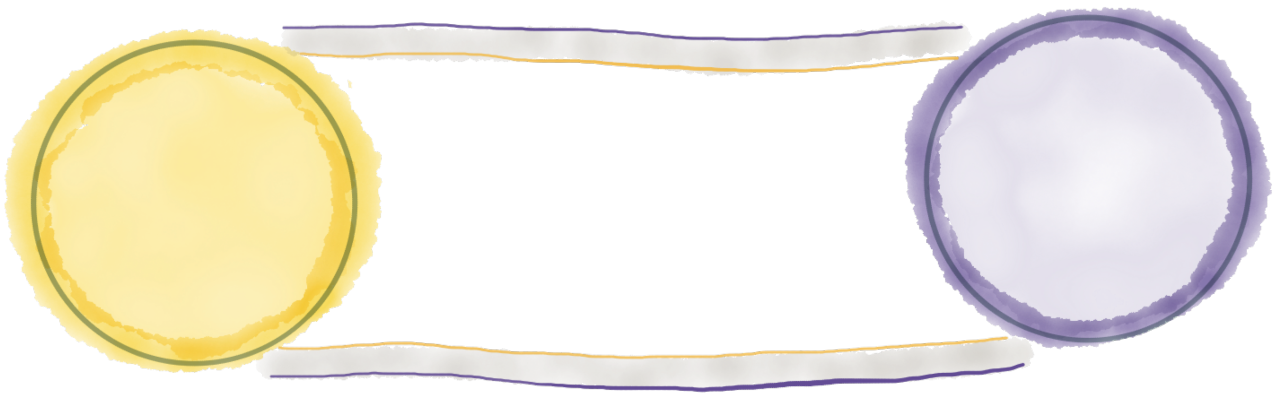
Surface states



Anomalies

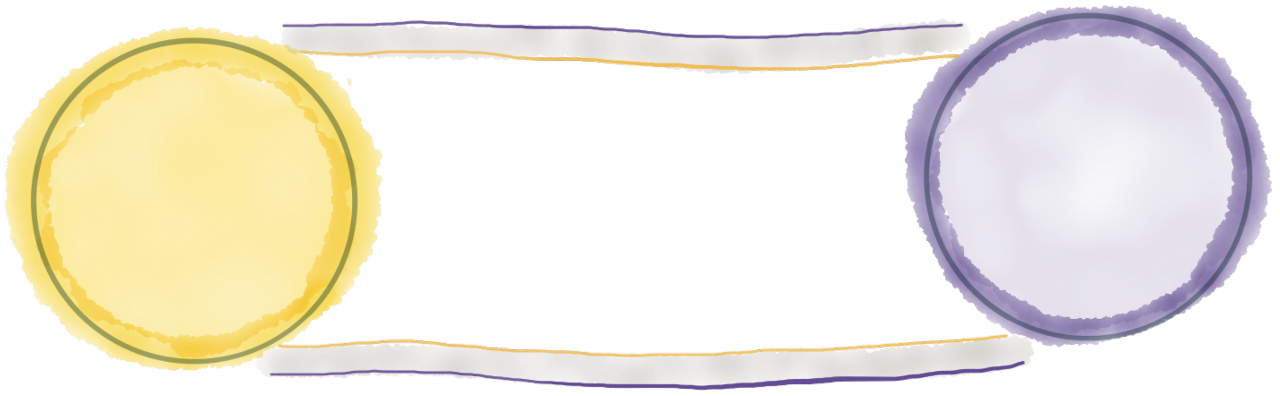


Surface states



Fermi arcs

Surface states

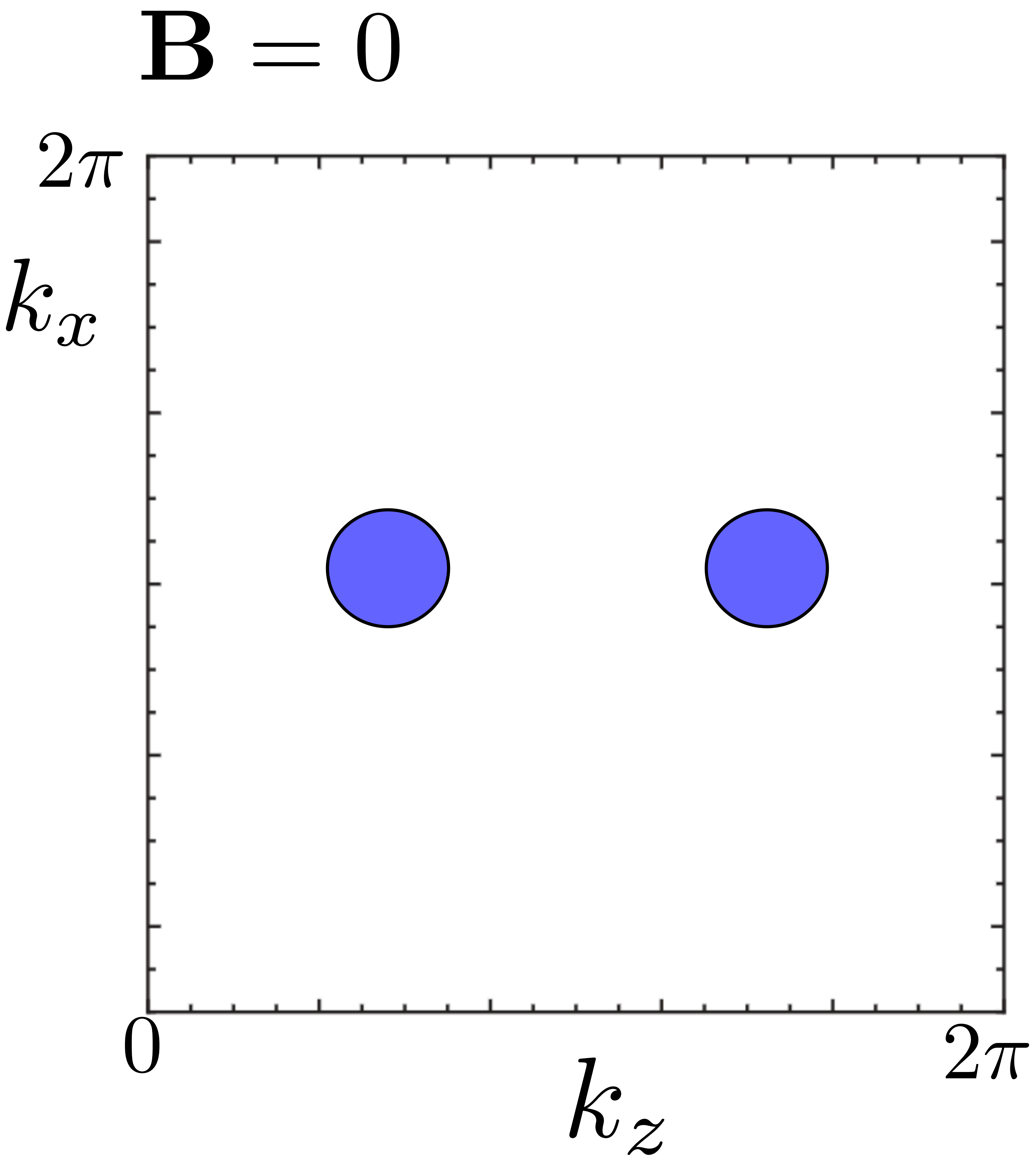


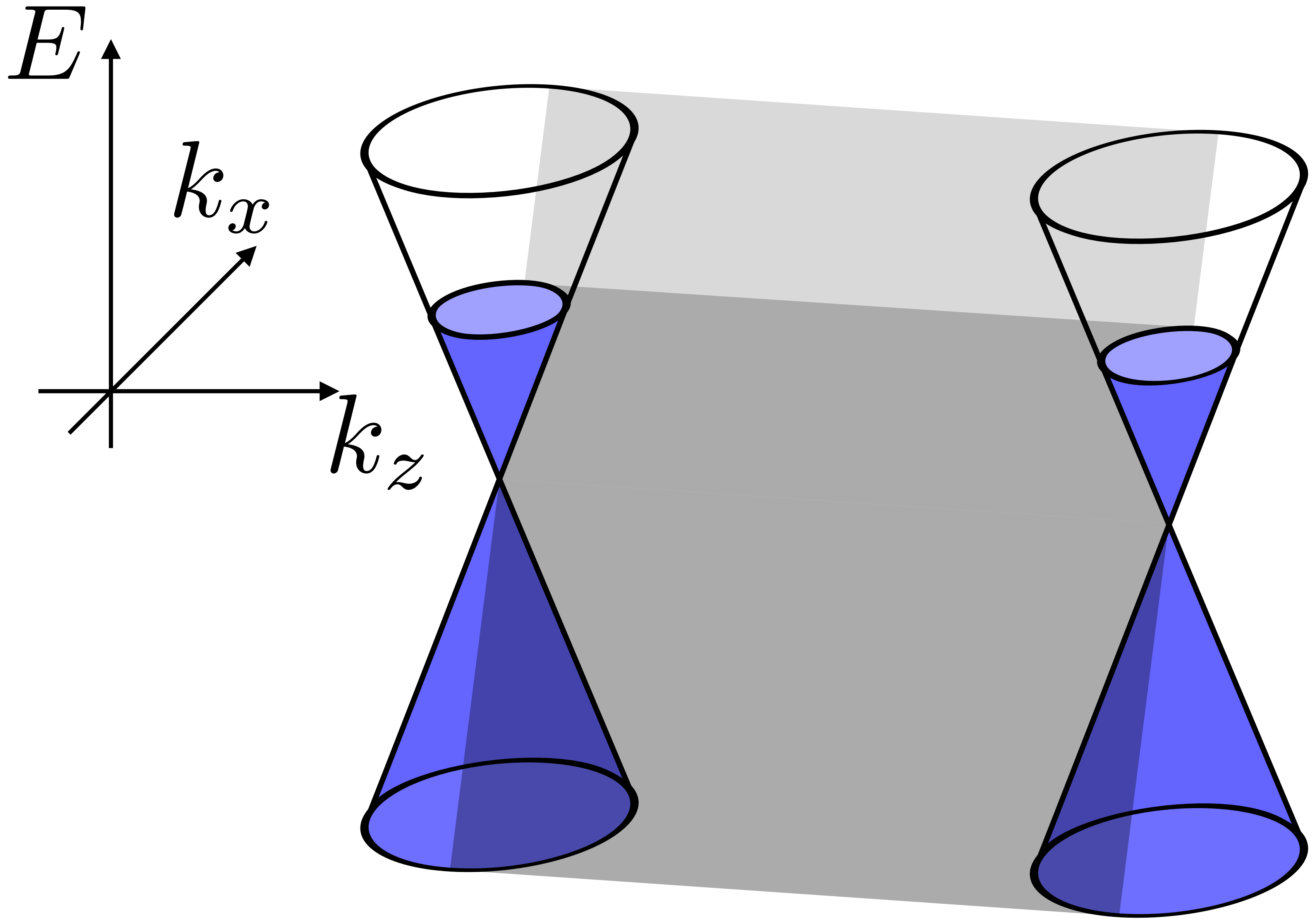
Fermi arcs

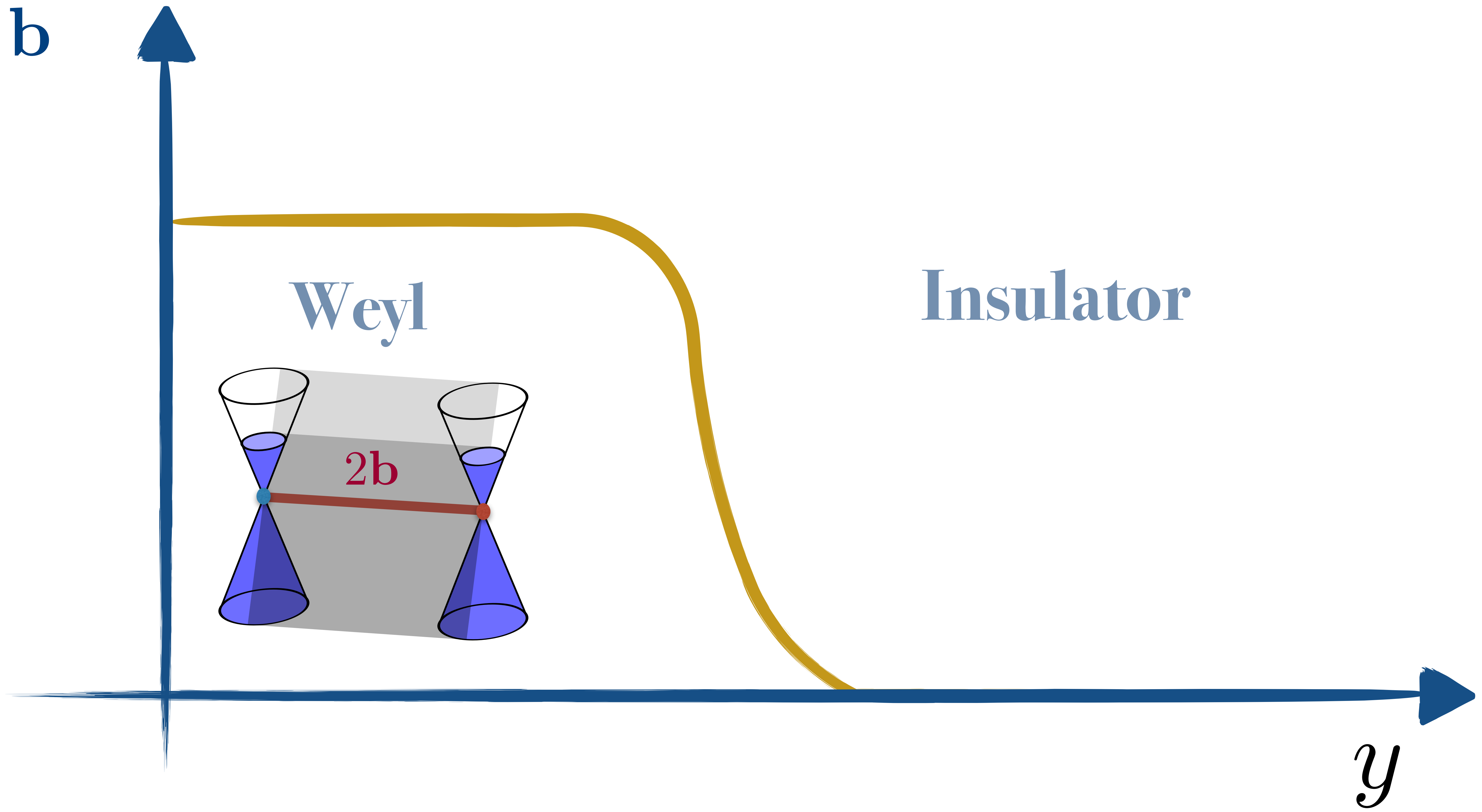
are

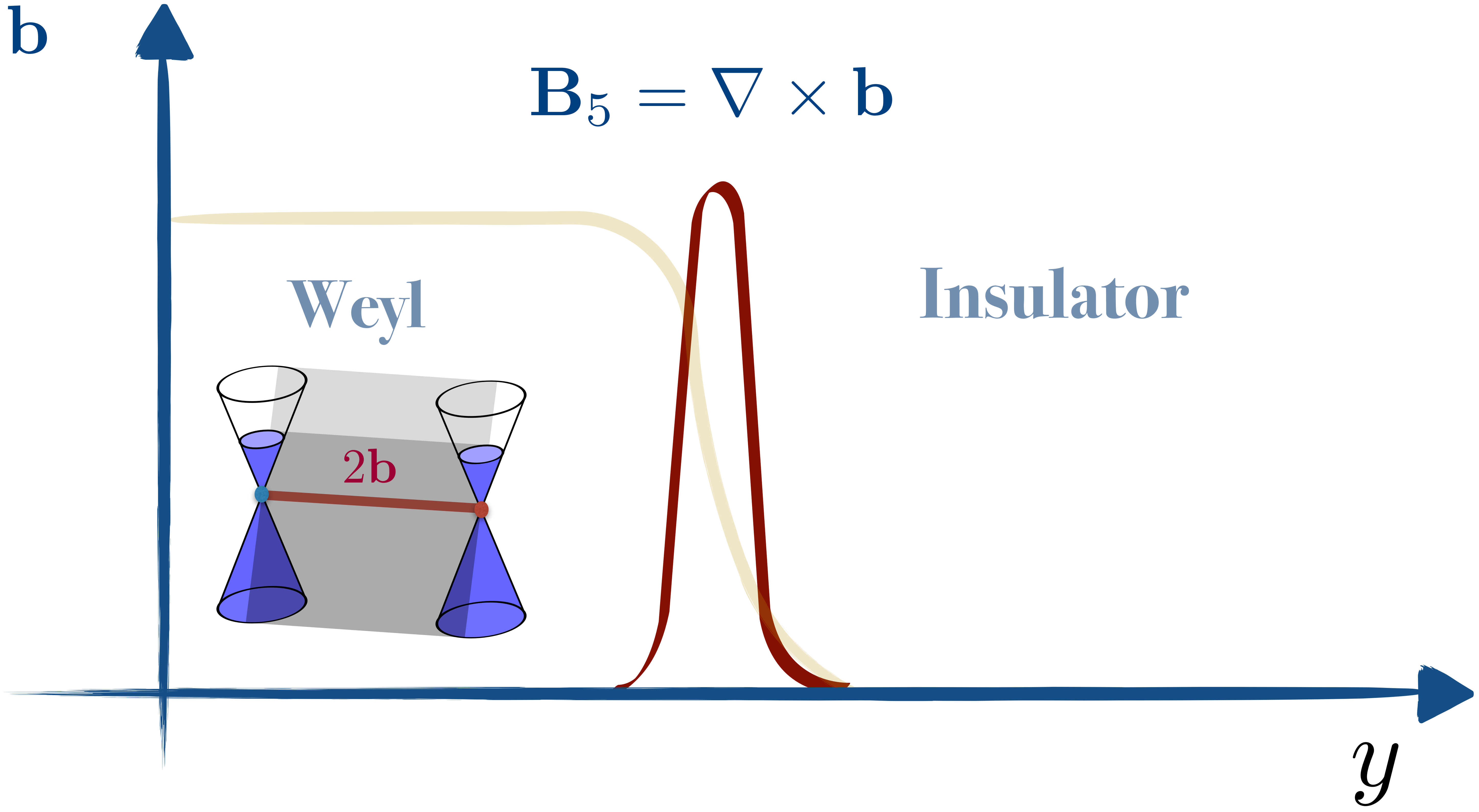
Landau levels

Zero magnetic field, periodic boundary conditions

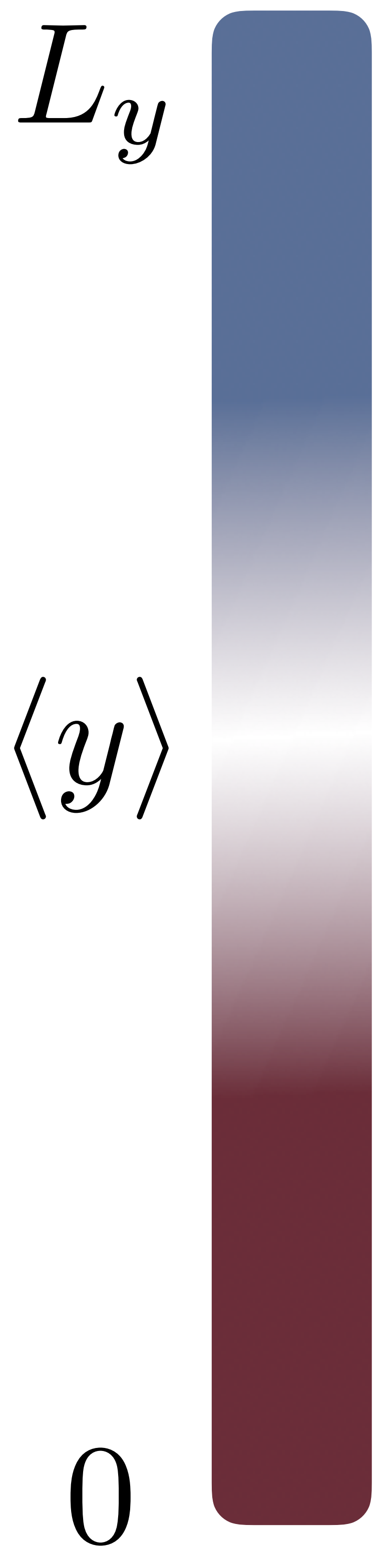


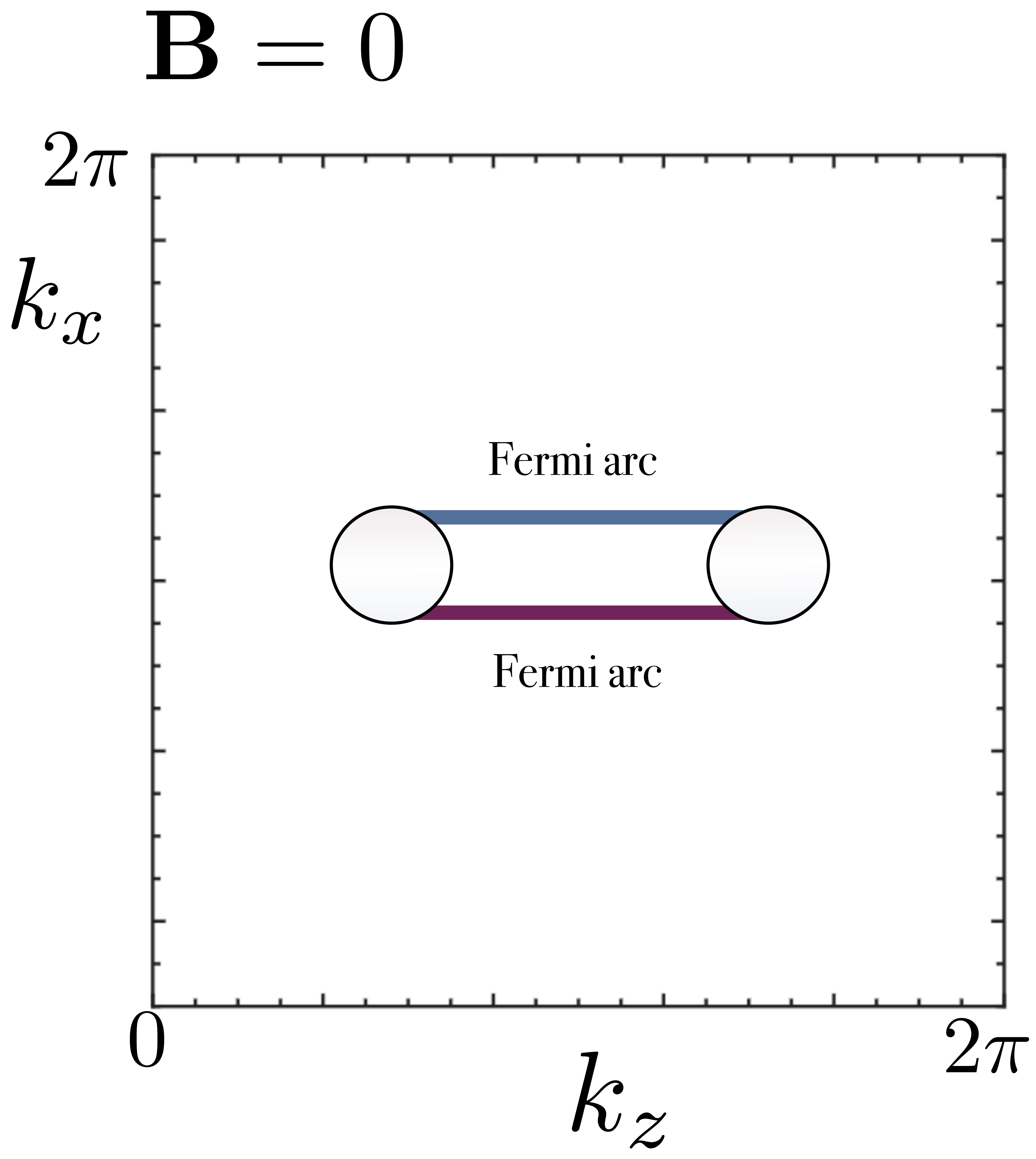


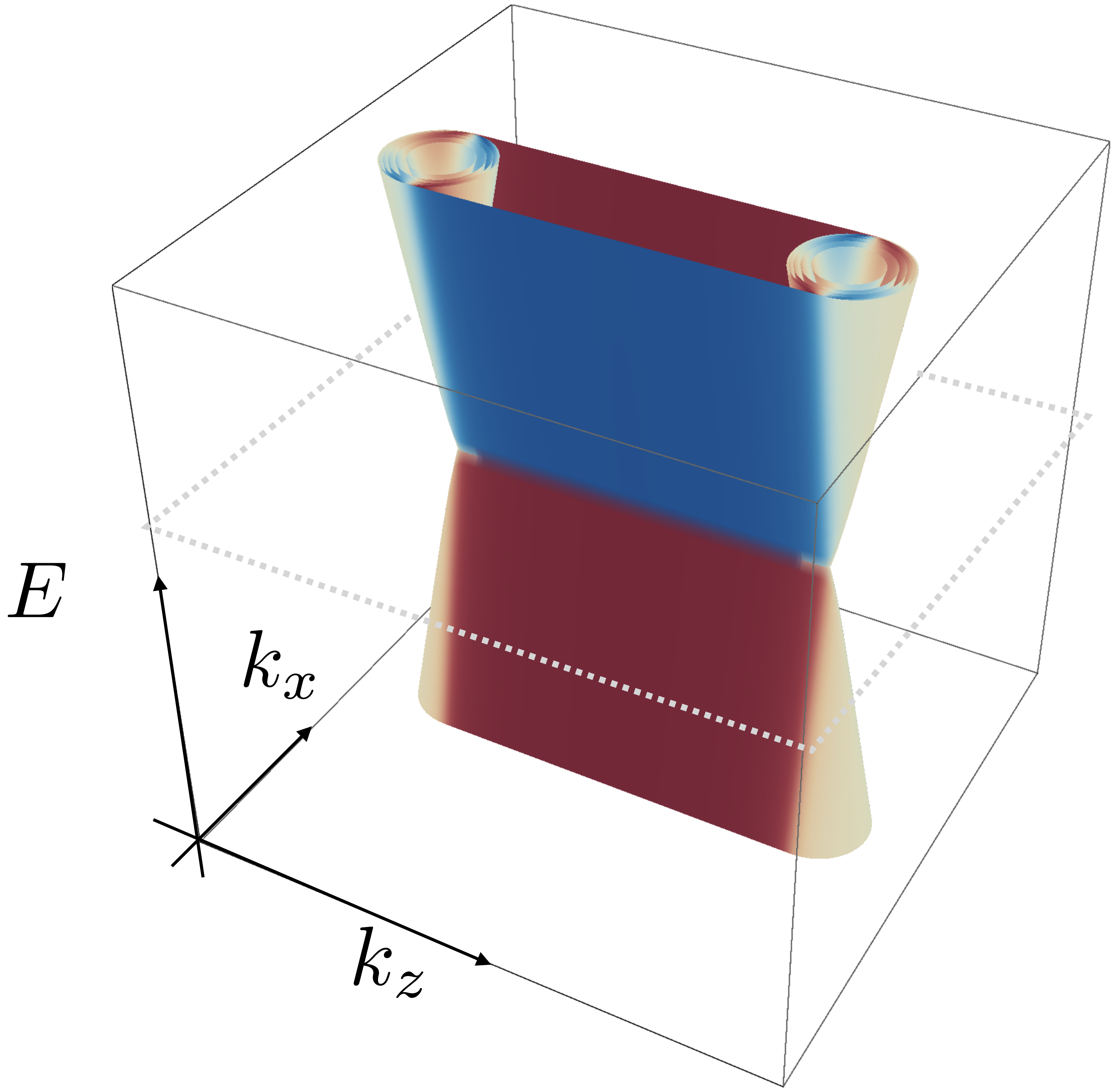

b
Weyl
2b
Insulator
y


b
$\mathbf{B}_5 = \nabla \times \mathbf{b}$
Weyl
2b
Insulator
y

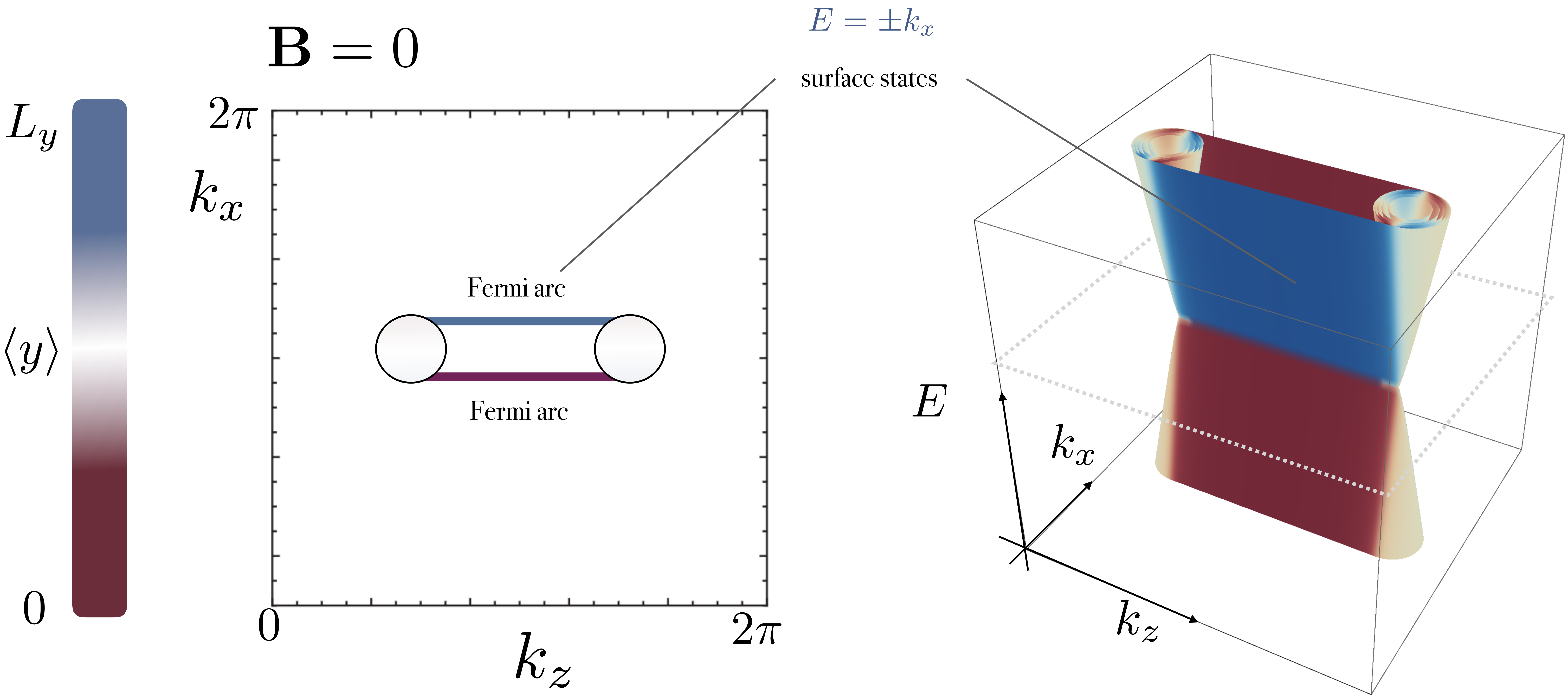
Zero magnetic field, open boundary conditions



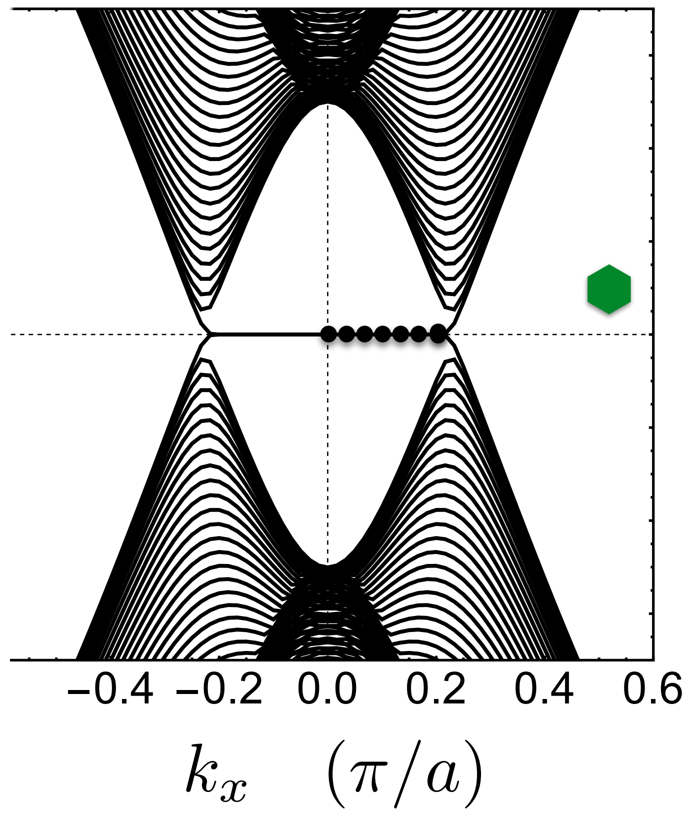


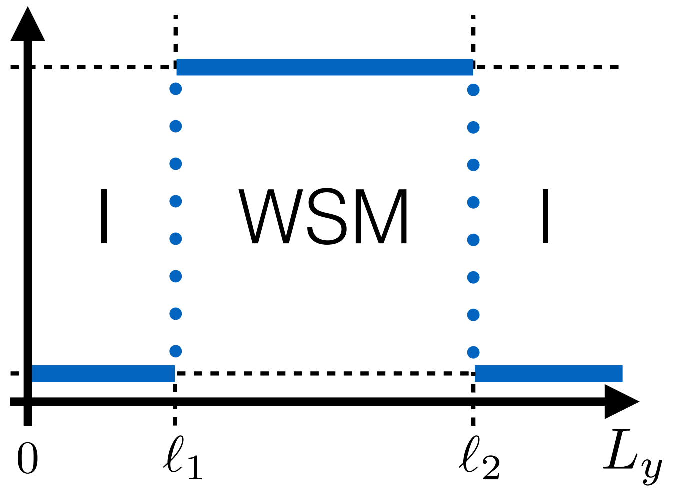


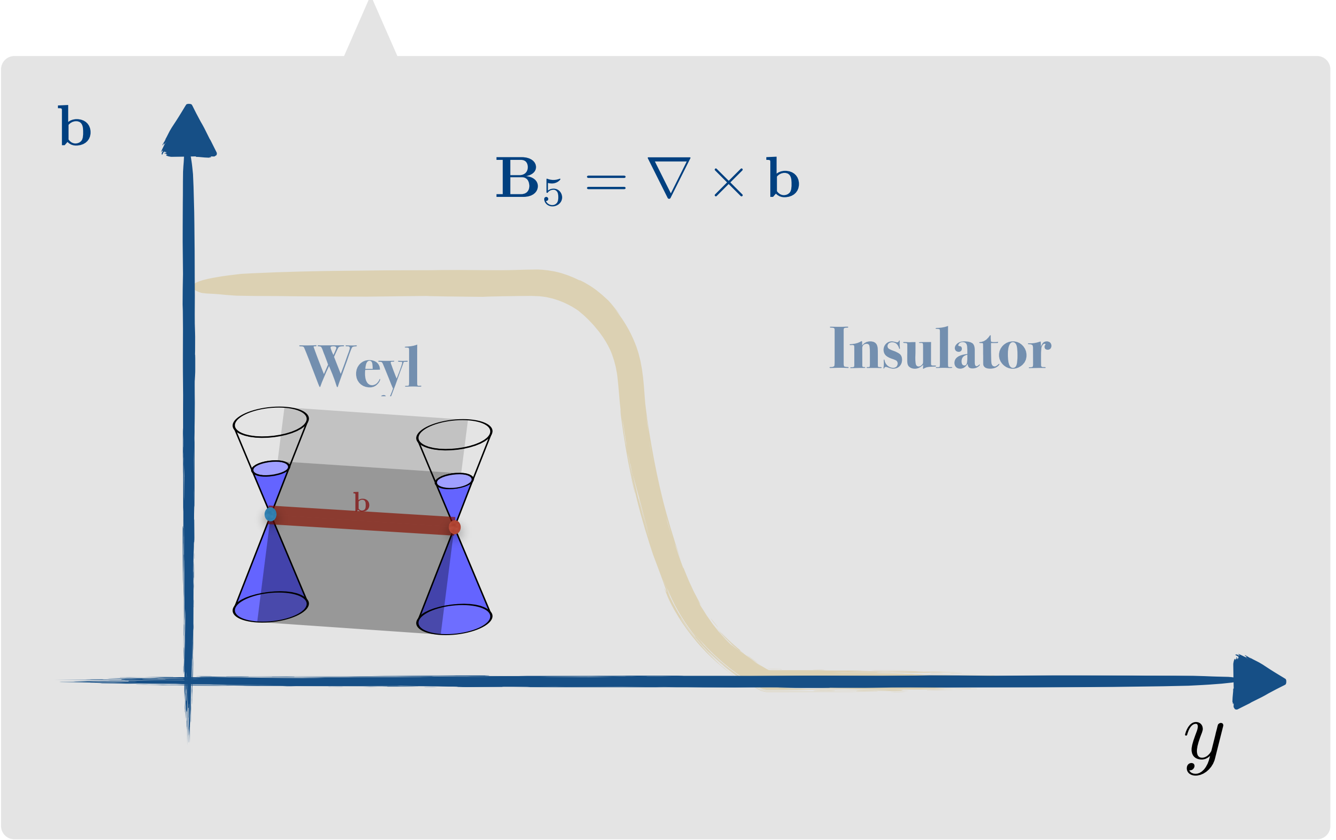
Zero magnetic field, open boundary conditions



Fermi arcs are 0th pseudo Landau Levels

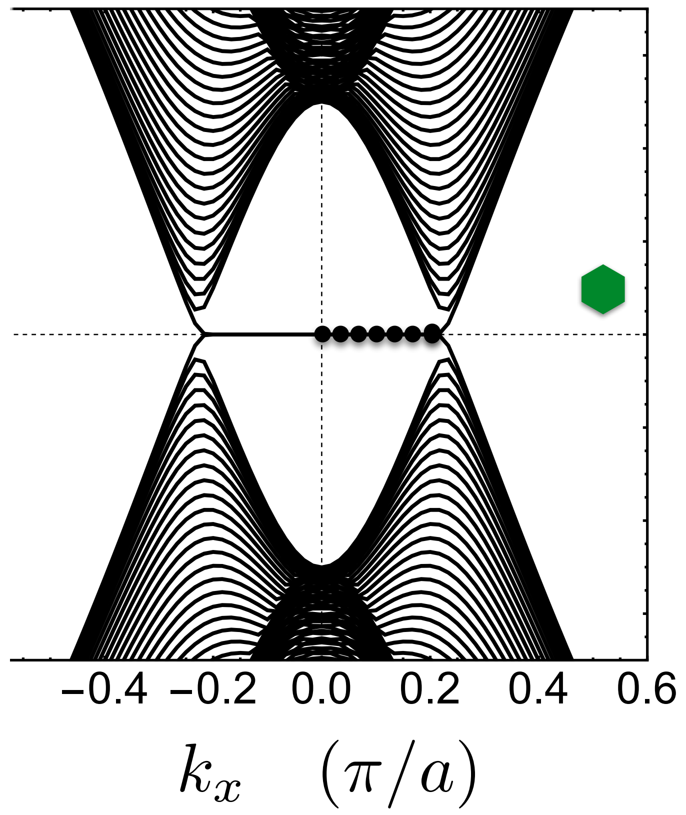


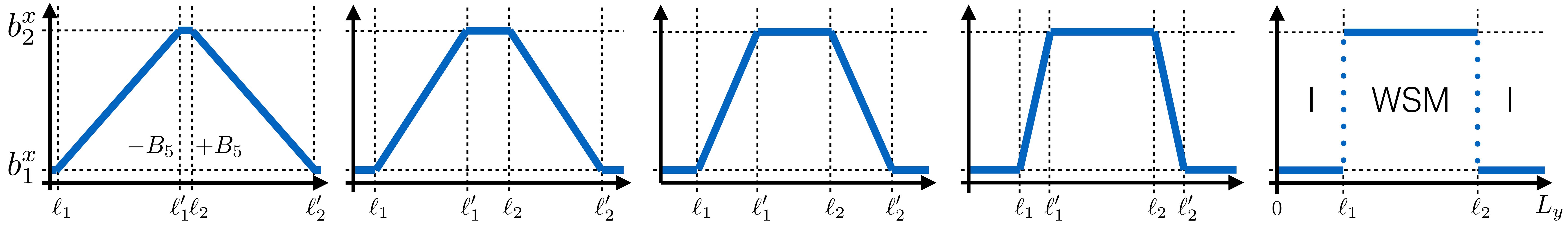


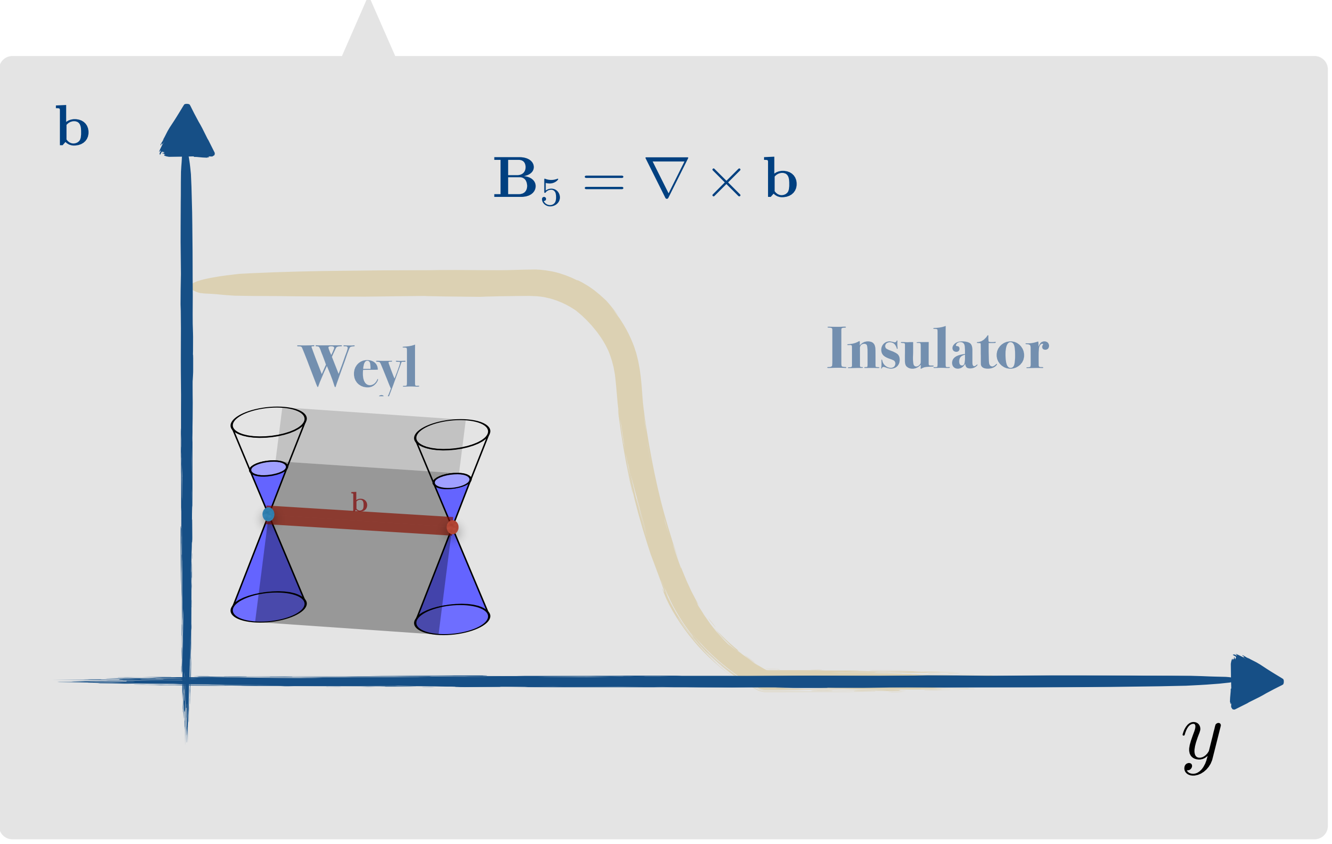


AGG, J. Venderbos, A. Vishwanath, R. Ilan PRX (2016)

Fermi arcs are 0th pseudo Landau Levels

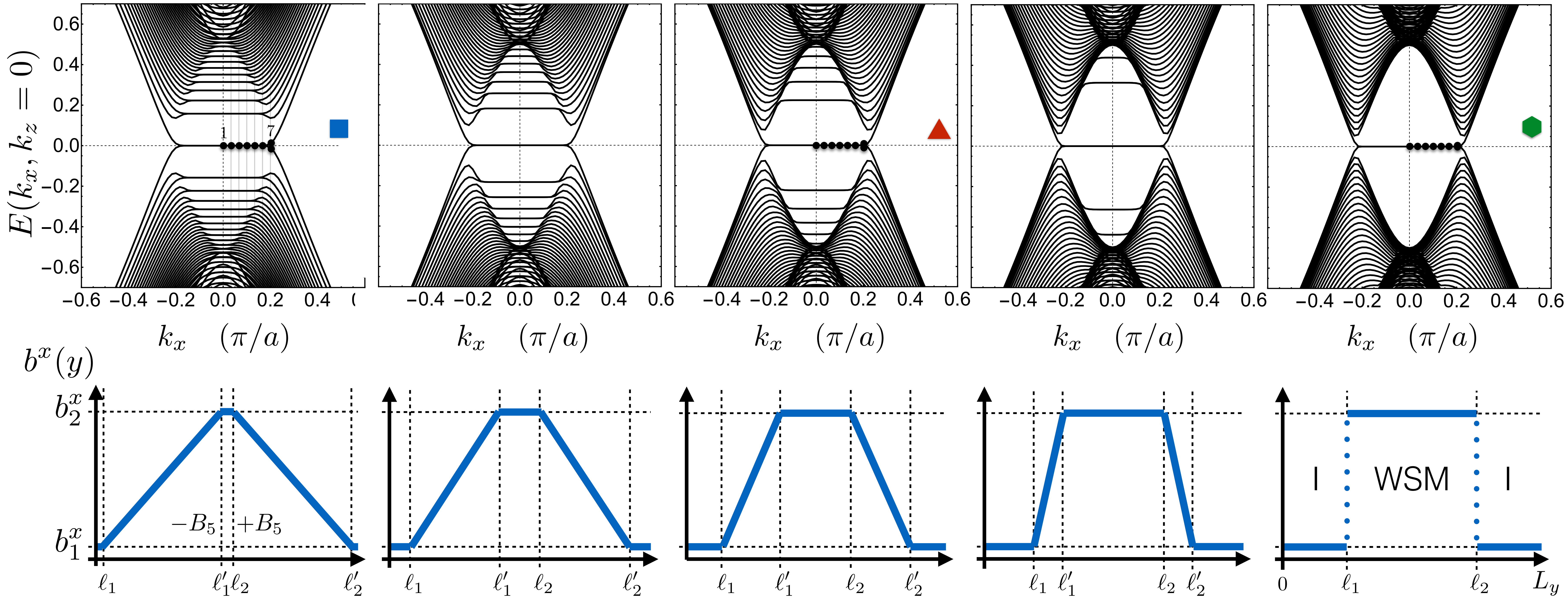


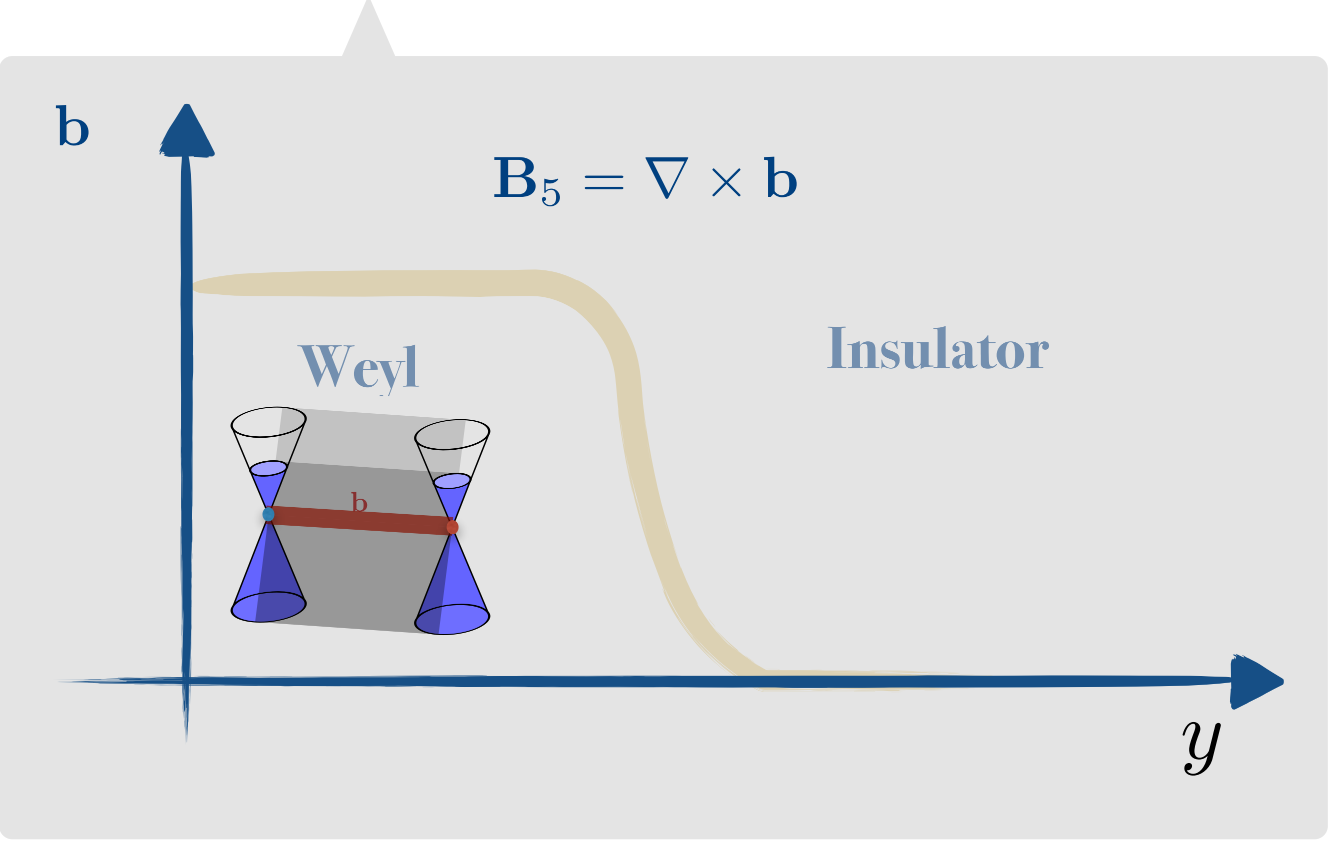




AGG, J. Venderbos, A. Vishwanath, R. Ilan PRX (2016)

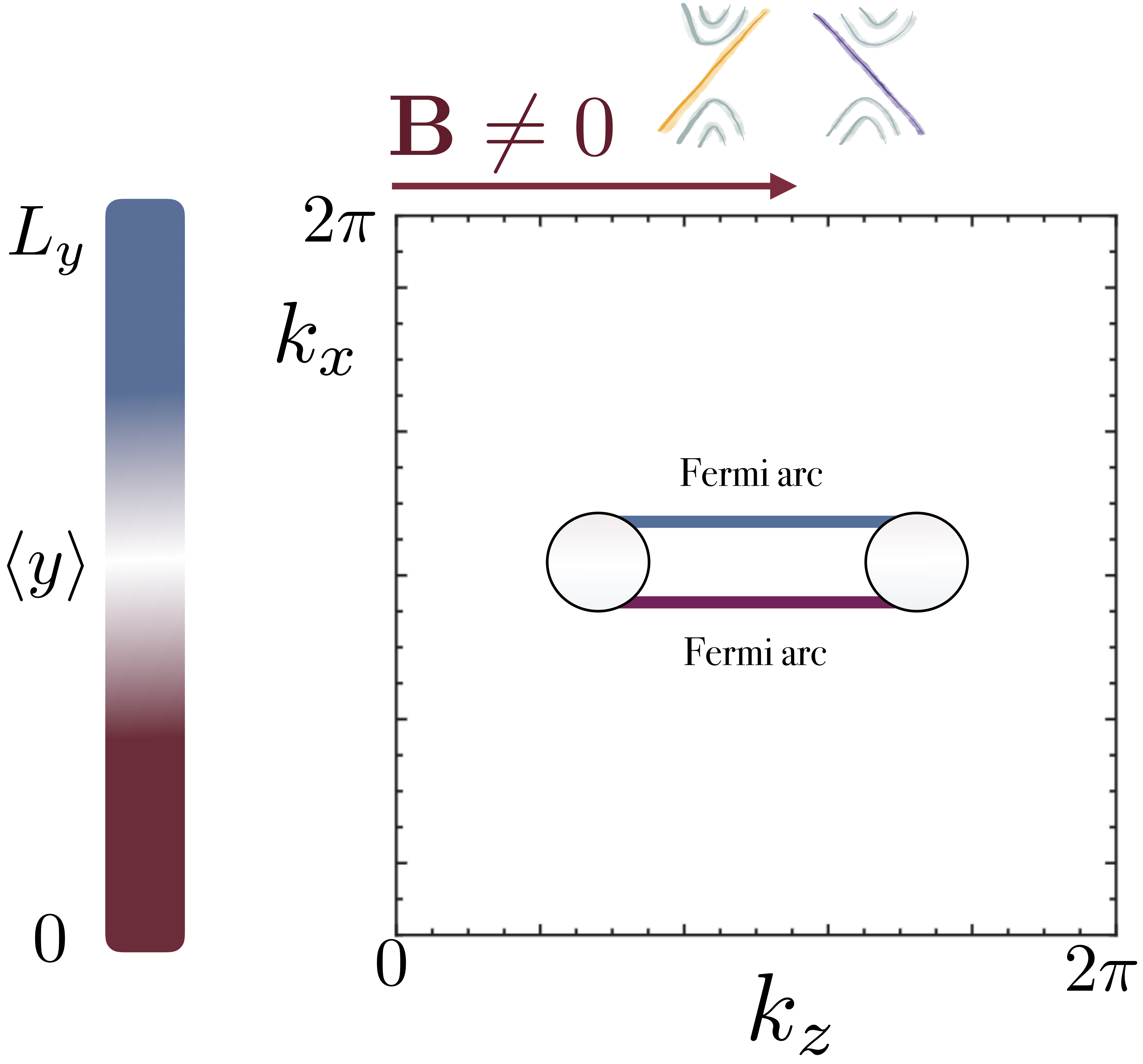
Fermi arcs are 0th pseudo Landau Levels

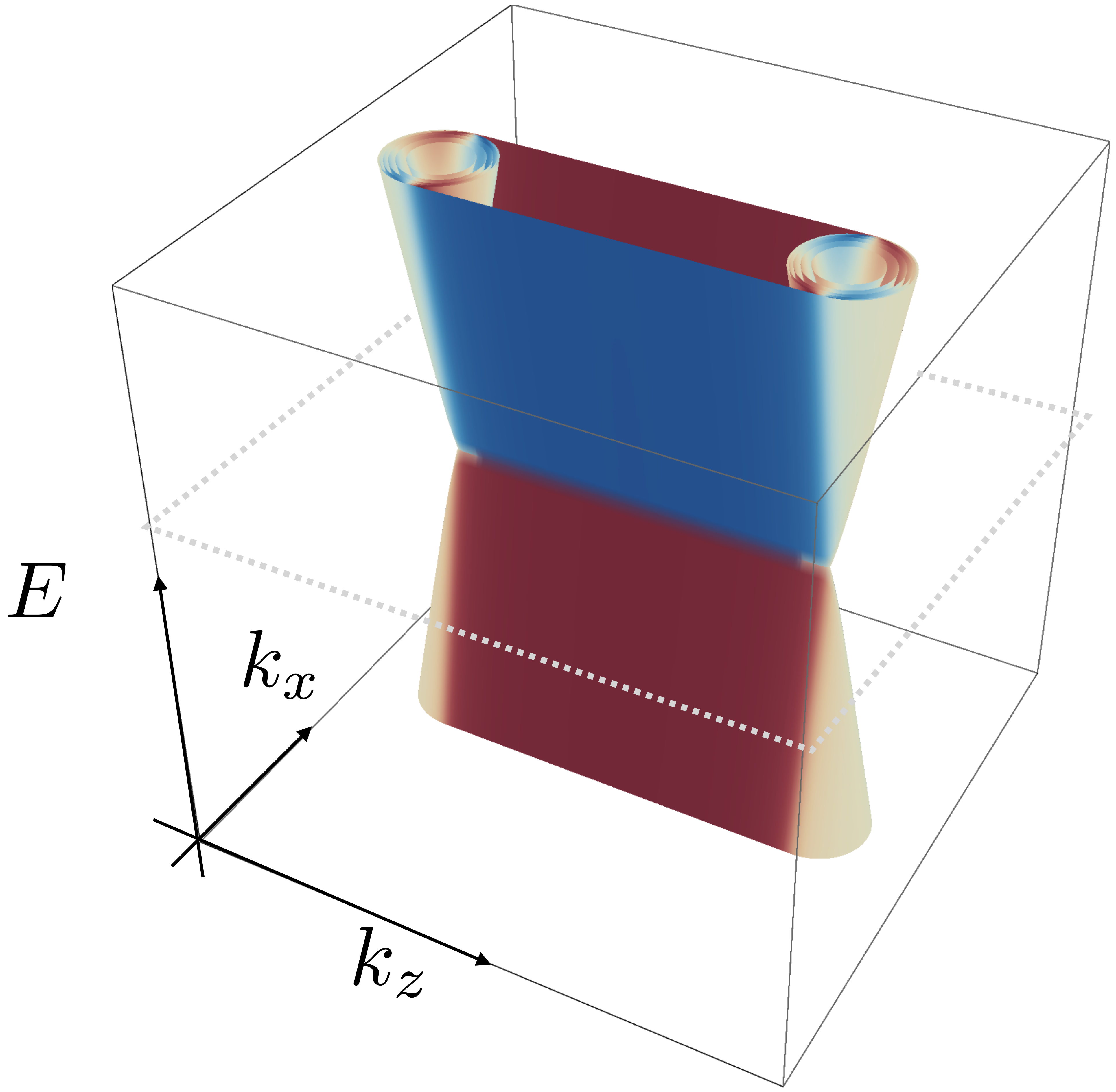




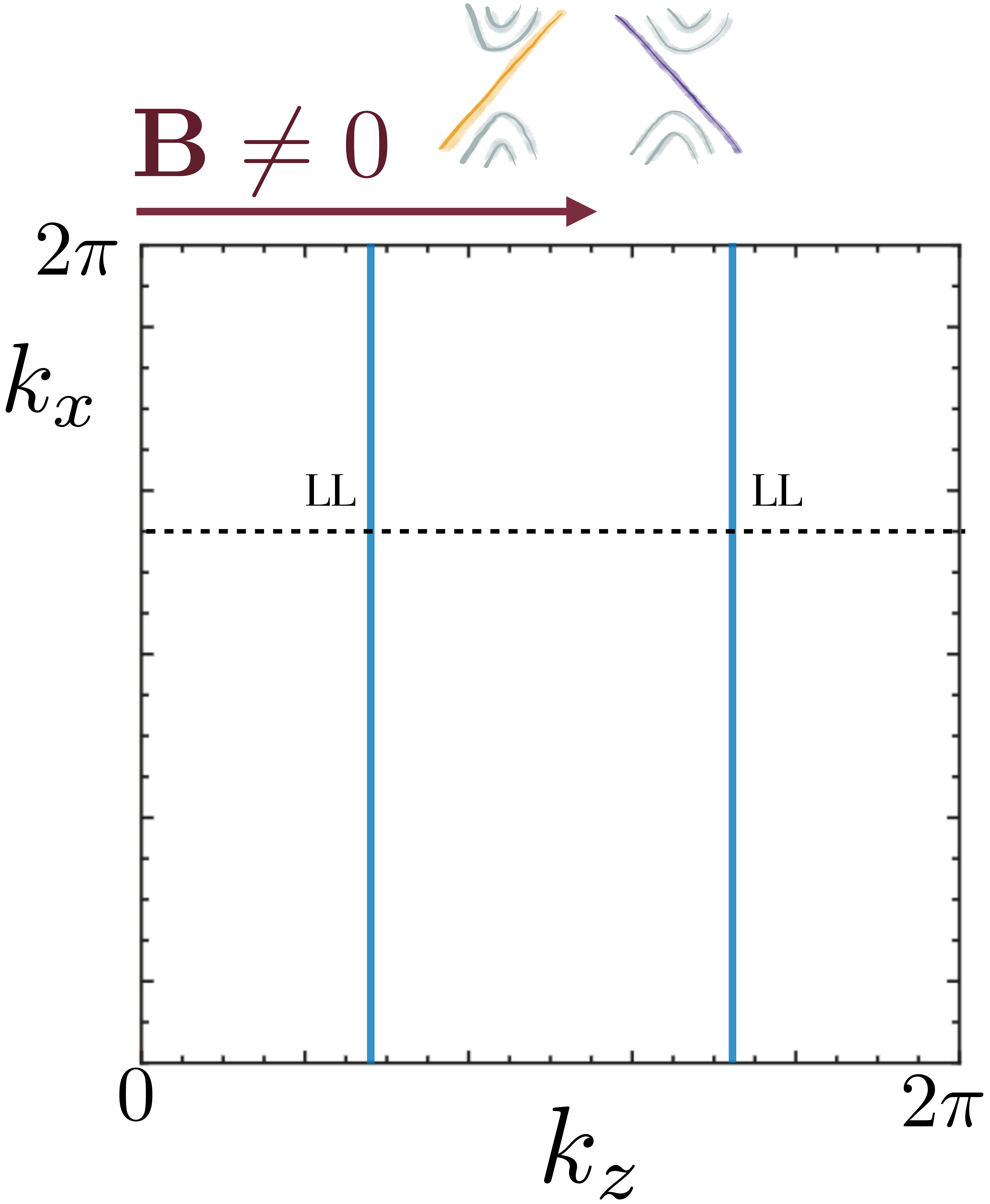
AGG, J. Venderbos, A. Vishwanath, R. Ilan PRX (2016)

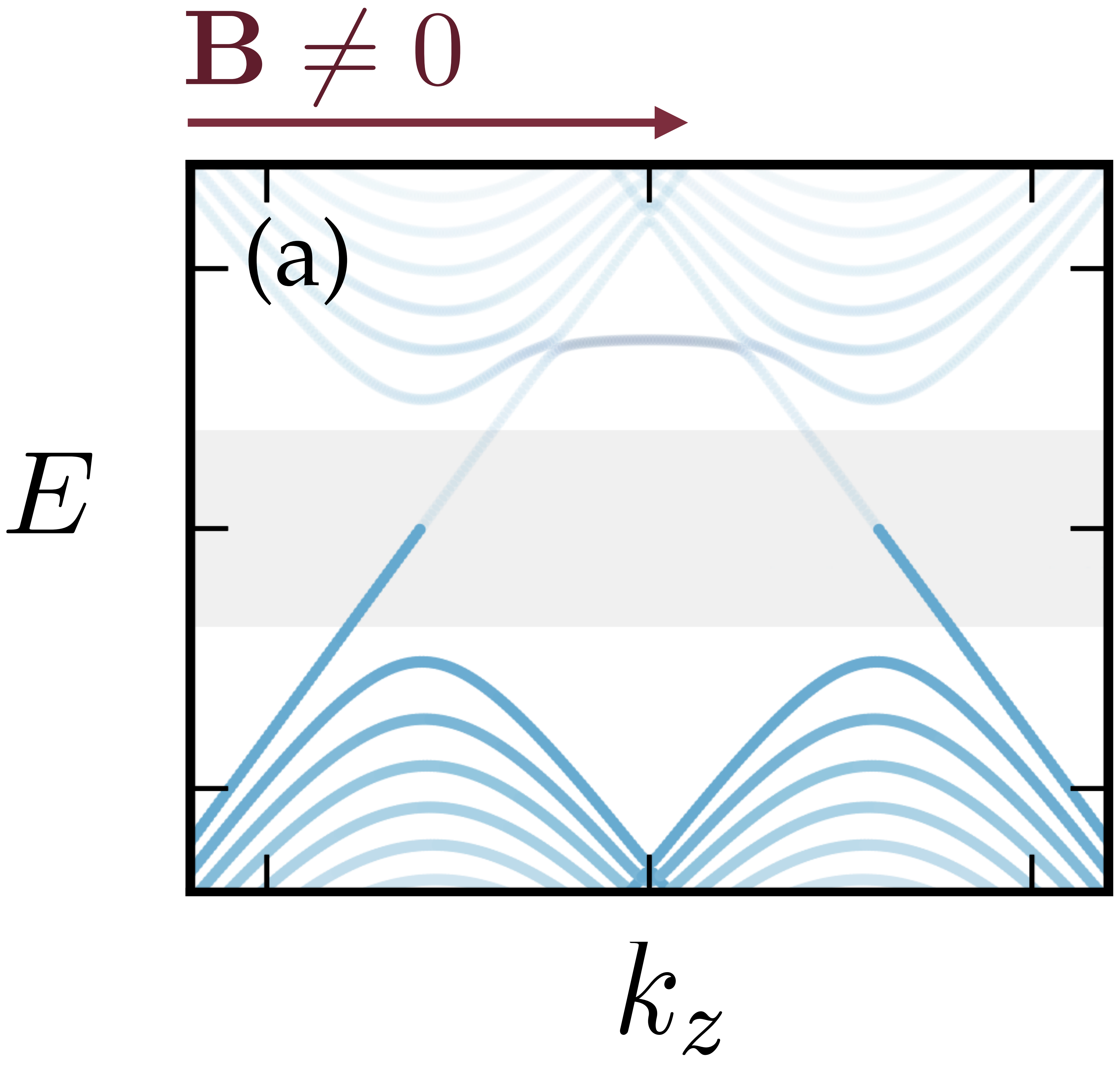
Finite magnetic field



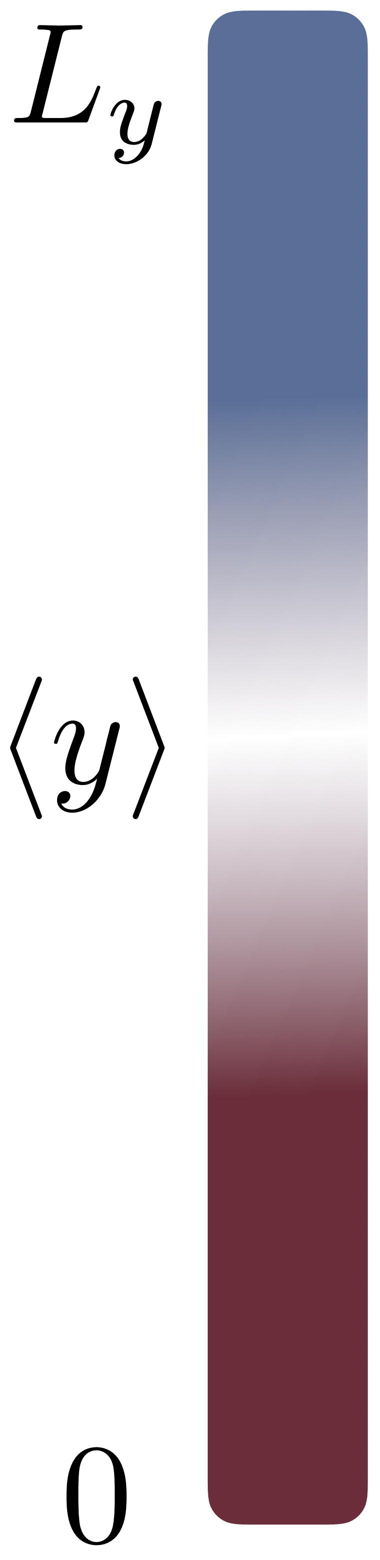


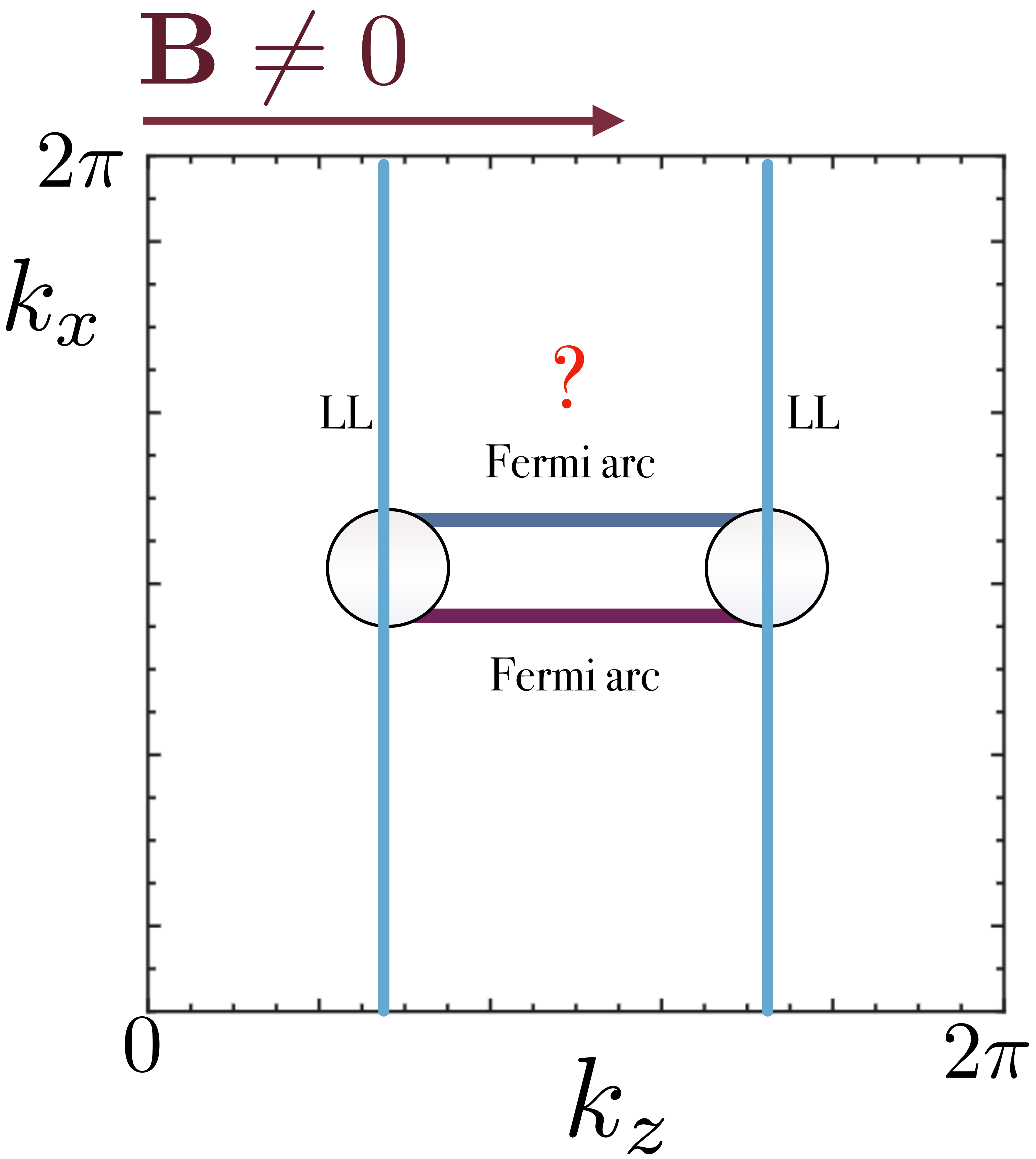
Finite magnetic field, periodic boundary conditions

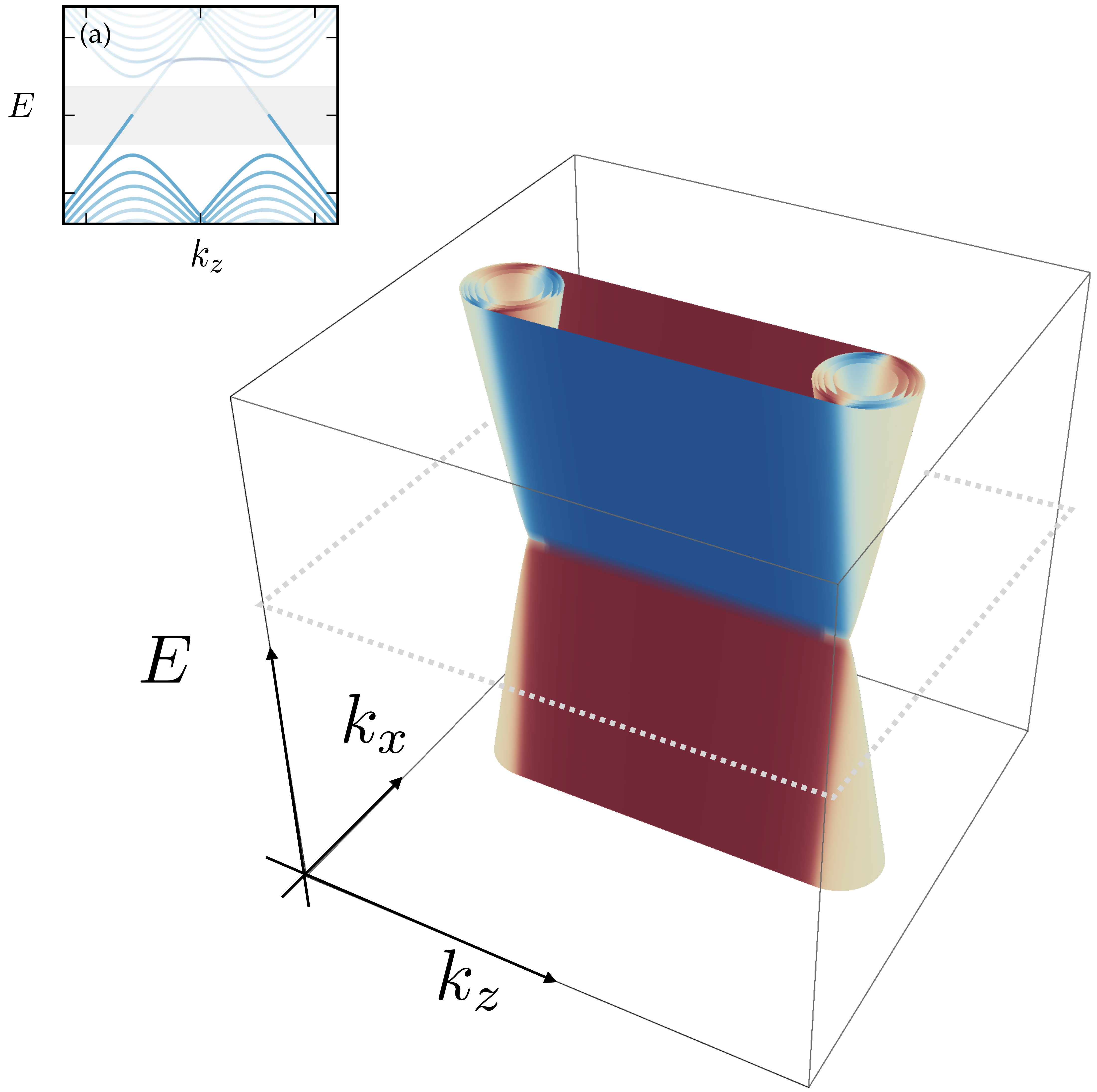




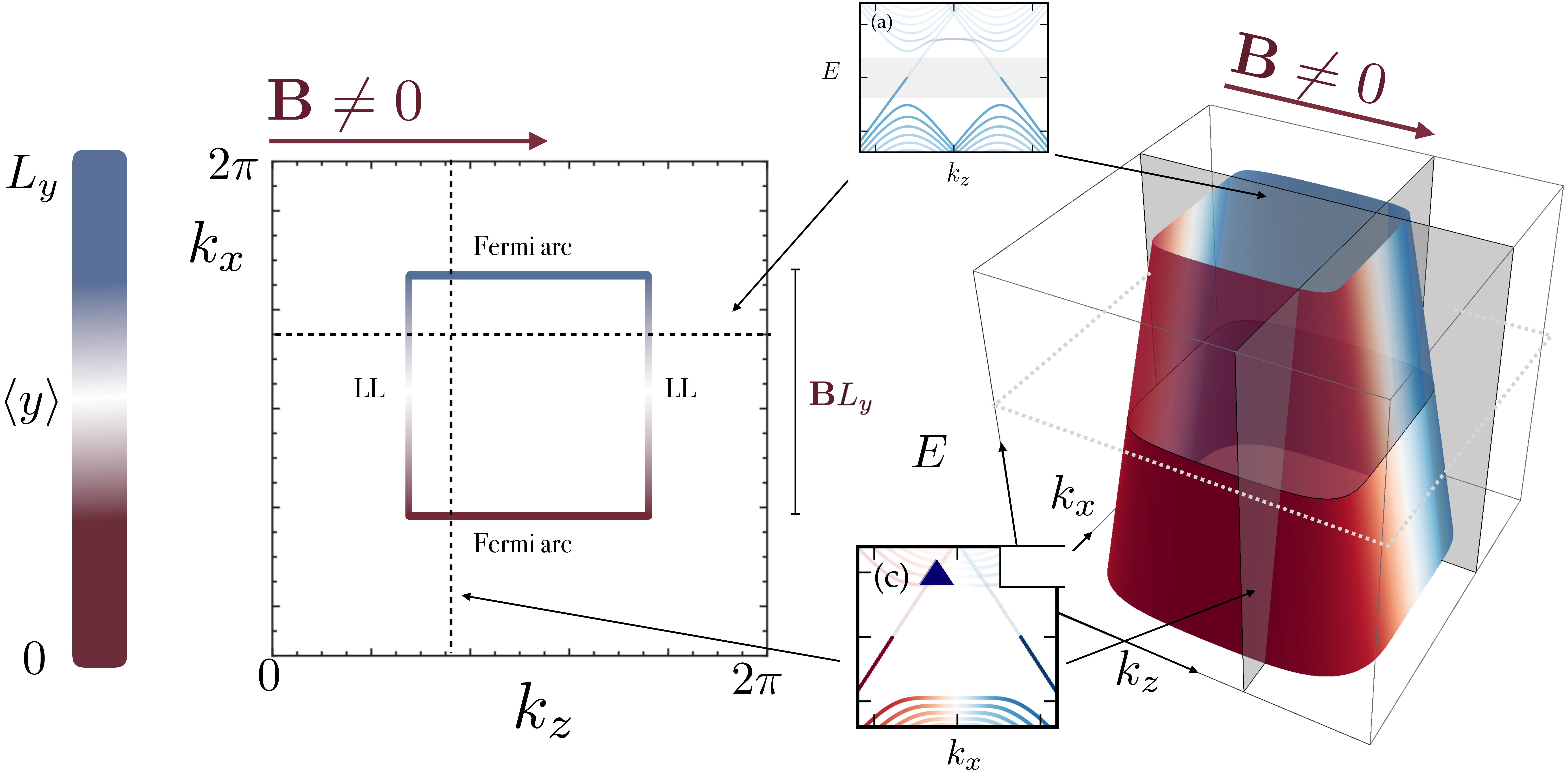
Finite magnetic field, open boundary conditions





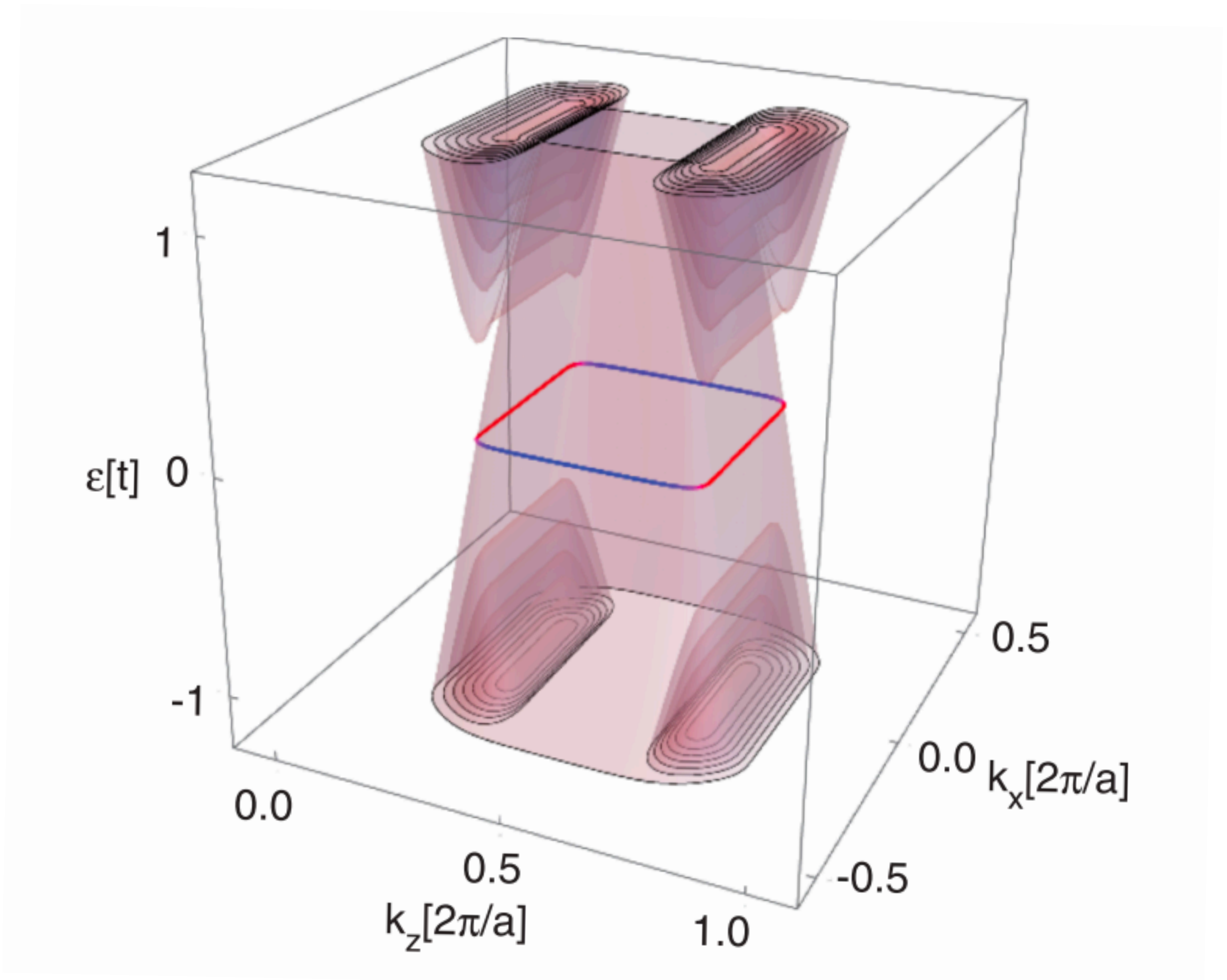


Finite magnetic field, periodic boundary conditions



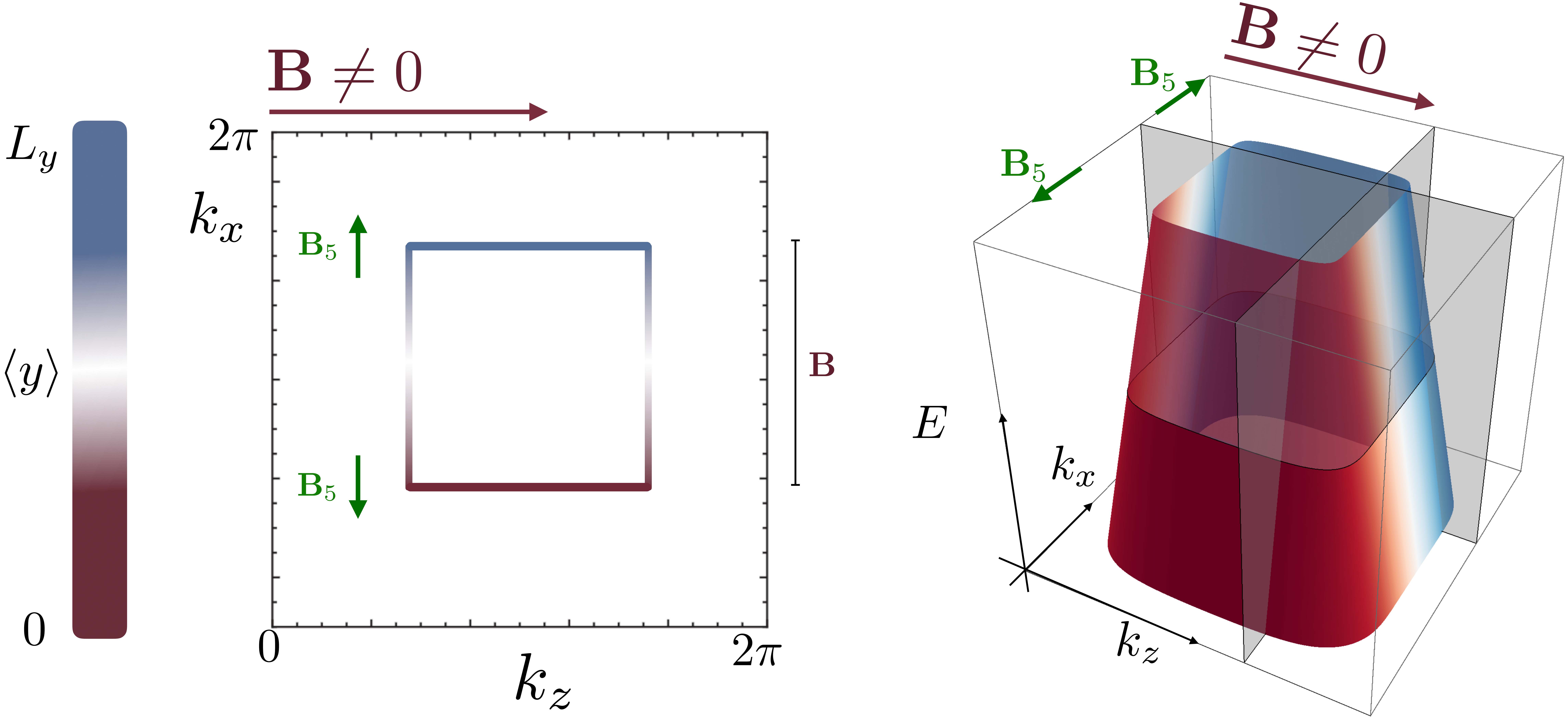
Position momentum locking $k_x \to k_x + B_z y$ $\langle y \rangle \propto k_x$

Ominato, Koshino '14, Bulmash, Qi '14



Ominato, Koshino ’14, Bulmash, Qi ‘14

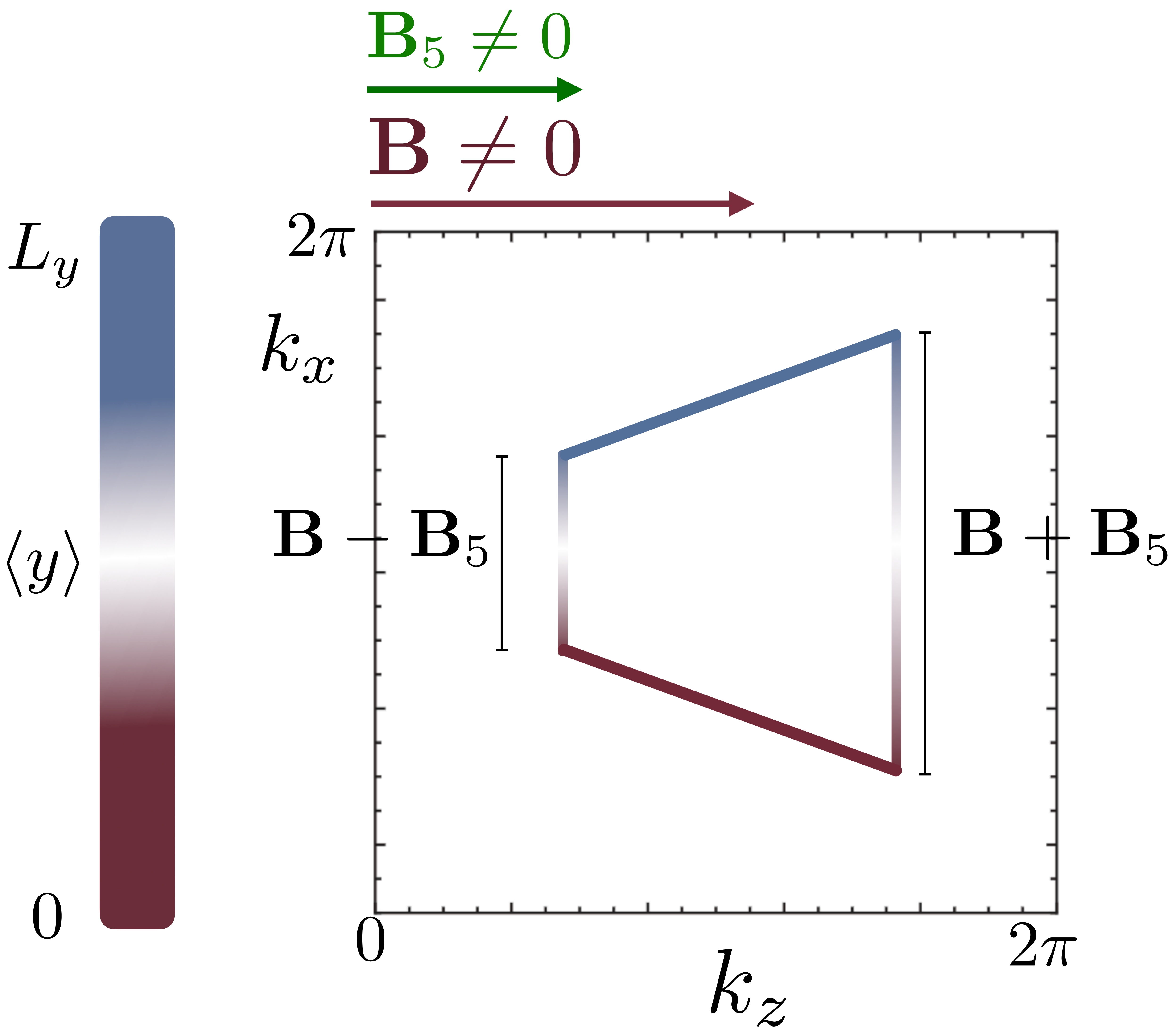
Bulk magnetic and pseudo magnetic fields

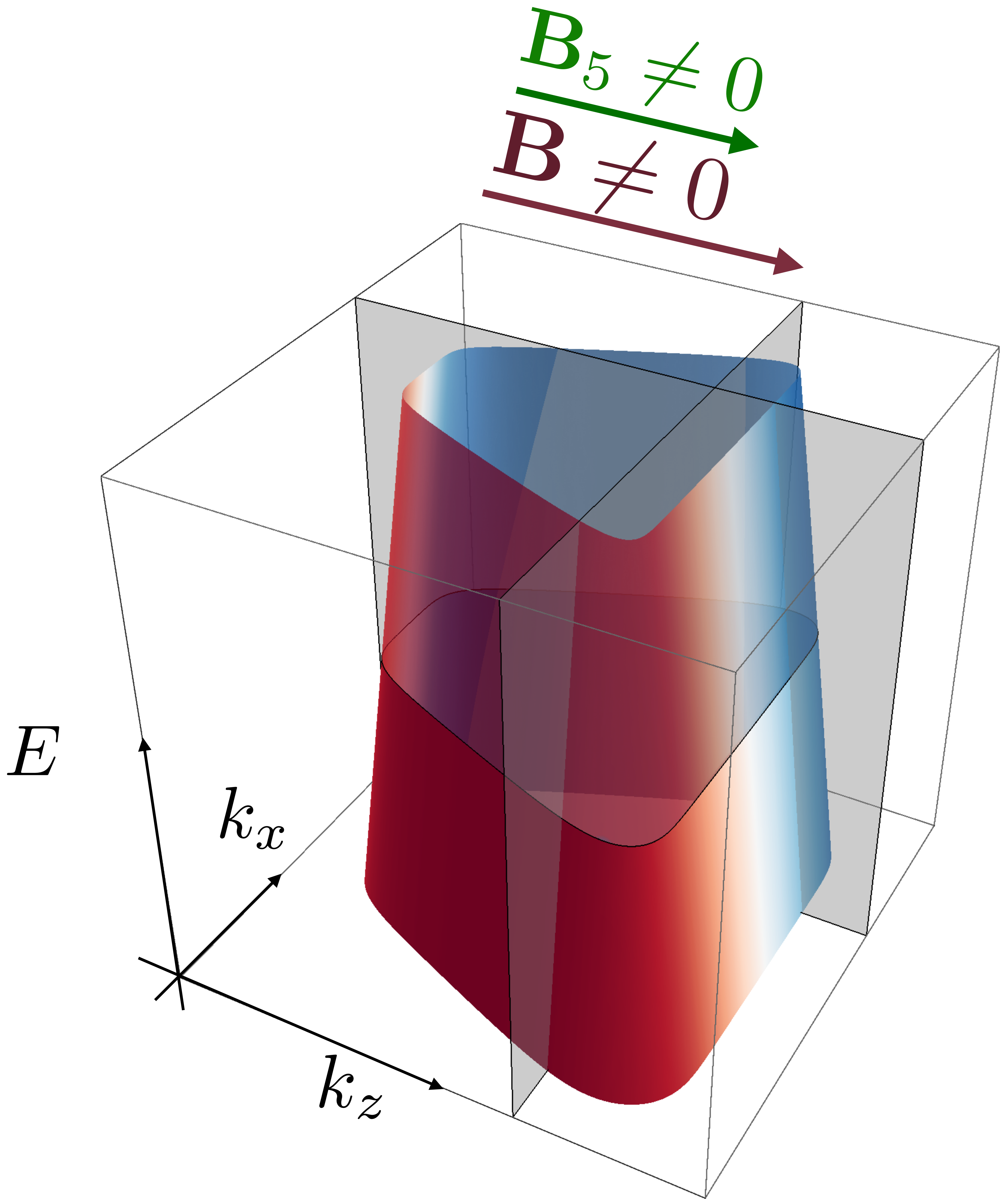


Position momentum locking $k_x \to k_x + B_z y$

$\langle y \rangle \propto k_x$

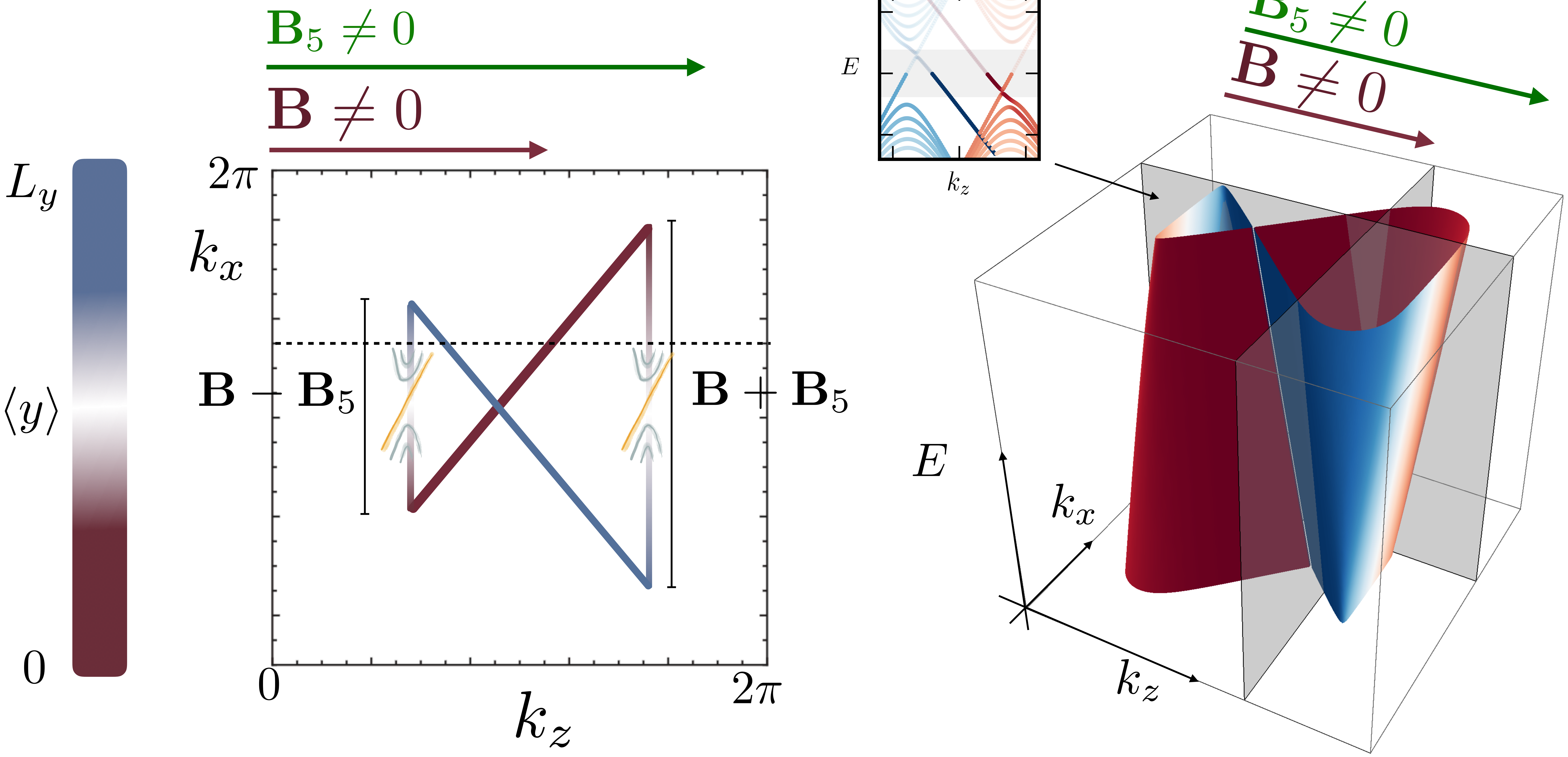
Bulk magnetic and pseudo magnetic fields





Position momentum locking $k_x \to k_x + (B_z \pm B_z^5)y$ $\langle y \rangle \propto k_x$

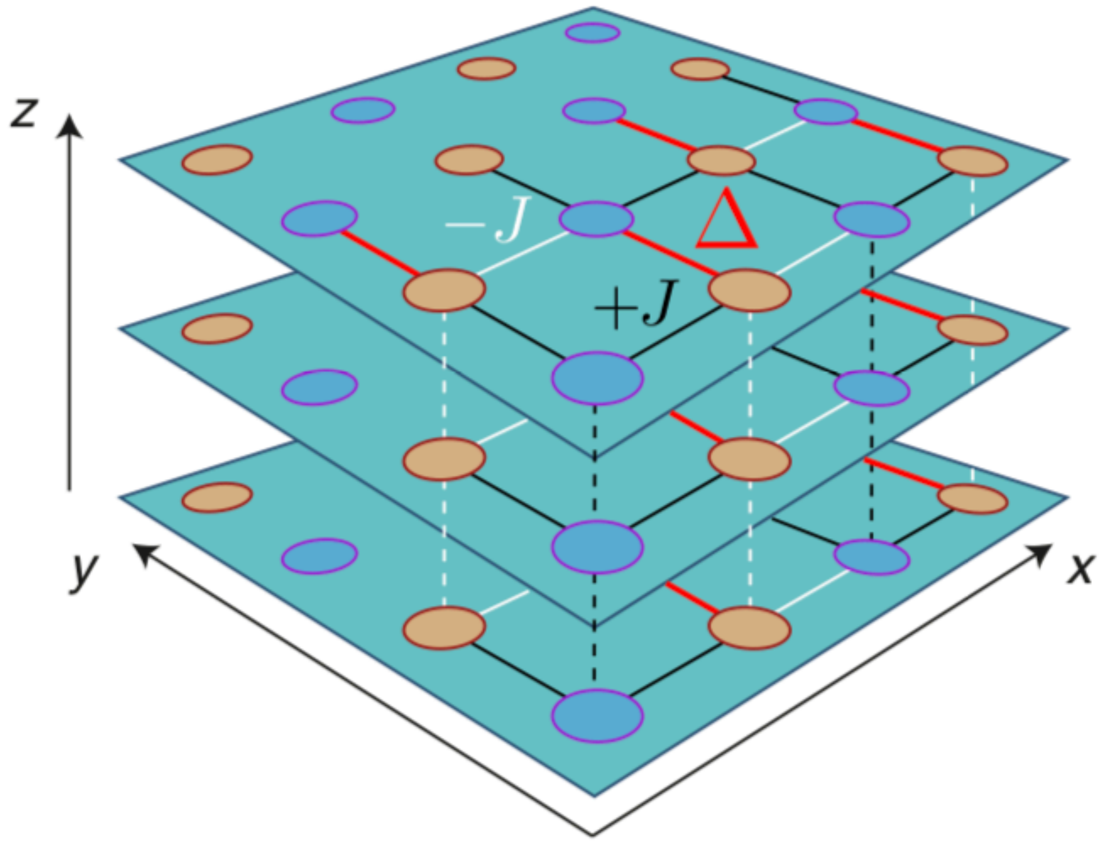
Bulk magnetic and pseudo magnetic fields



Position momentum locking $k_x \to k_x + (B_z \pm B_z^5)y$ $\langle y \rangle \propto k_x$

Resembles: N. Bovenzi, et. al. NJP (2018)!

Modulated optical lattices



S. Roy, M. Kolodrubetz, N. Goldman, AGG 2D Mat. (2018)

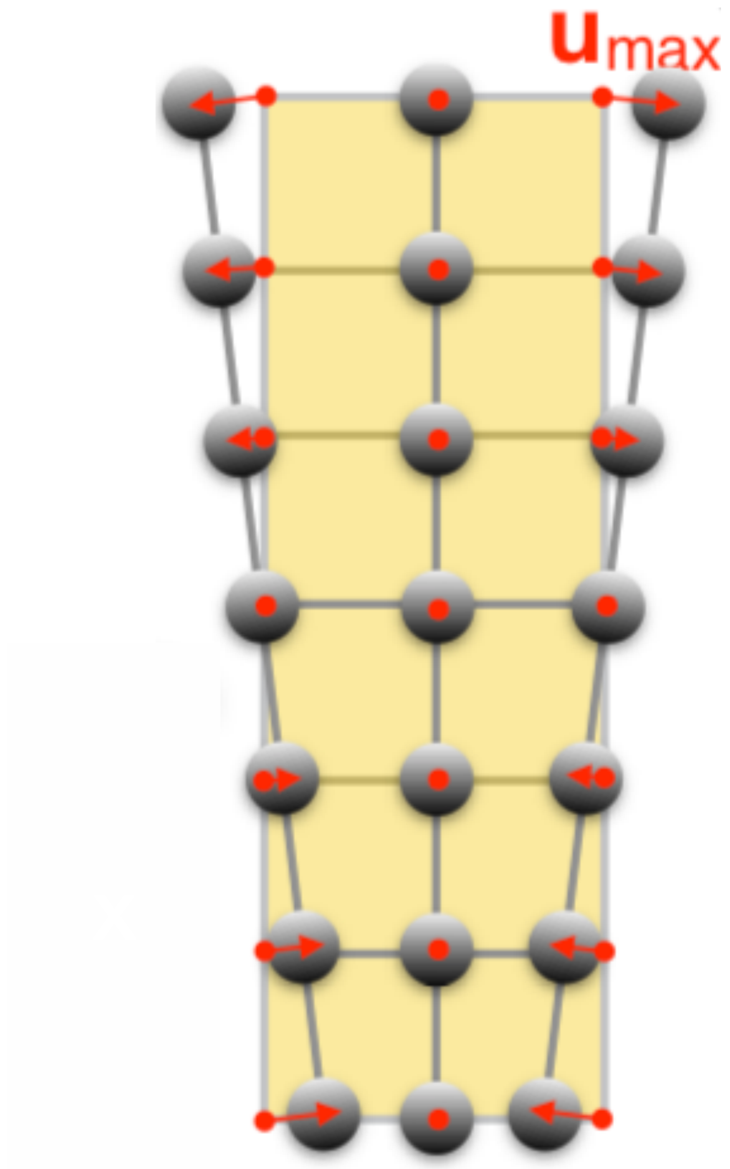
space dependent node separation

$$\mathbf{B}_5 = \nabla \times \mathbf{b}$$

time dependent node separation

$$\mathbf{E}_5 = \partial_t \mathbf{b}$$

Strain



C. X. Liu, P. Ye, X. L.Qi PRB (2013)

A. Cortijo, Y. Ferreiros, K. Landsteiner,
B. M. A. H. Vozmediano PRL (2016)

M. Chernodub, A. Cortijo, AGG,
K. Landsteiner, M. A. H. Vozmediano PRB (2014)

Ramamurthy, Hughes PRB (2015)

D. Pikulin, A. Chen, M. Franz PRX (2016)

AGG, J. Venderbos, A. Vishwanath,
R. Ilan PRX (2016)
Gorbar et. al PRL (2017), PRB (2017) (2018)…

hep:

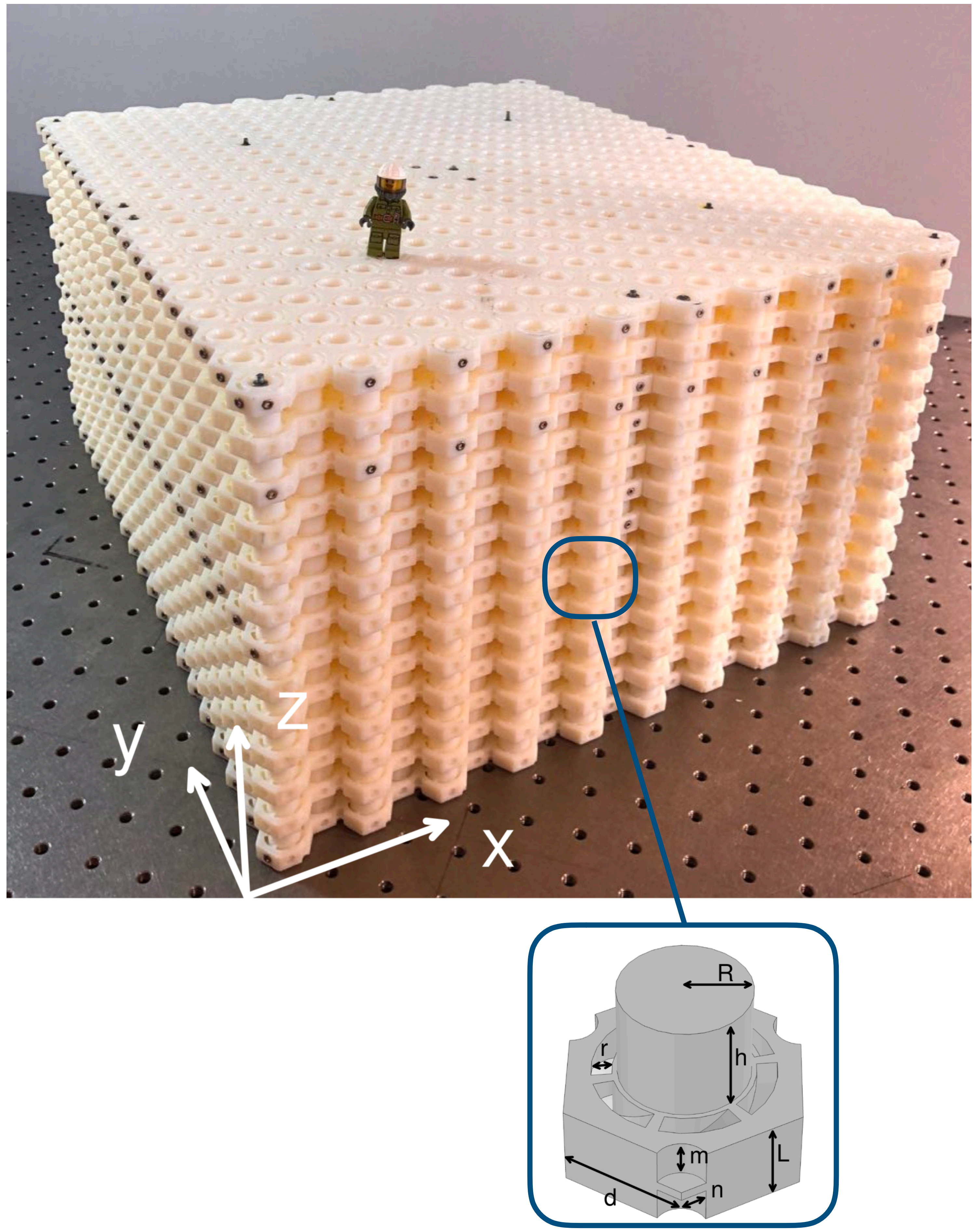
Bertlmann, Fujikawa, ABJ….

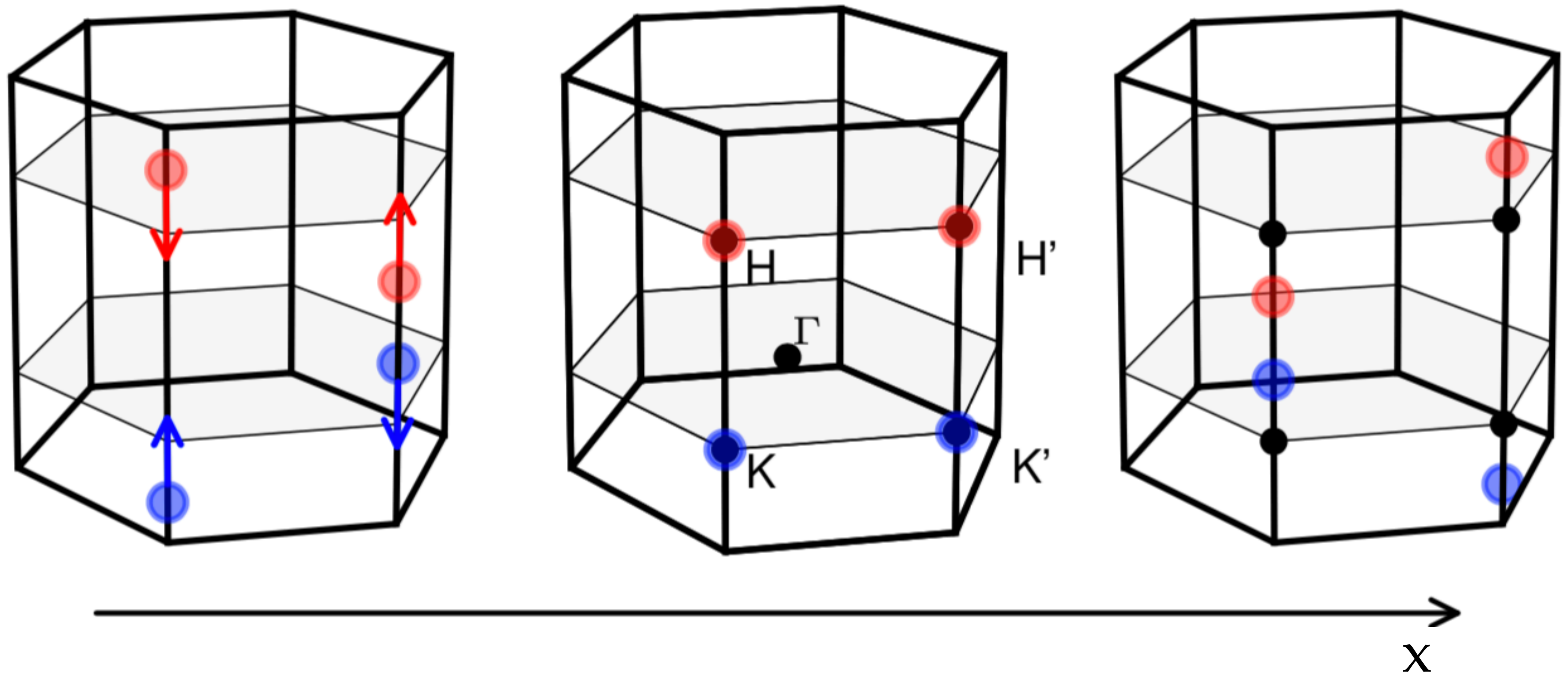
Bardeen, Zumino Nuc. Phys. B (1984)

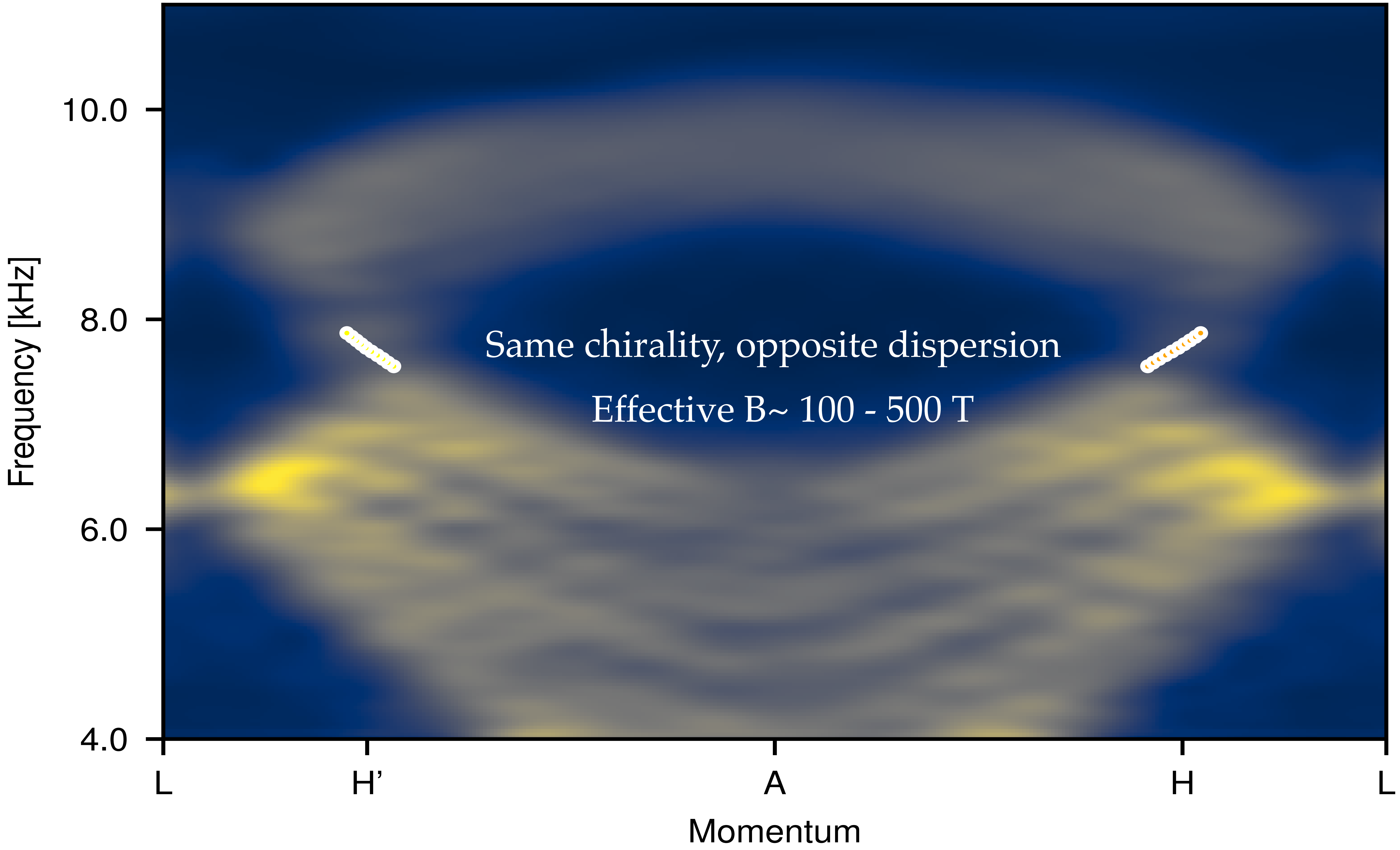
K. Landsteiner PRB (2014)

Kharzeev PRB (2014)

Strained acoustic crystals





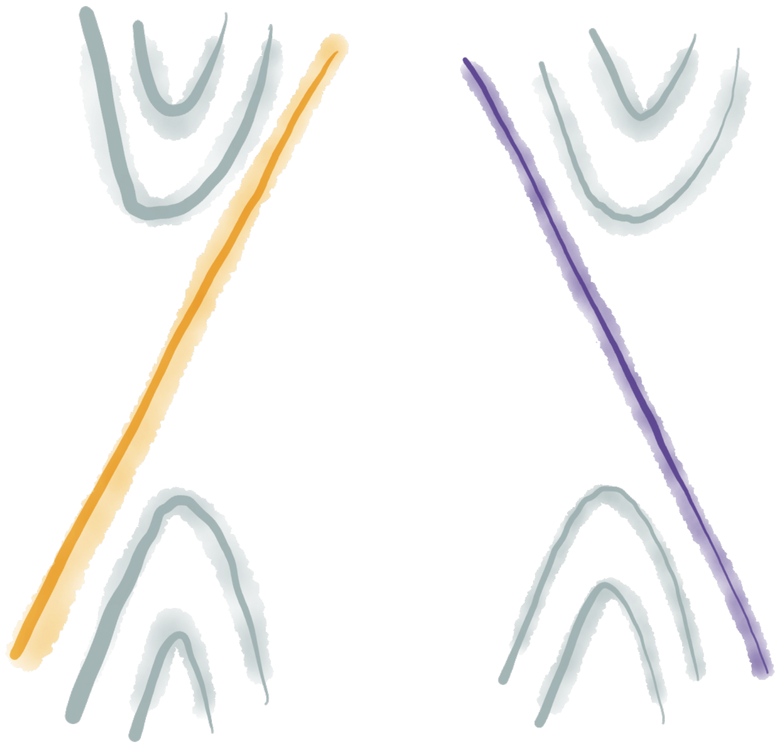


Same chirality, opposite dispersion

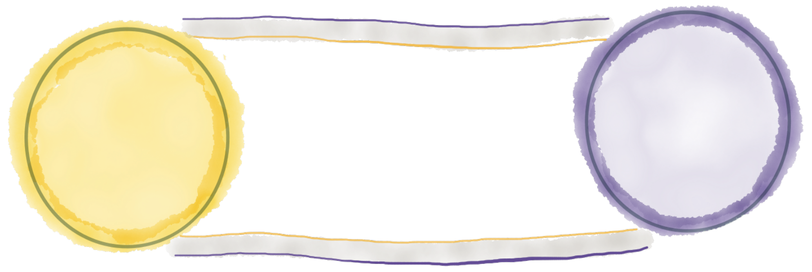
Effective B~ 100 - 500 T

Peri et. al. Nat. Phys. (in press)
Jia et. al. Science 363, 148 (2019)

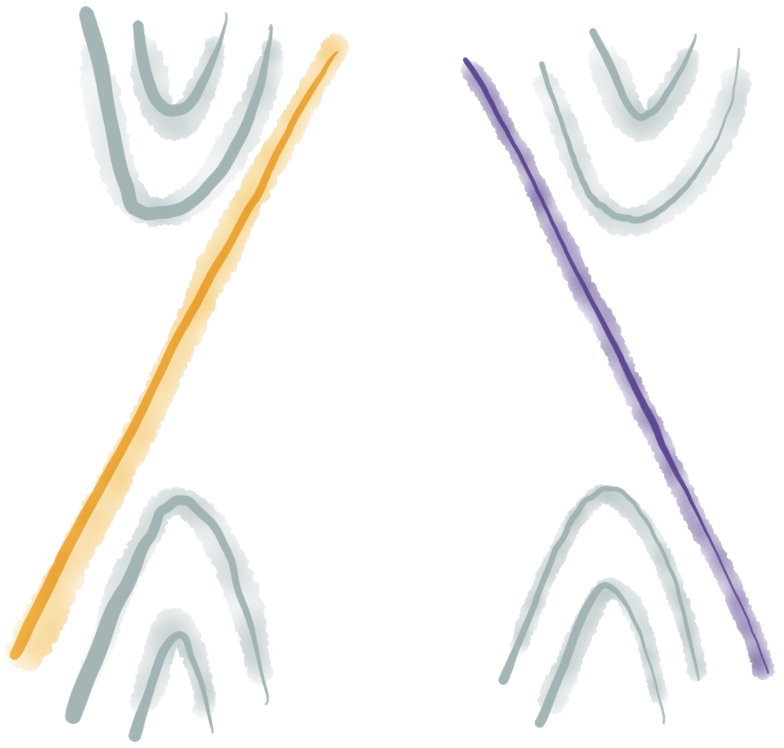
Light induced strain



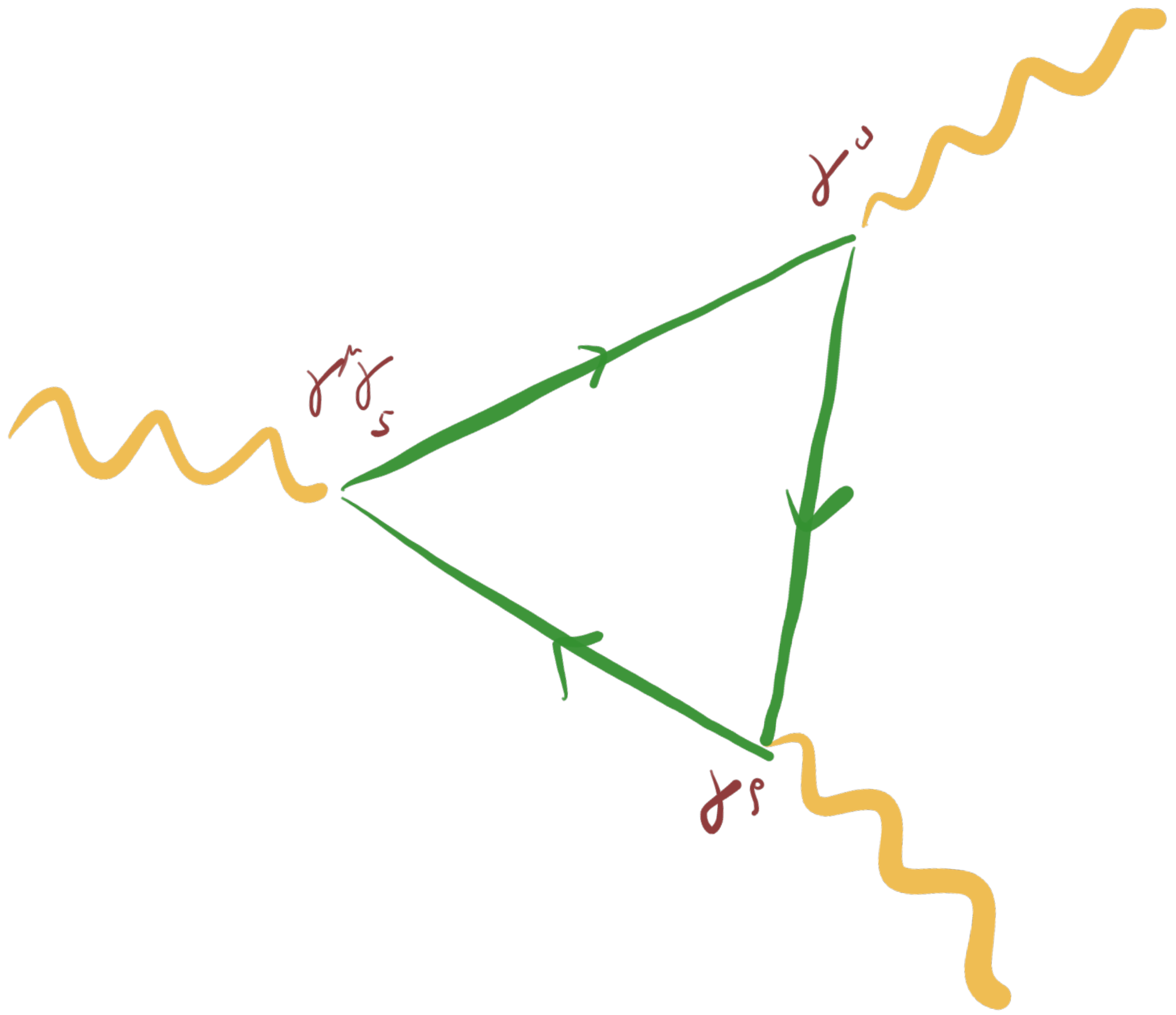
Landau levels



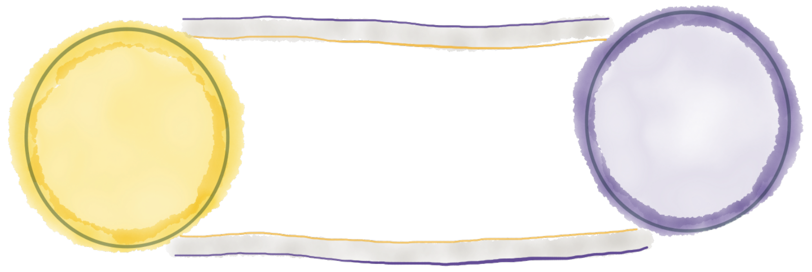
Fermi arcs



Landau levels

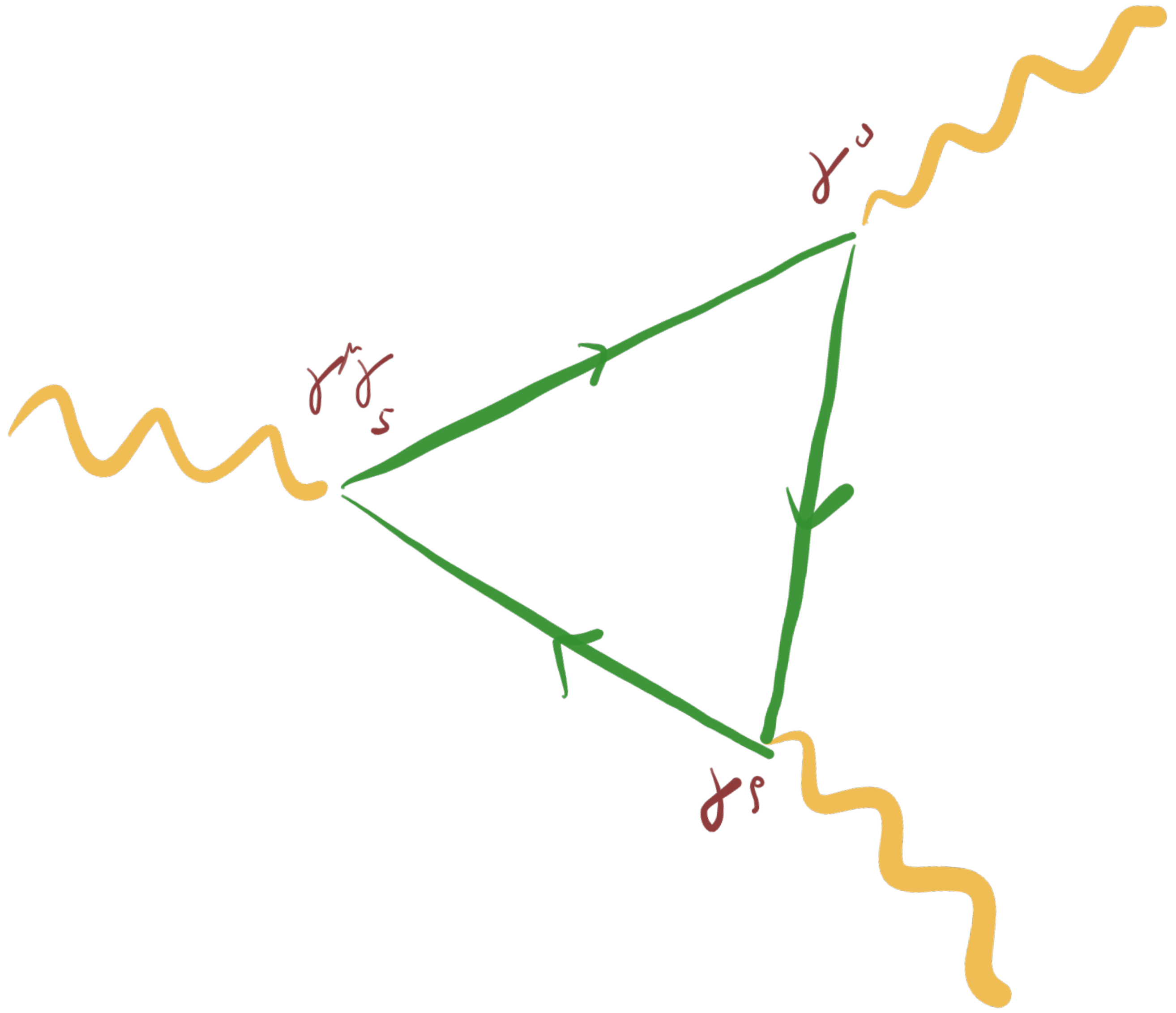


Bardeen polynomials



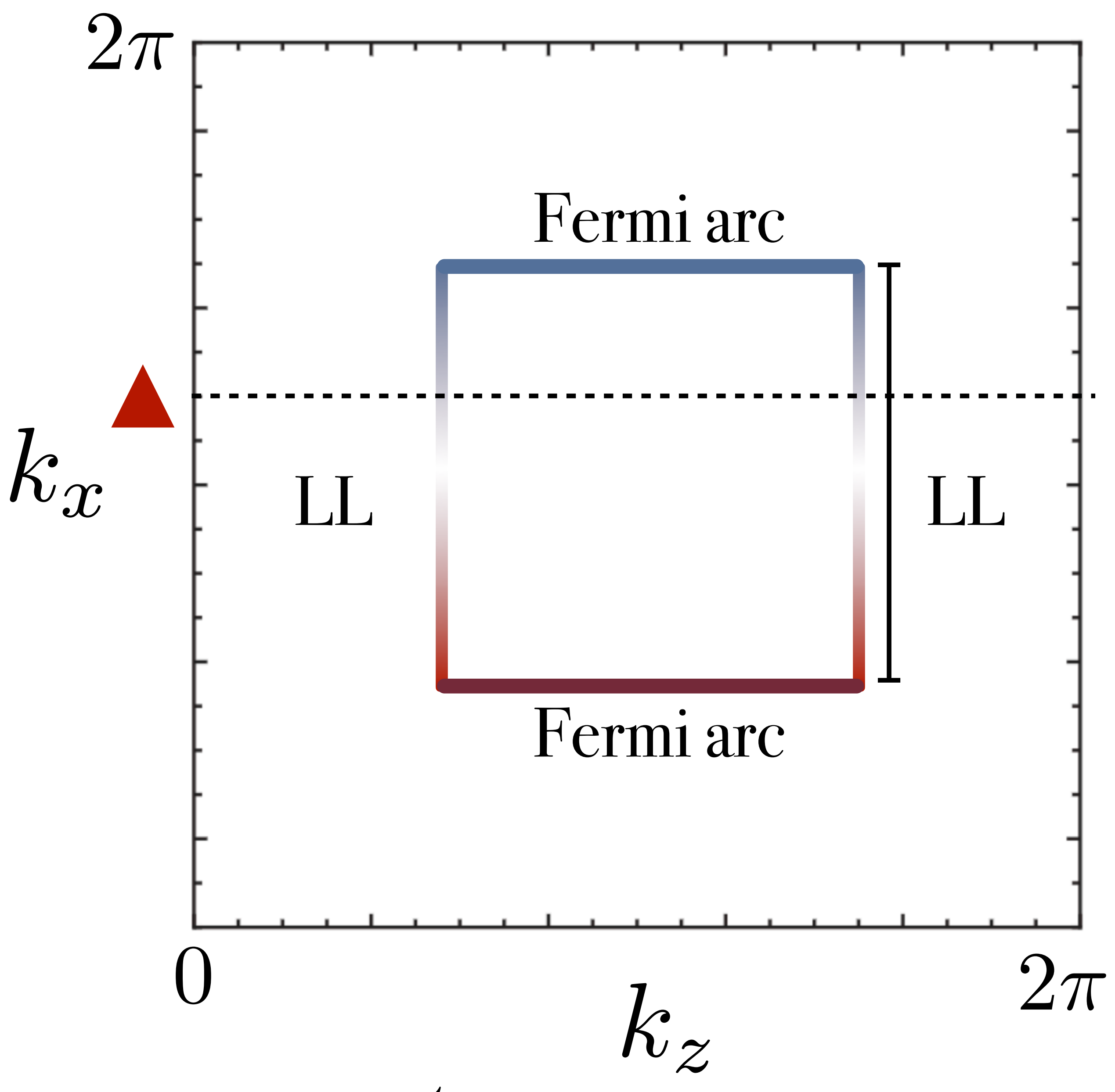
Fermi arcs

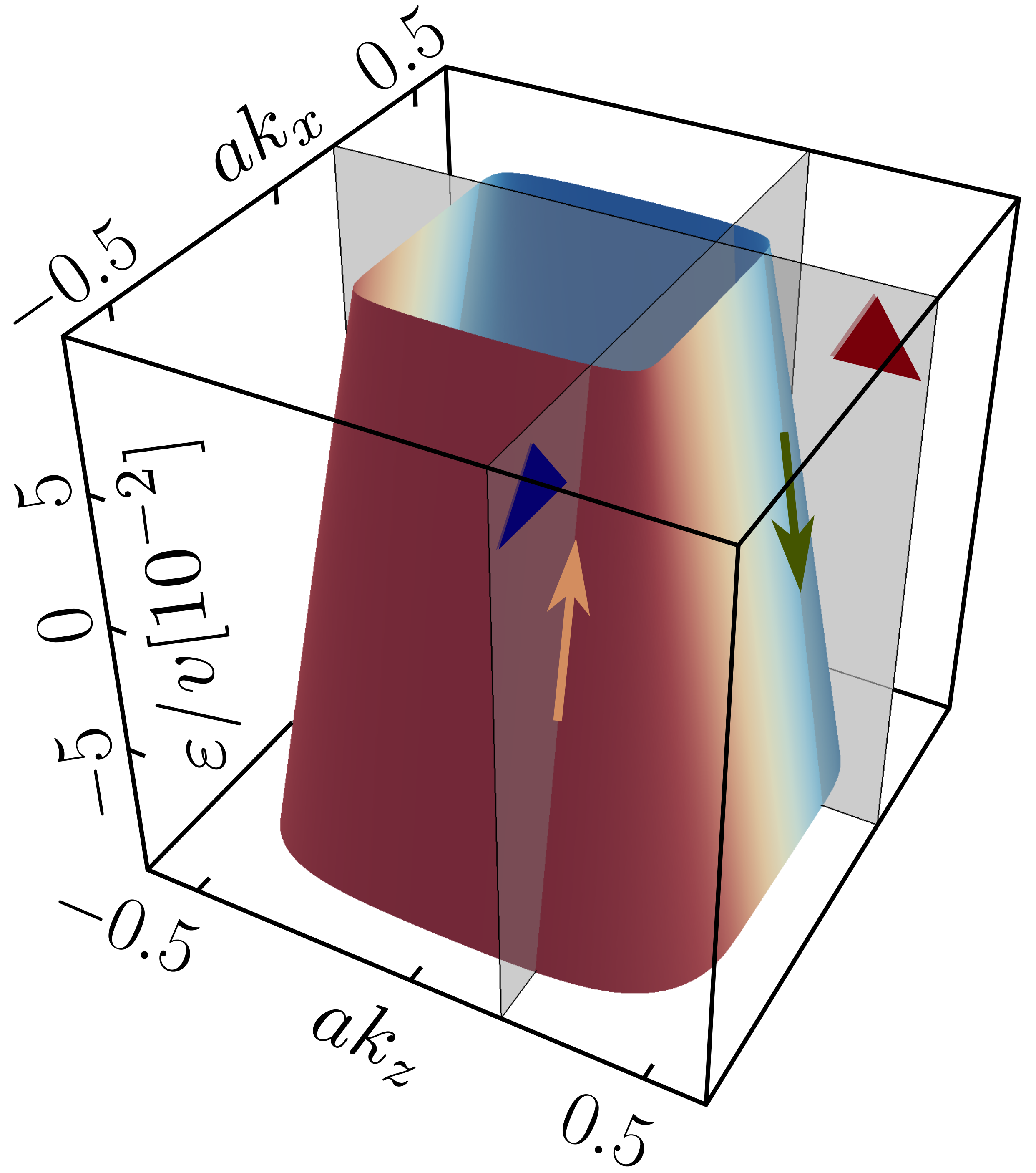
Anomalies



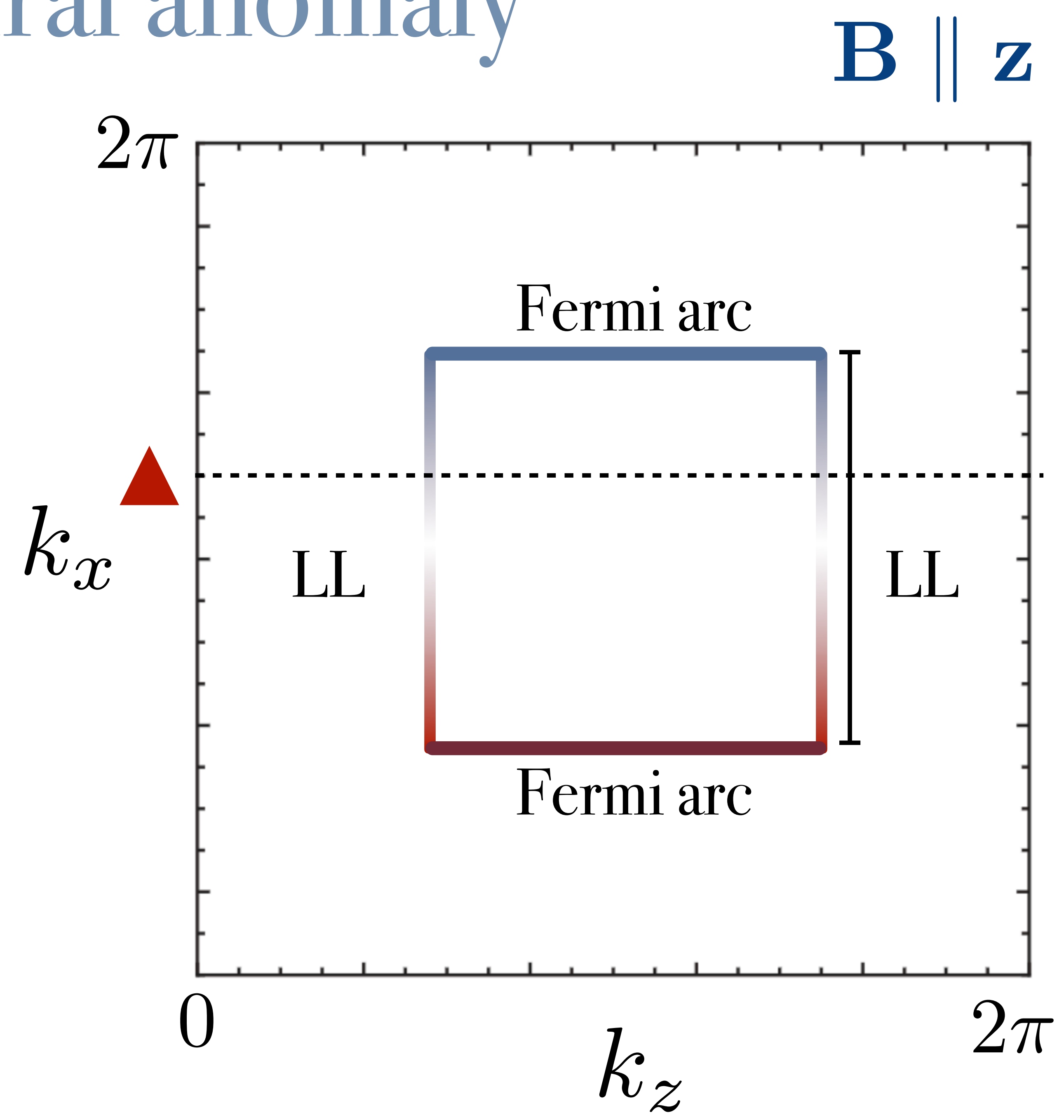
Chiral anomaly

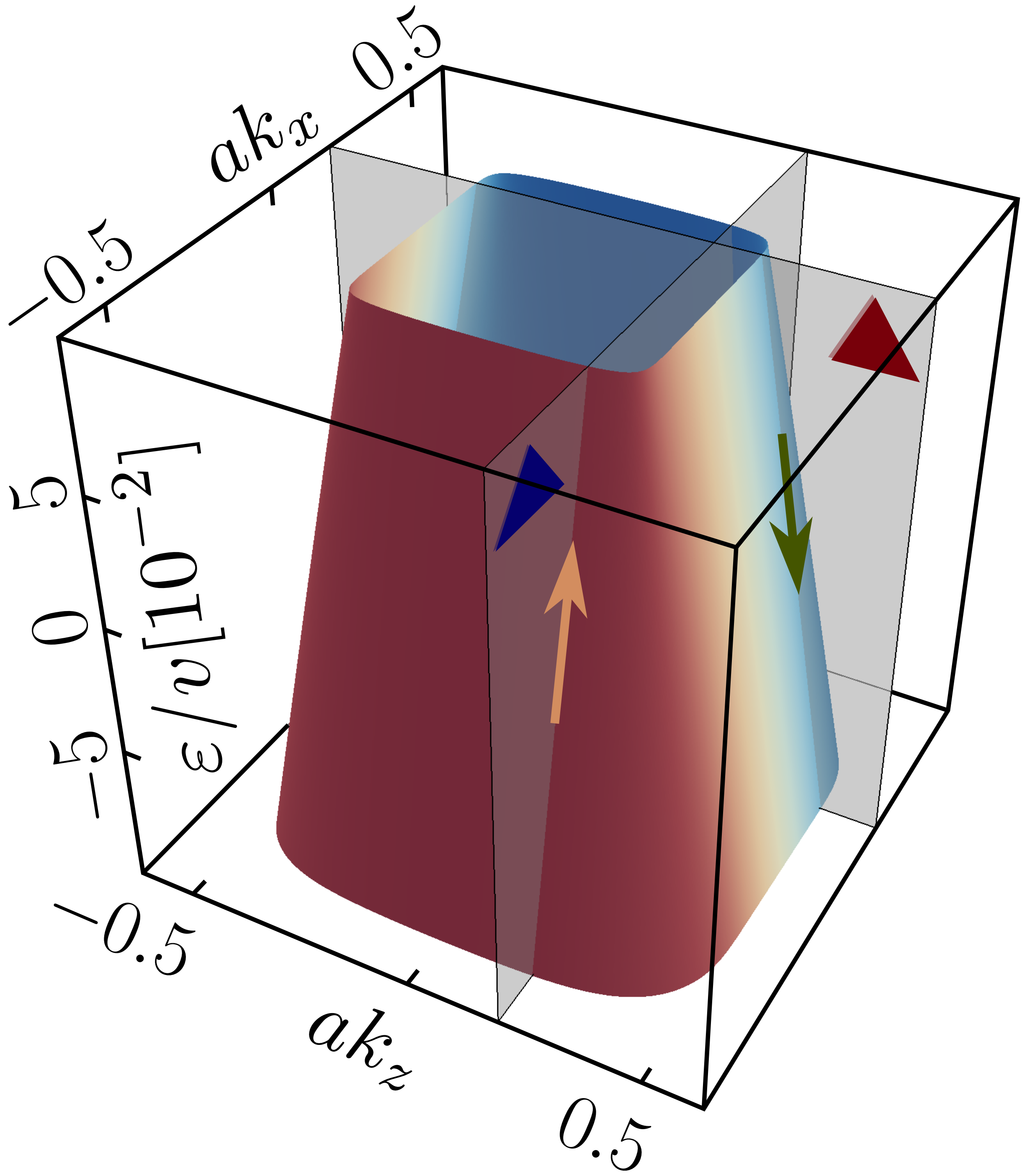
$\mathbf{B} \parallel \mathbf{z}$

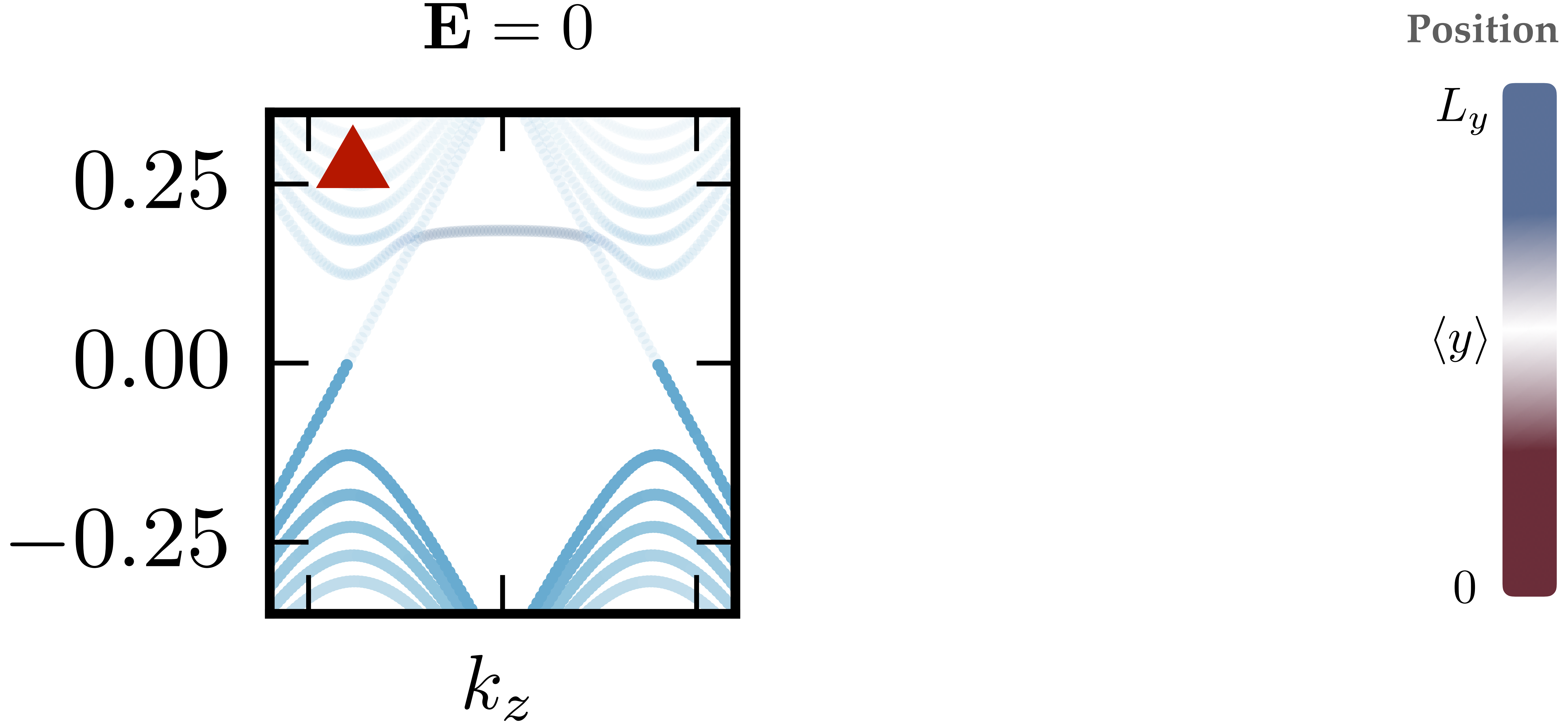




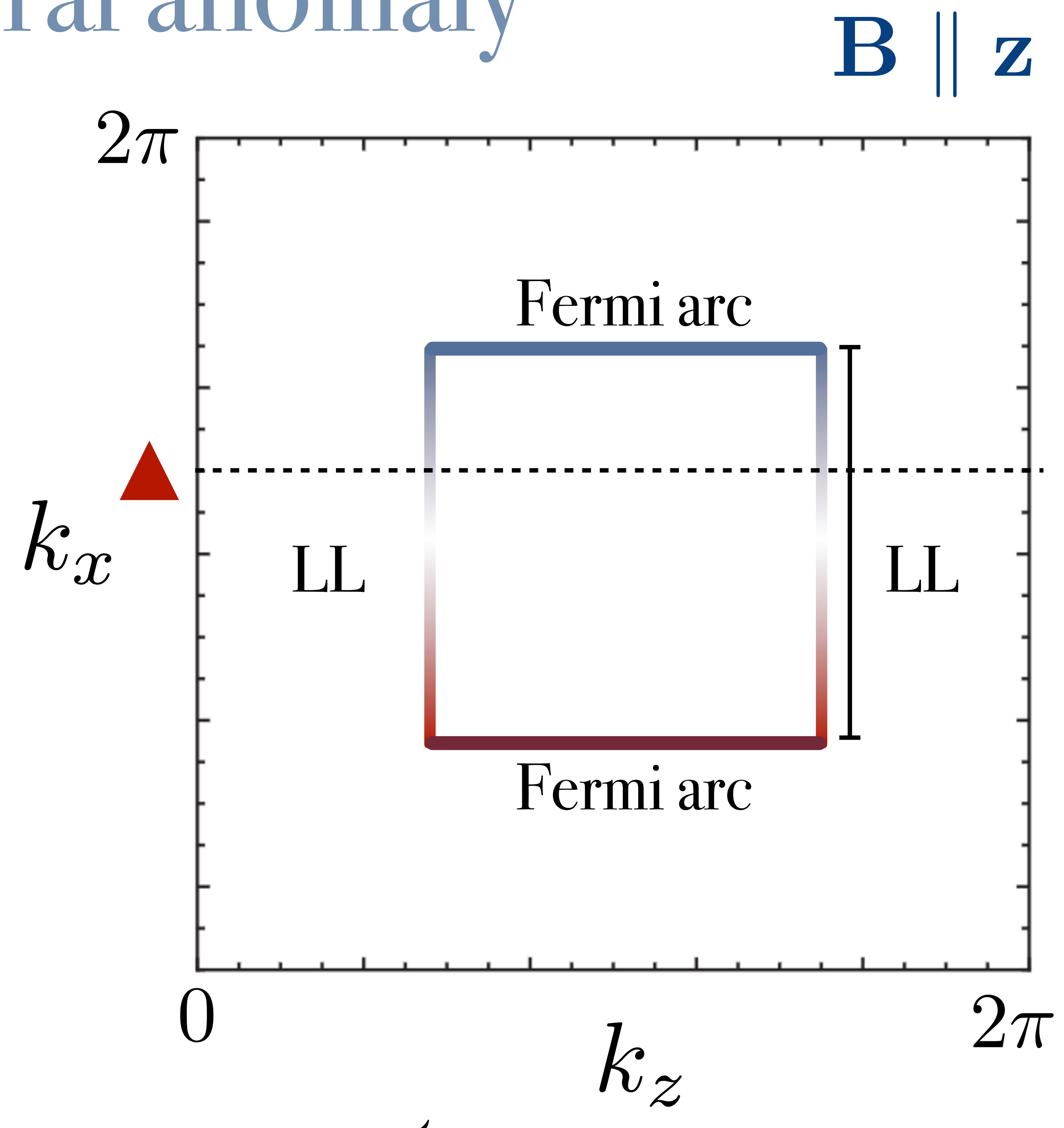
Chiral anomaly

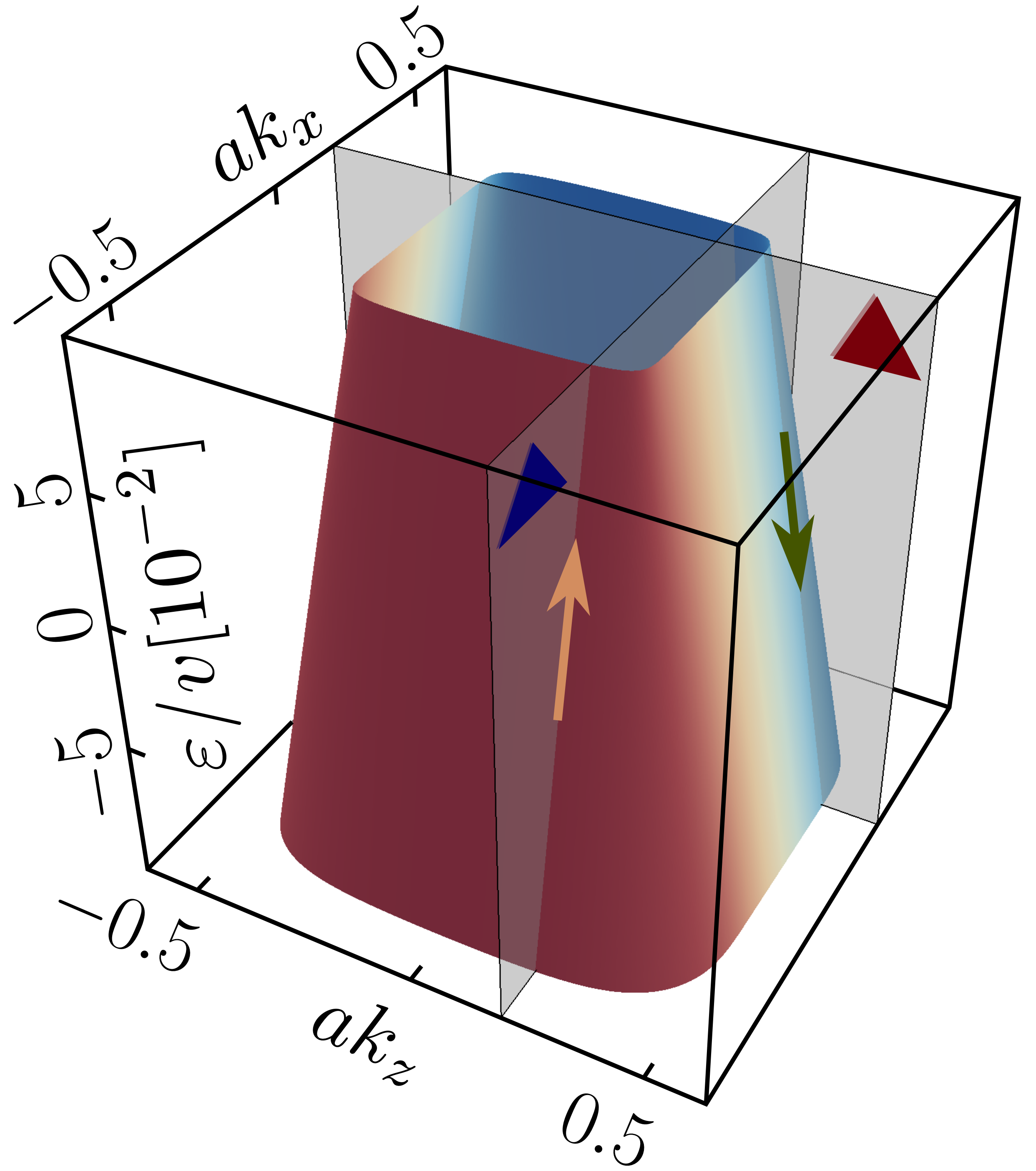


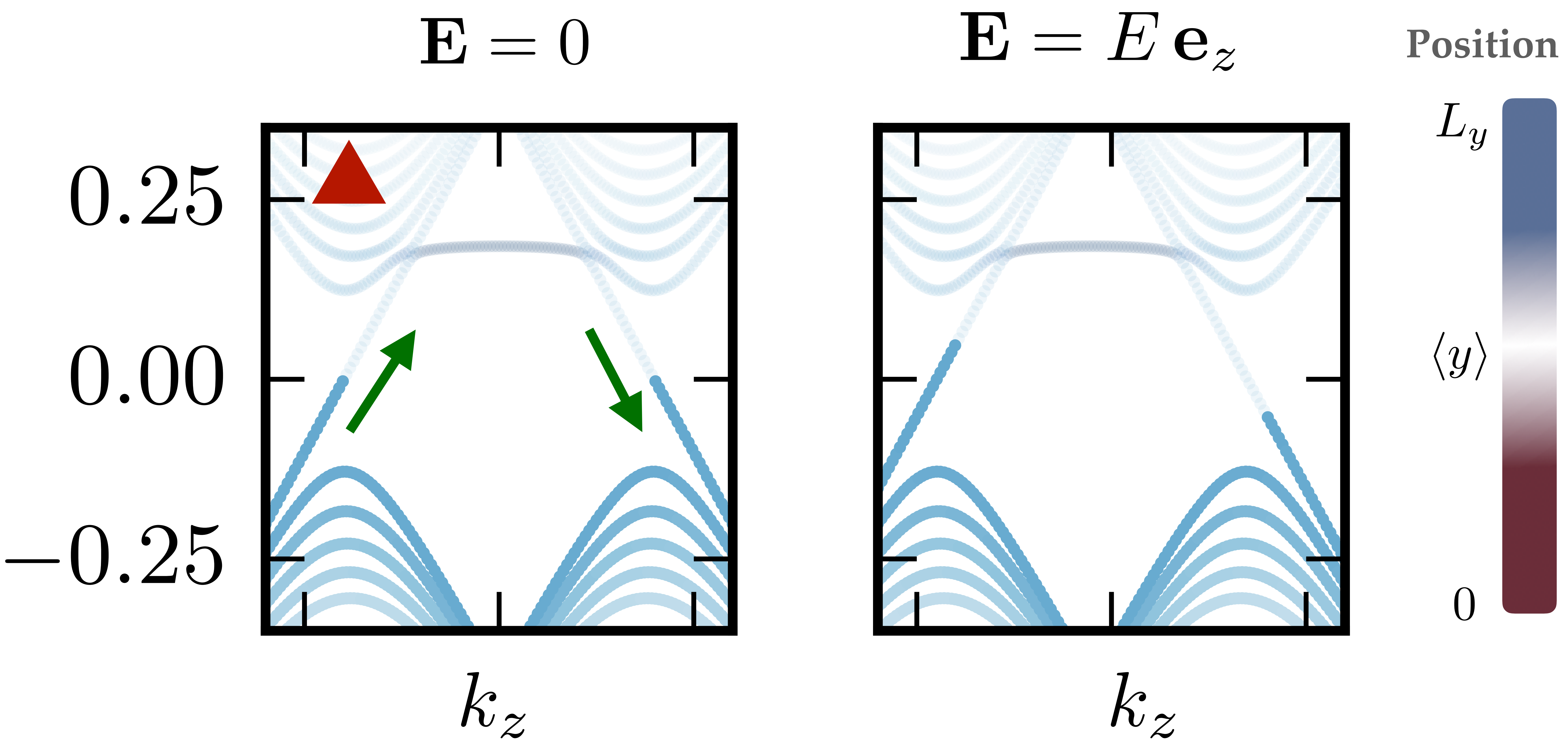




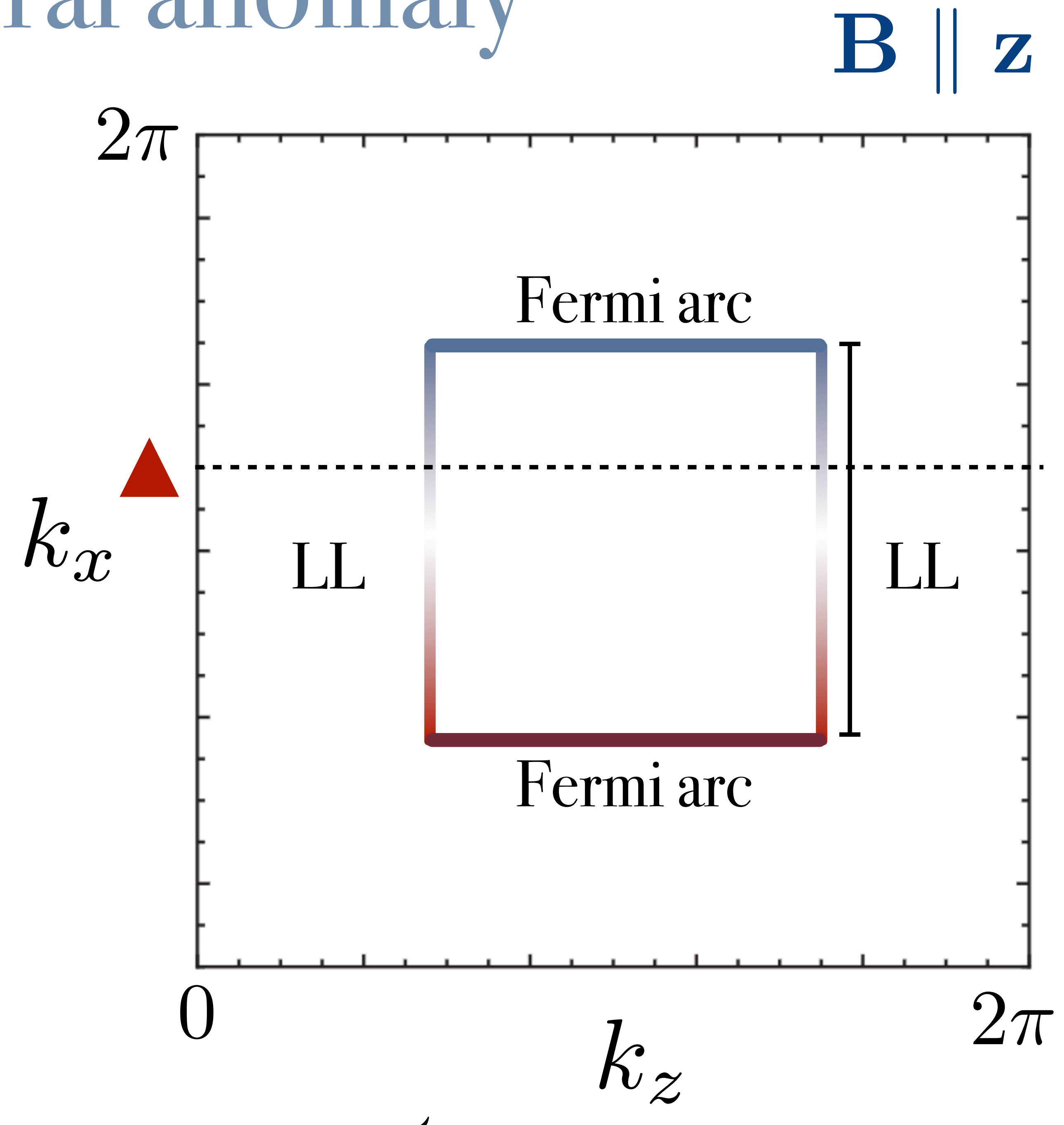
Chiral anomaly

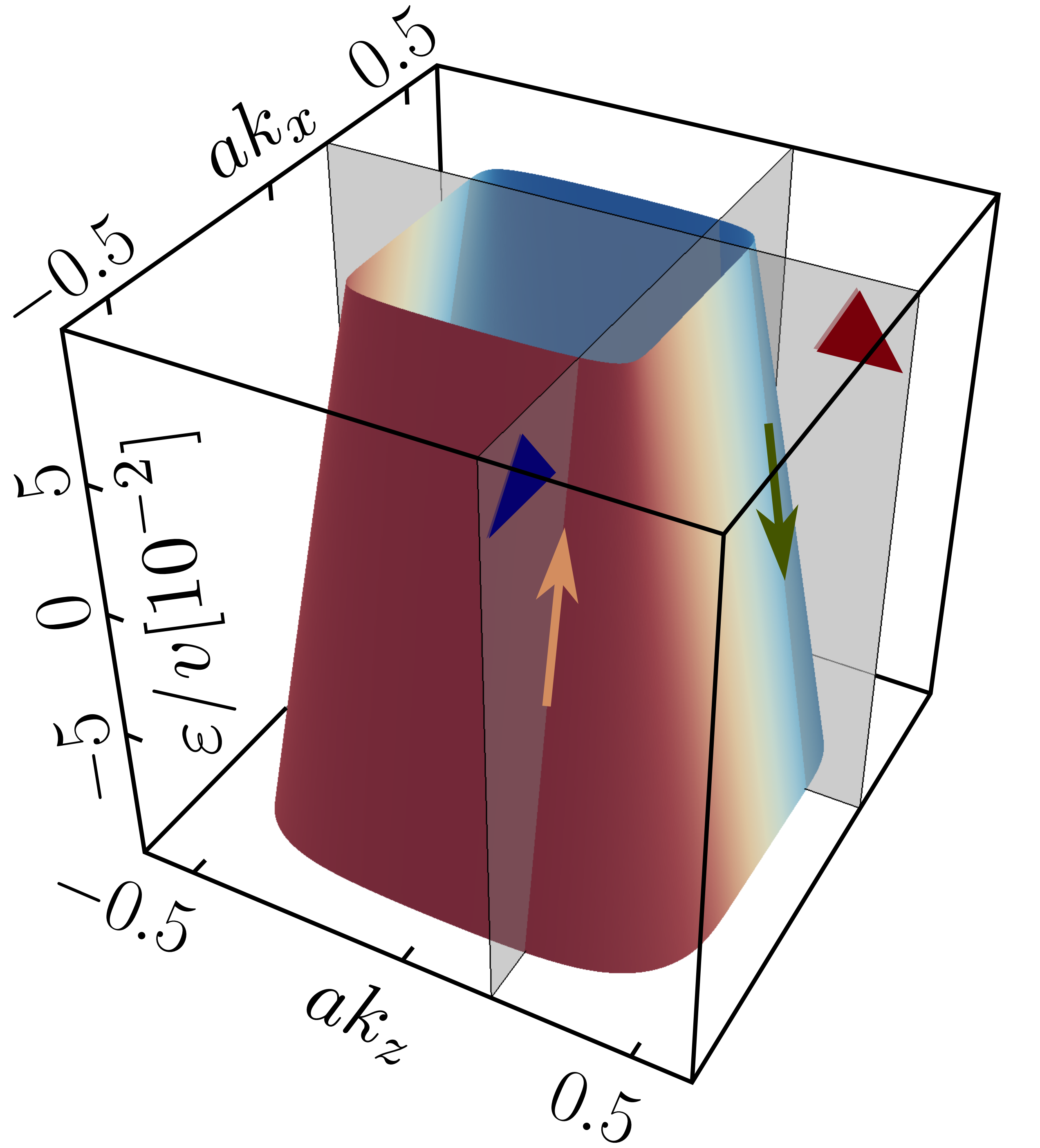


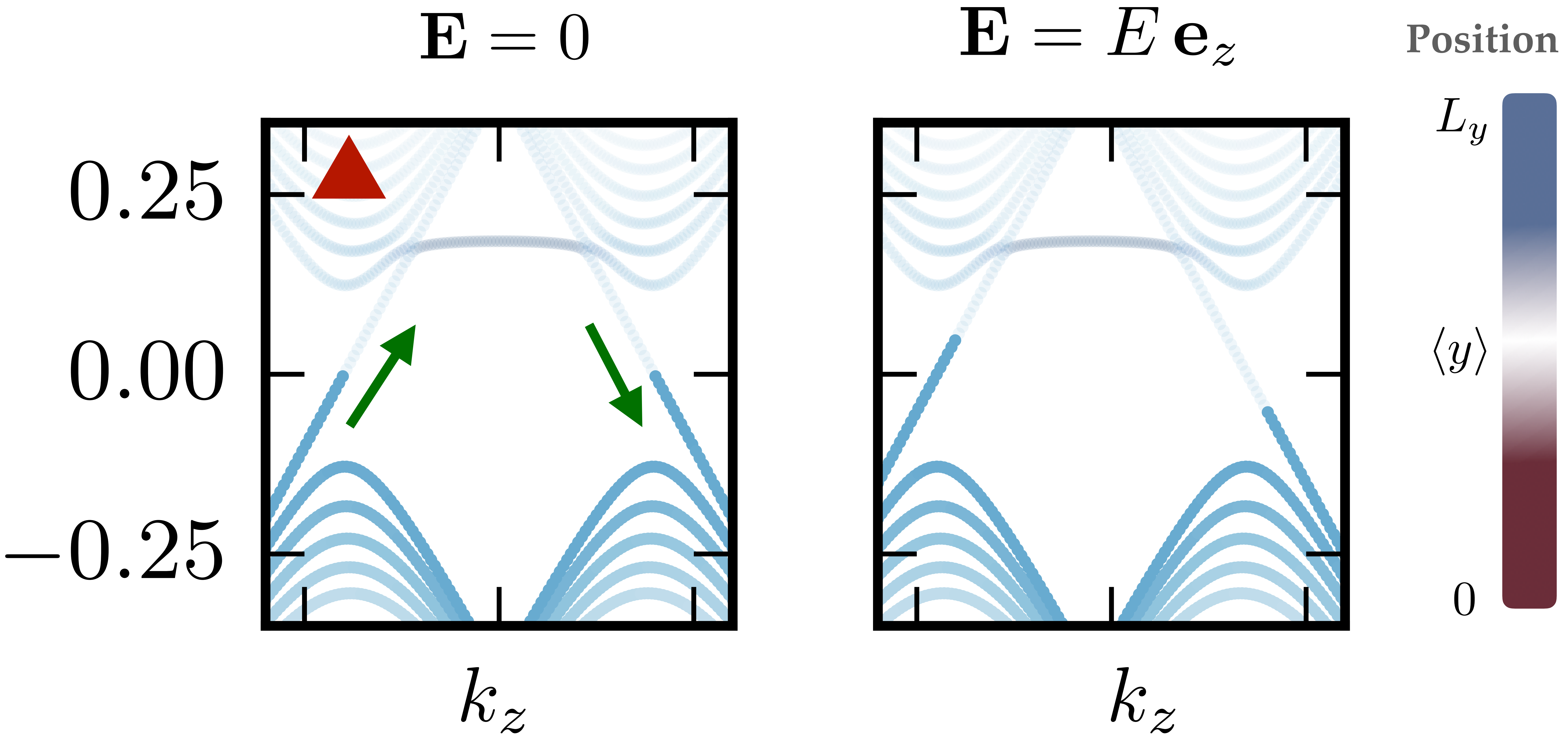


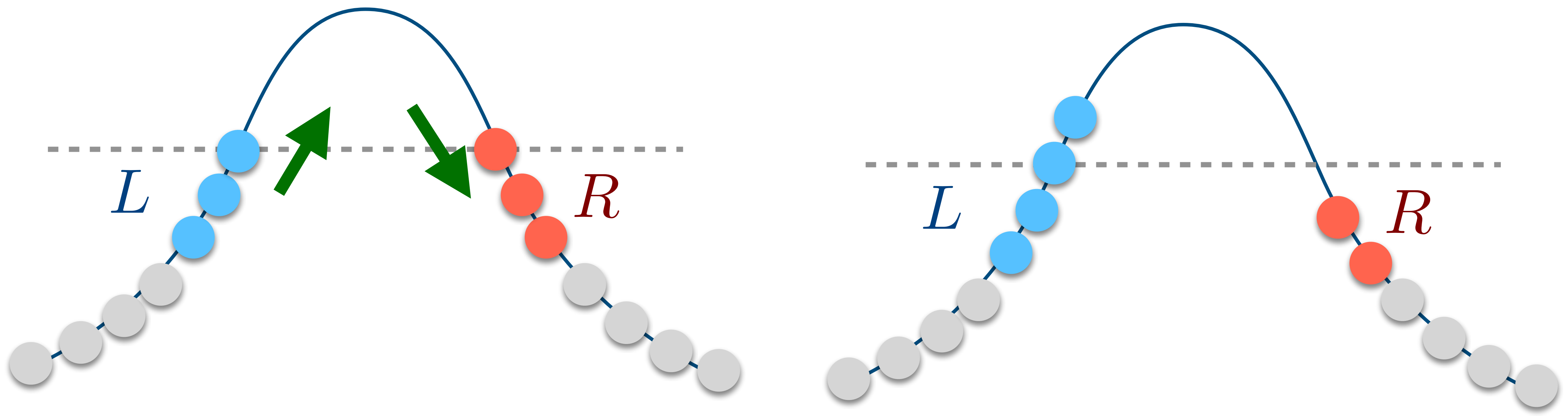


Chiral anomaly



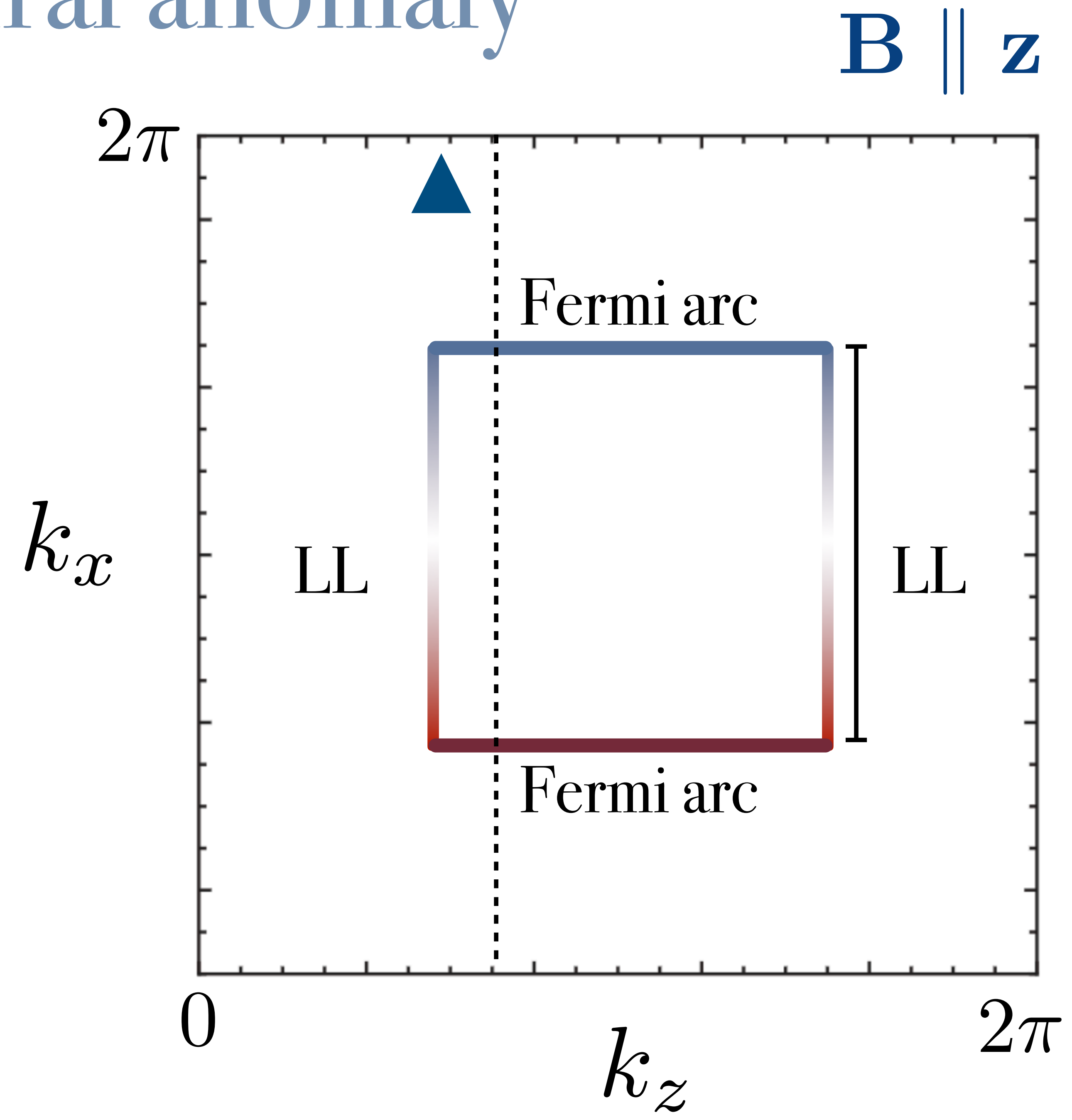




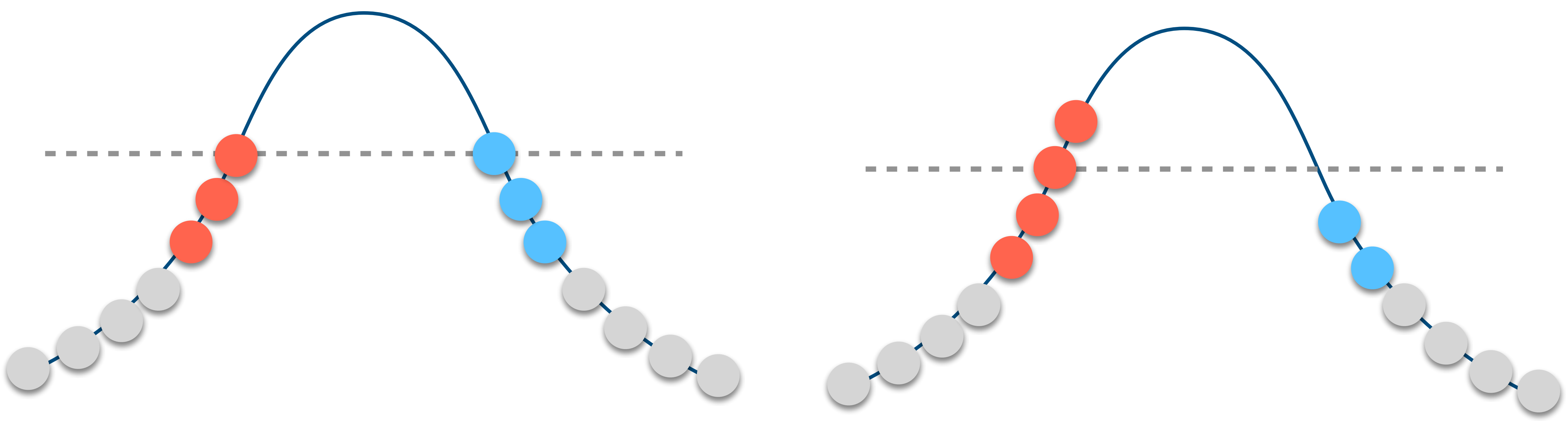


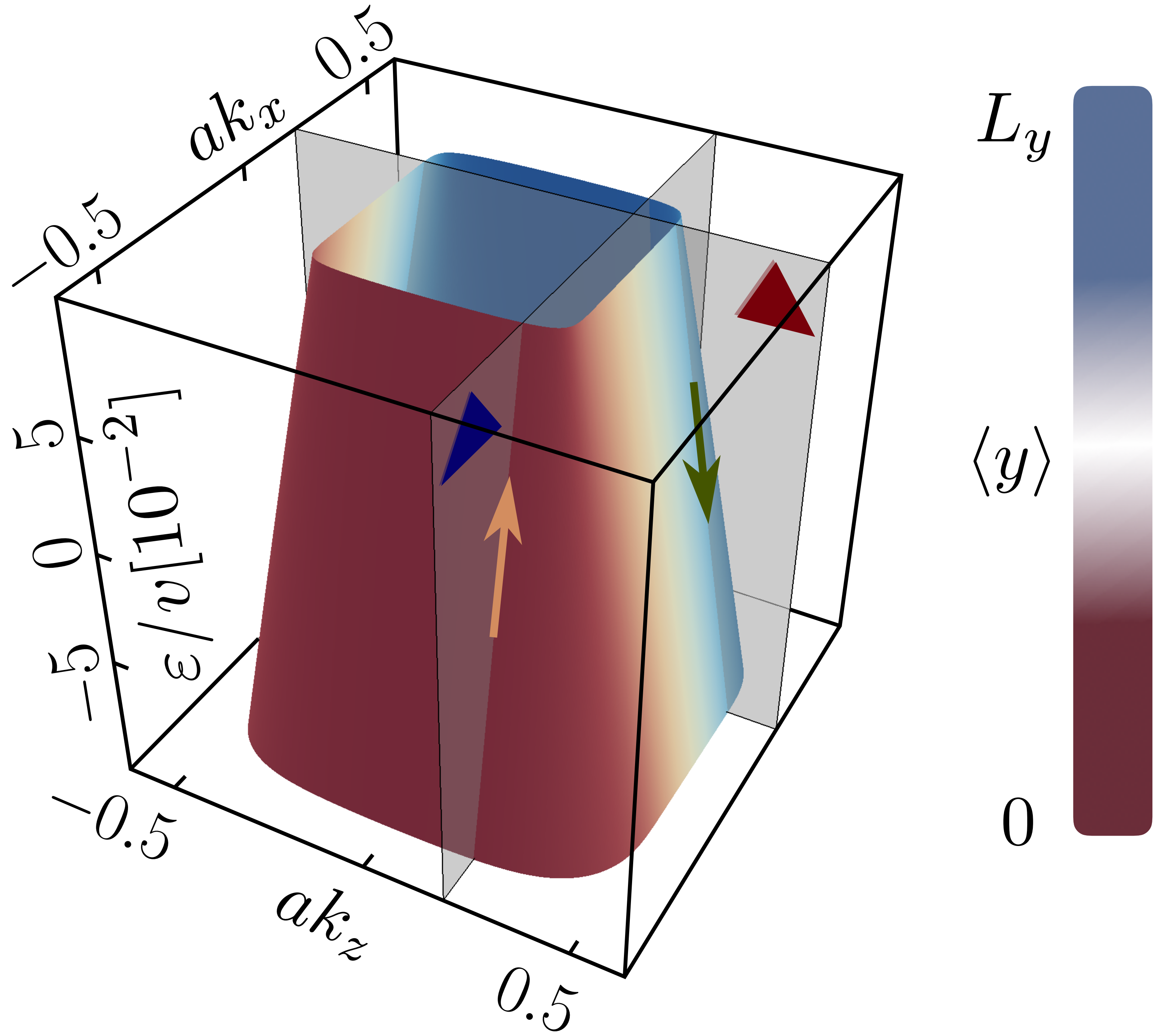
$$\partial_t(n_R - n_L) = \frac{e^2}{4\pi^2\hbar}\mathbf{E}\cdot\mathbf{B}$$

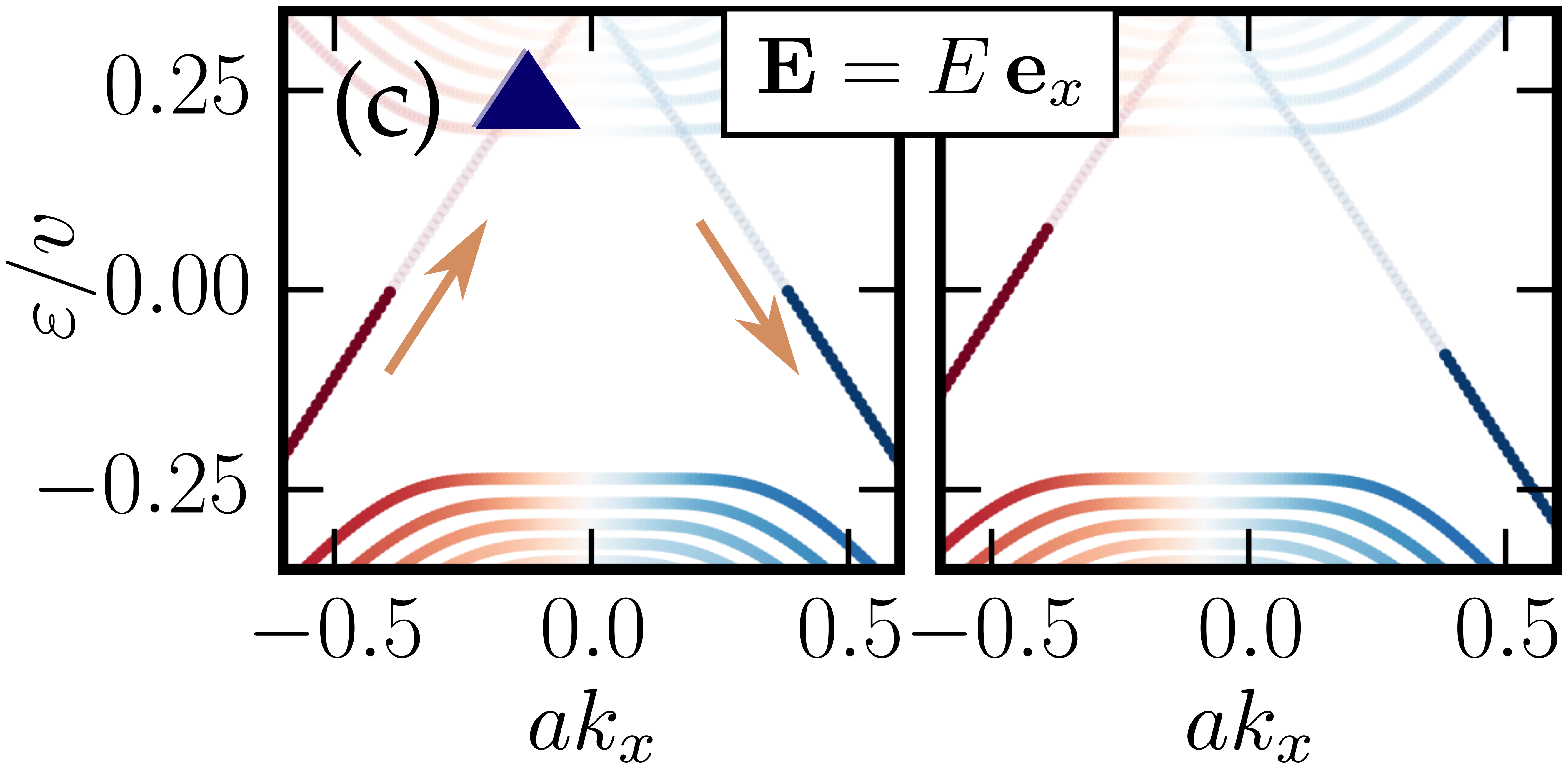
Chiral anomaly



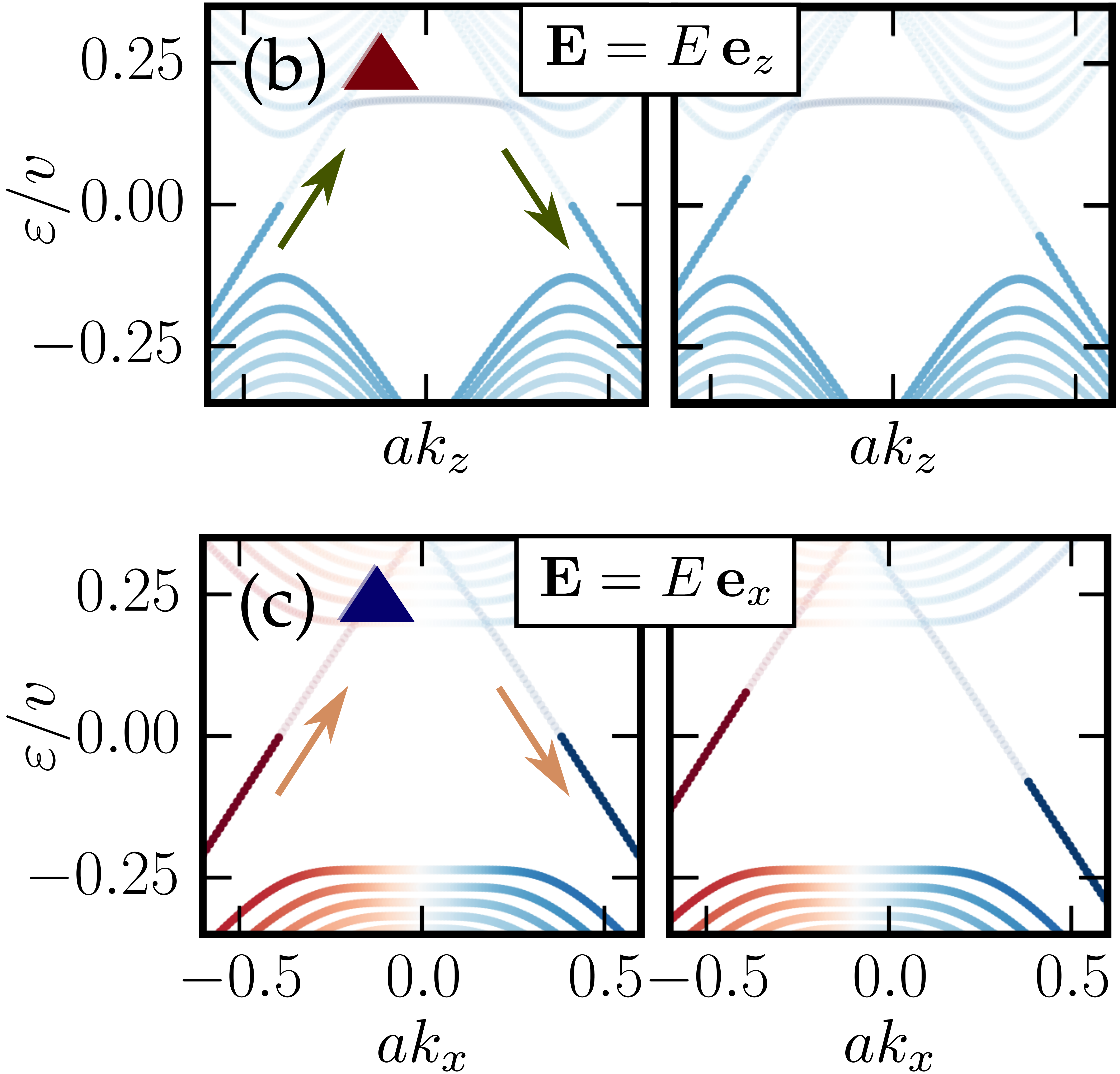
Two "chiralities" live at different surfaces!







Chiral anomaly



$$\partial_\mu J_5^\mu = \frac{e^2}{4\pi^2\hbar}\mathbf{E}\cdot\mathbf{B}$$

?

Quantum Field theory

The chiral anomaly with chiral fields

Covariant anomaly: the Fermi surface contribution

$$\partial_\mu J_5^\mu = \frac{e^2}{4\pi^2\hbar}\mathbf{E}\cdot\mathbf{B}$$

$$\partial_\mu J^\mu = 0$$

🟦 = enhancement of magneto-conductivity

D. T. Son, B. Spivak PRB (2013)
Many experiments (Ong, Hasan, Felser…)

Covariant anomaly: the Fermi surface contribution

$$\partial_\mu J_5^\mu = \frac{e^2}{4\pi^2\hbar}\left(\mathbf{E}\cdot\mathbf{B} + \mathbf{E}_5\cdot\mathbf{B}_5\right)$$

$$\partial_\mu J^\mu \overset{!}{=} \frac{e^2}{4\pi^2\hbar}\left(\mathbf{E}\cdot\mathbf{B}_5 + \mathbf{E}_5\cdot\mathbf{B}\right)$$

🟦 = enhancement of magneto-conductivity

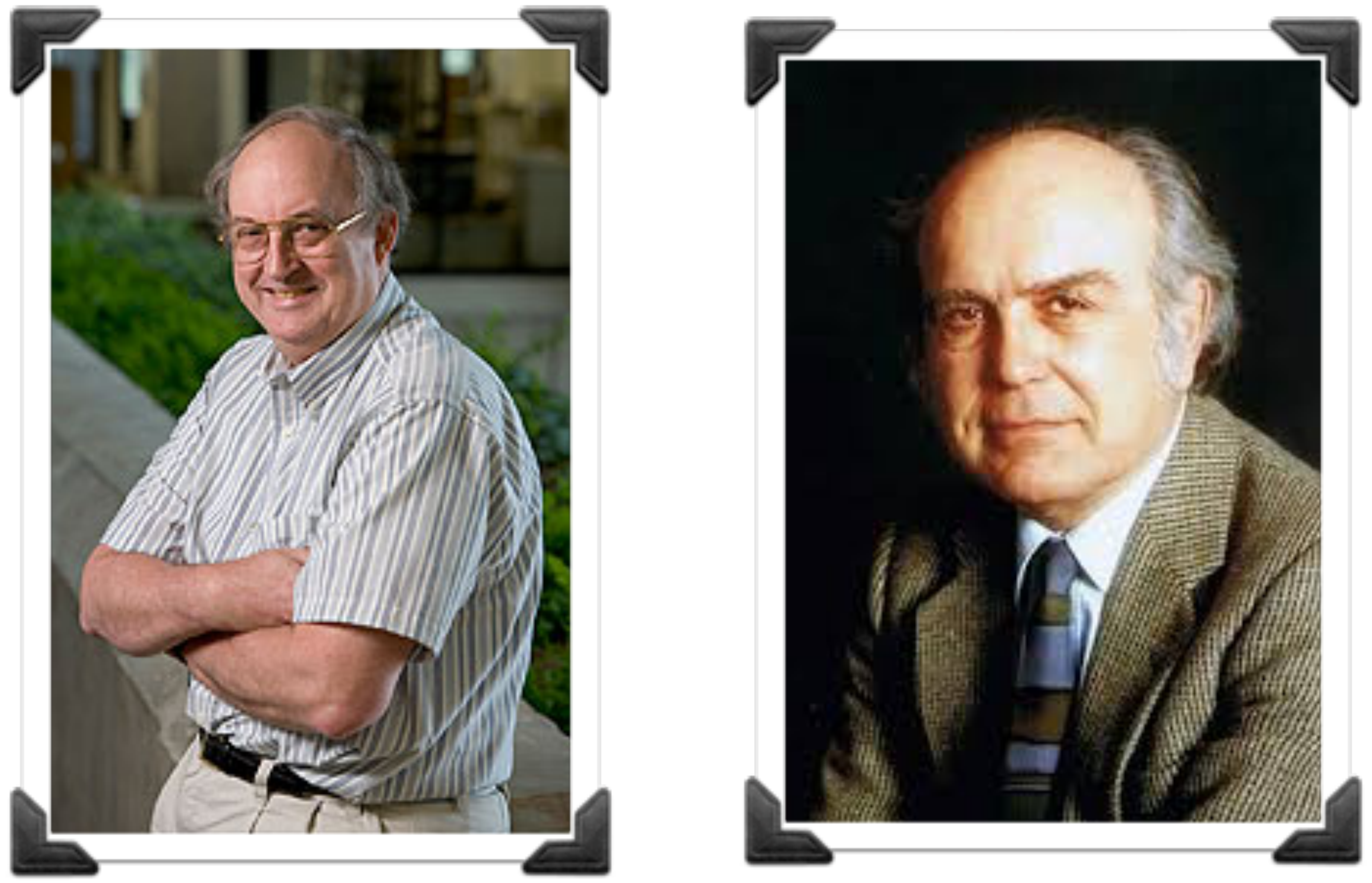
D. T. Son, B. Spivak PRB (2013)
Many experiments (Ong, Hasan, Felser…)

🟨 🟩 🟪 = strained induced enhancement of conductivity ⚠

D. Pikulin, A. Chen, M. Franz PRX (2016)

AGG, J. Venderbos, A. Vishwanath, R. Ilan PRX (2016)

Covariant anomaly: the Fermi surface contribution



Bardeen, Zumino Nuc. Phys. B (1984)

$$\partial_\mu J^\mu_5 = \frac{e^2}{4\pi^2\hbar}\left(\mathbf{E}\cdot\mathbf{B} + \mathbf{E}_5\cdot\mathbf{B}_5\right)$$

$$\partial_\mu J^\mu \overset{!}{=} \frac{e^2}{4\pi^2\hbar}\left(\mathbf{E}\cdot\mathbf{B}_5 + \mathbf{E}_5\cdot\mathbf{B}\right)$$

$$J^\mu_{\text{cons}} = \underset{\substack{\nearrow \\ \text{Fermi surface}}}{J^\mu} + \underset{\substack{\nwarrow \\ \text{Bardeen Polynomials}}}{\delta J^\mu}$$

e.g. $\delta\mathbf{j} = \mathbf{b}\times\mathbf{E}$

🟦 = enhancement of magneto-conductivity

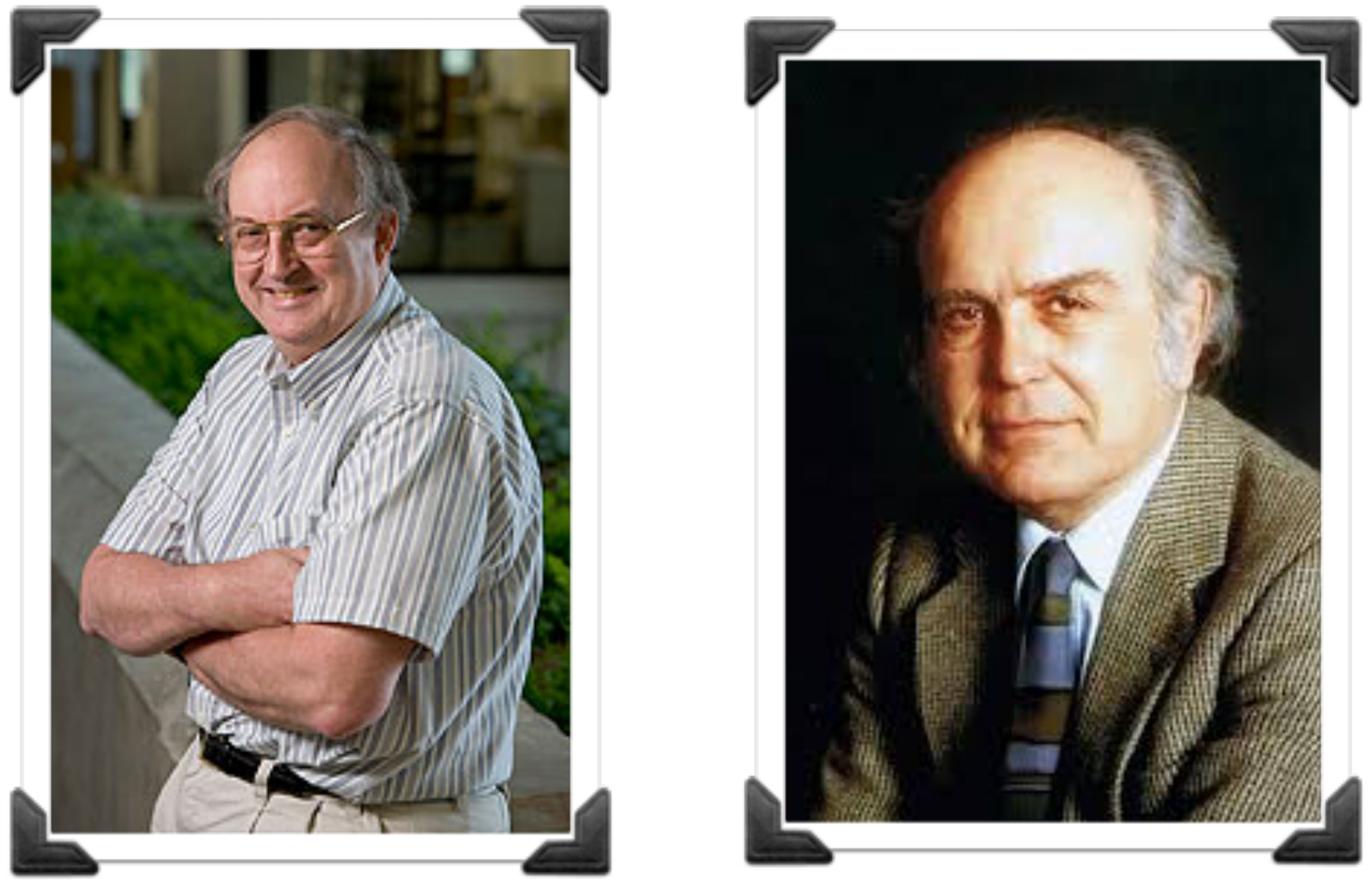
D. T. Son, B. Spivak PRB (2013)
Many experiments (Ong, Hasan, Felser…)

🟨 🟩 🟪 = strained induced enhancement of conductivity ⚠

D. Pikulin, A. Chen, M. Franz PRX (2016)

AGG, J. Venderbos, A. Vishwanath, R. Ilan PRX (2016)

Consistent anomaly: Fermi surface + bottom of the band



Bardeen, Zumino Nuc. Phys. B (1984)

$$\partial_\mu J^\mu_{5\,\mathrm{cons}} = \frac{e^2}{4\pi^2\hbar}\left(\mathbf{E}\cdot\mathbf{B} + \frac{1}{3}\mathbf{E}_5\cdot\mathbf{B}_5\right)$$

$$\partial_\mu J^\mu_{\mathrm{cons}} = 0$$

$$J^\mu_{\mathrm{cons}} = J^\mu + \delta J^\mu$$

Fermi surface   Bardeen Polynomials

🟪 has different coefficient

🟨 🟩 terms are gone

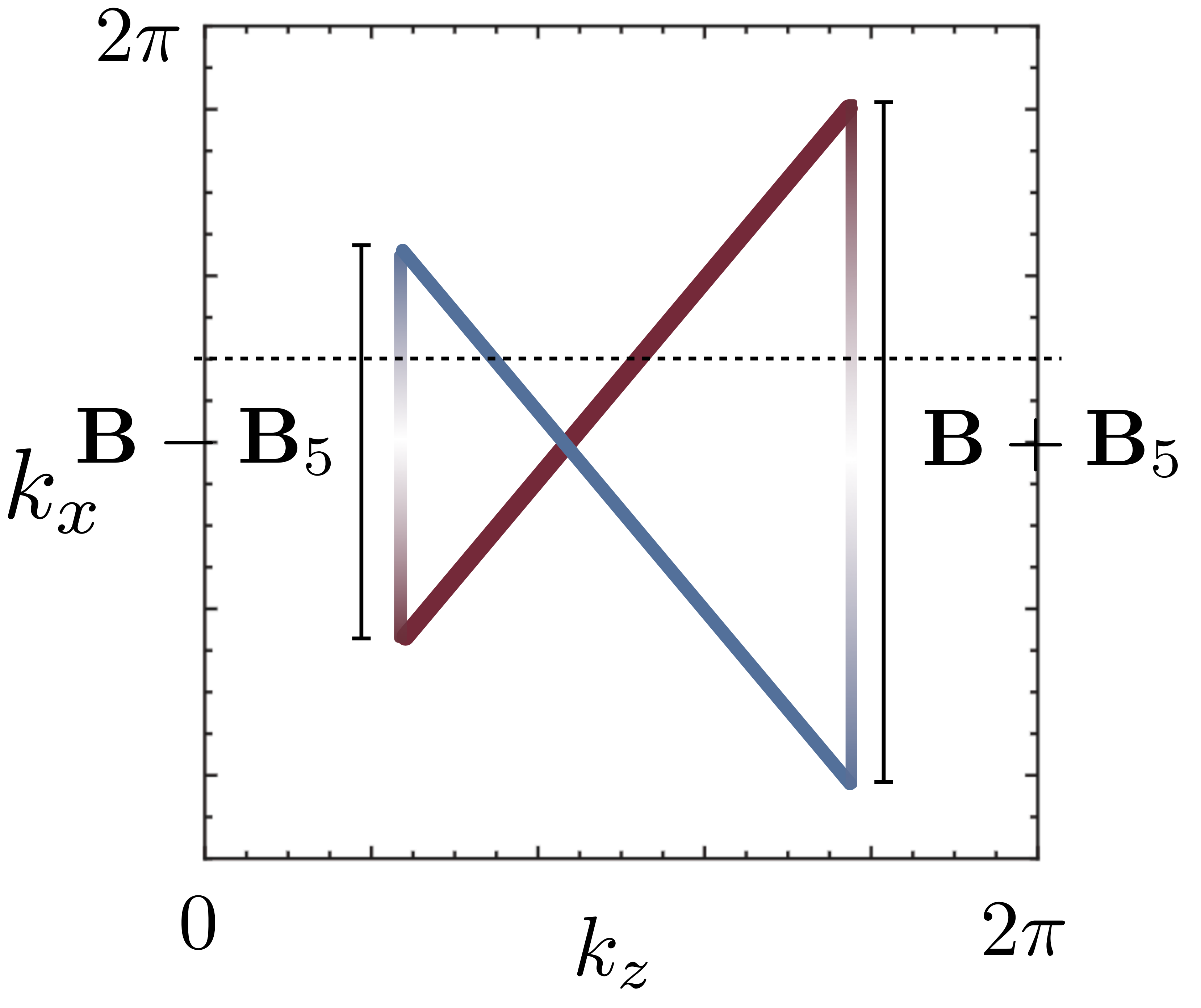
Bardeen, Zumino Nuc. Phys. B (1984)
Landsteiner PRB (2014)
Gorbar et. al PRL (2017), PRB (2017)…

Consistent vs covariant pictures

Covariant anomaly	Consistent anomaly
$\partial_\mu J_5^\mu = \frac{e^2}{4\pi^2\hbar}\left(\mathbf{E}\cdot\mathbf{B} + \mathbf{E}_5\cdot\mathbf{B}_5\right)$	$\overbrace{\partial_\mu J^\mu_{5,\text{cons}}}^{\text{Defining 5? Fermi arcs?}} = \frac{e^2}{4\pi^2\hbar}\left(\underbrace{\mathbf{E}\cdot\mathbf{B} + \frac{1}{3}\mathbf{E}_5\cdot\mathbf{B}_5}_{\text{(When) Is this 1/3 factor observable?}}\right)$
$\partial_\mu J^\mu \overset{!}{=} \frac{e^2}{4\pi^2\hbar}\left(\mathbf{E}\cdot\mathbf{B}_5 + \mathbf{E}_5\cdot\mathbf{B}\right)$	$\partial_\mu J^\mu_{\text{cons}} = 0 \quad \underbrace{\boxed{}\quad\boxed{}}_{\text{physical meaning?}}$
"Fermi surface" contribution	"Fermi surface" contribution + "band bottom"

Landsteiner PRB (2014)
Gorbar et. al PRL (2017), PRB (2017)…

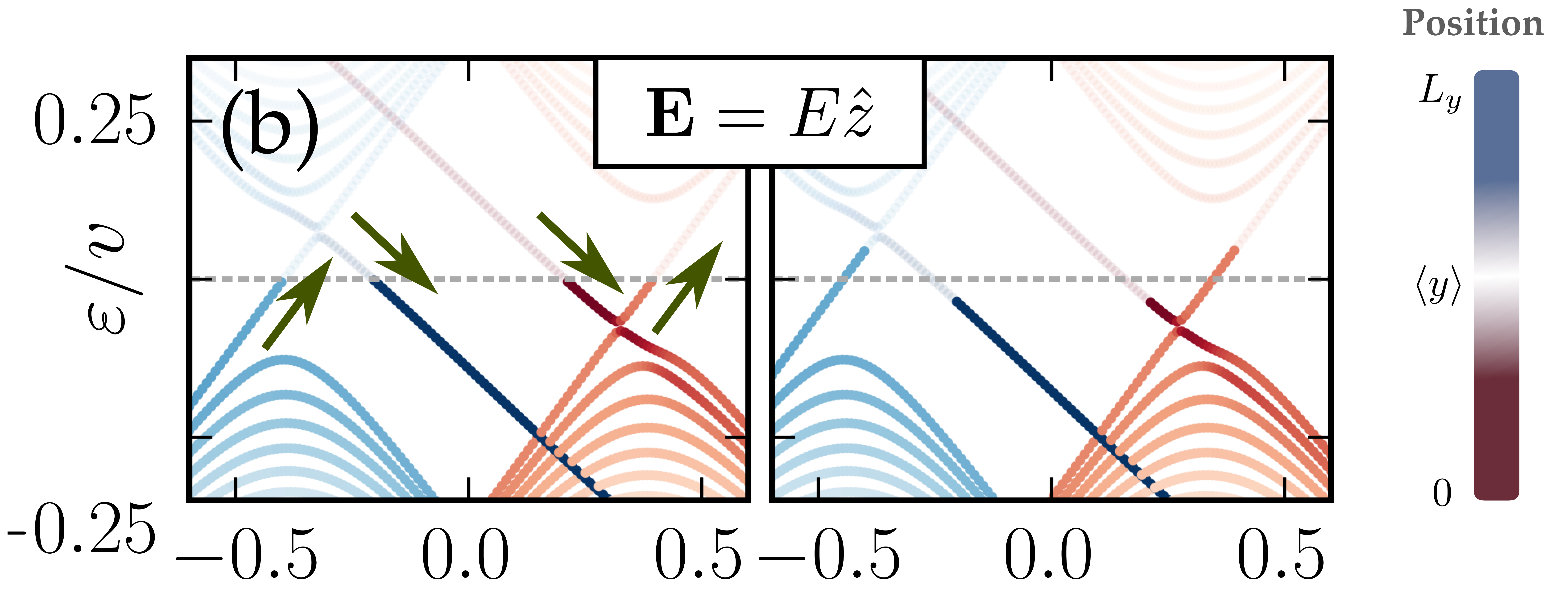
Example:

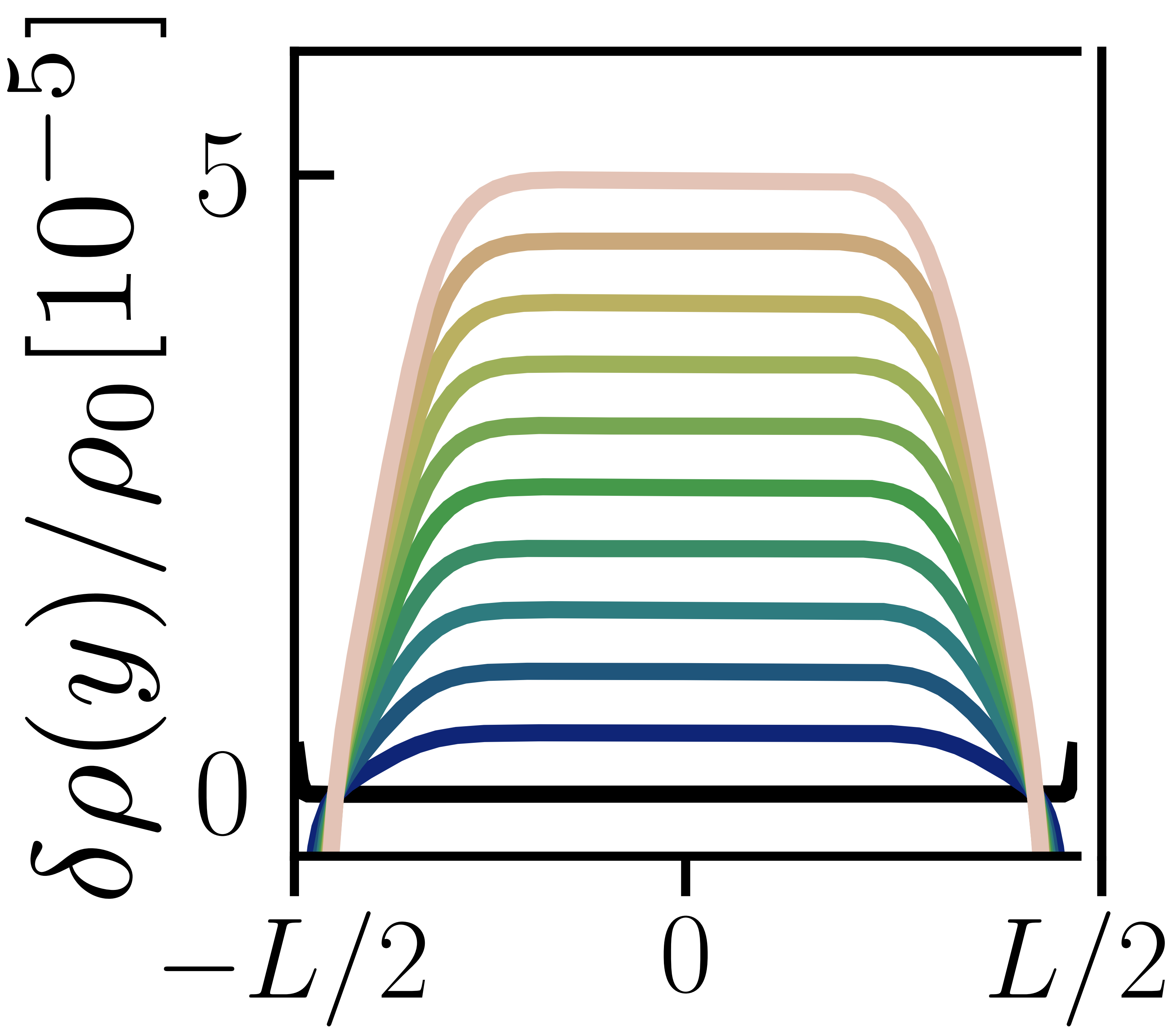


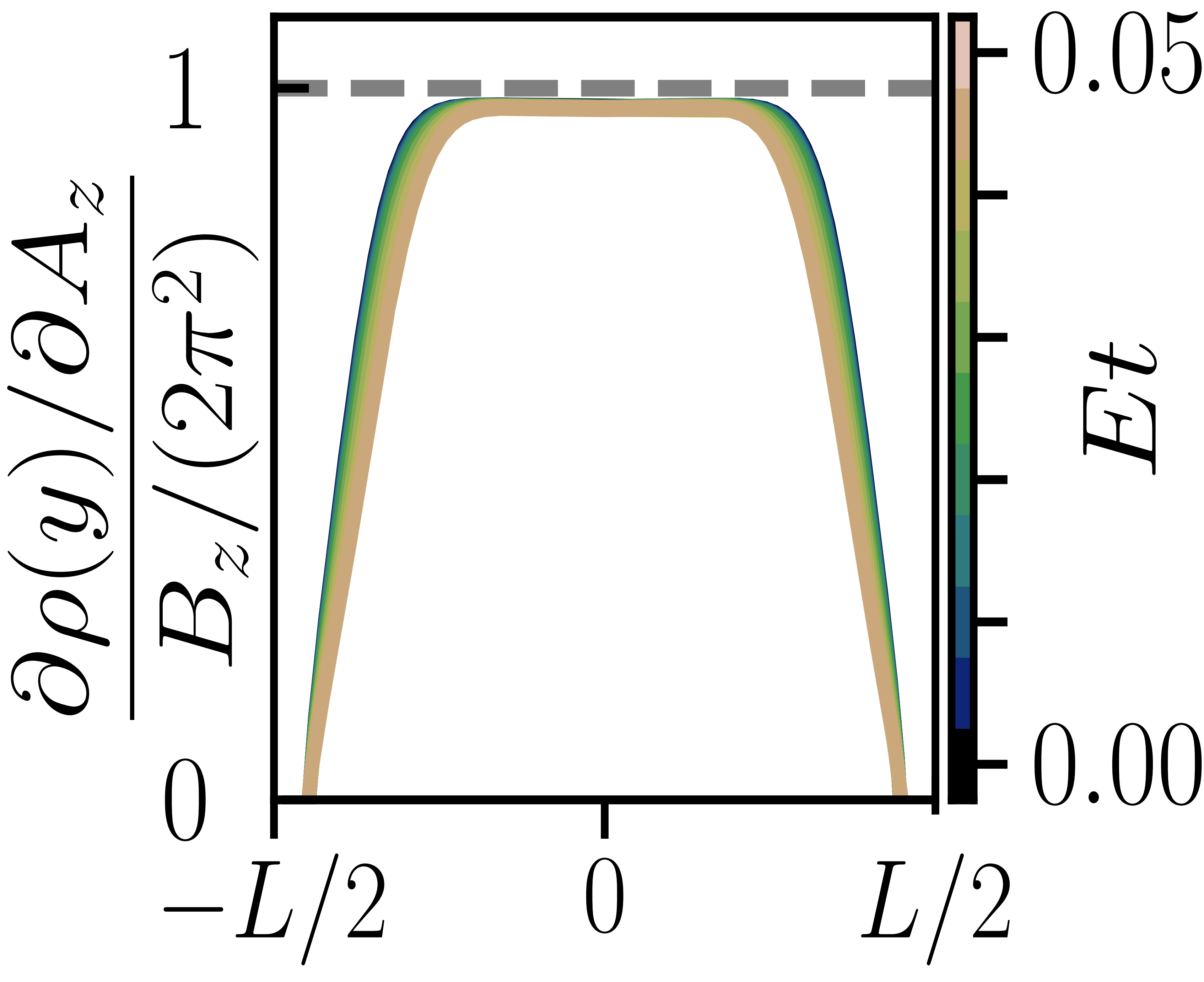
$$\partial_\mu J^\mu = \frac{e^2}{4\pi^2\hbar}\left(\mathbf{E}\cdot\mathbf{B}_5 + \text{🟩}\right)$$

Fermi arcs enable the **covariant** anomaly

🟨 🟩







The missing 1/3

Field theory prediction

Covariant anomaly

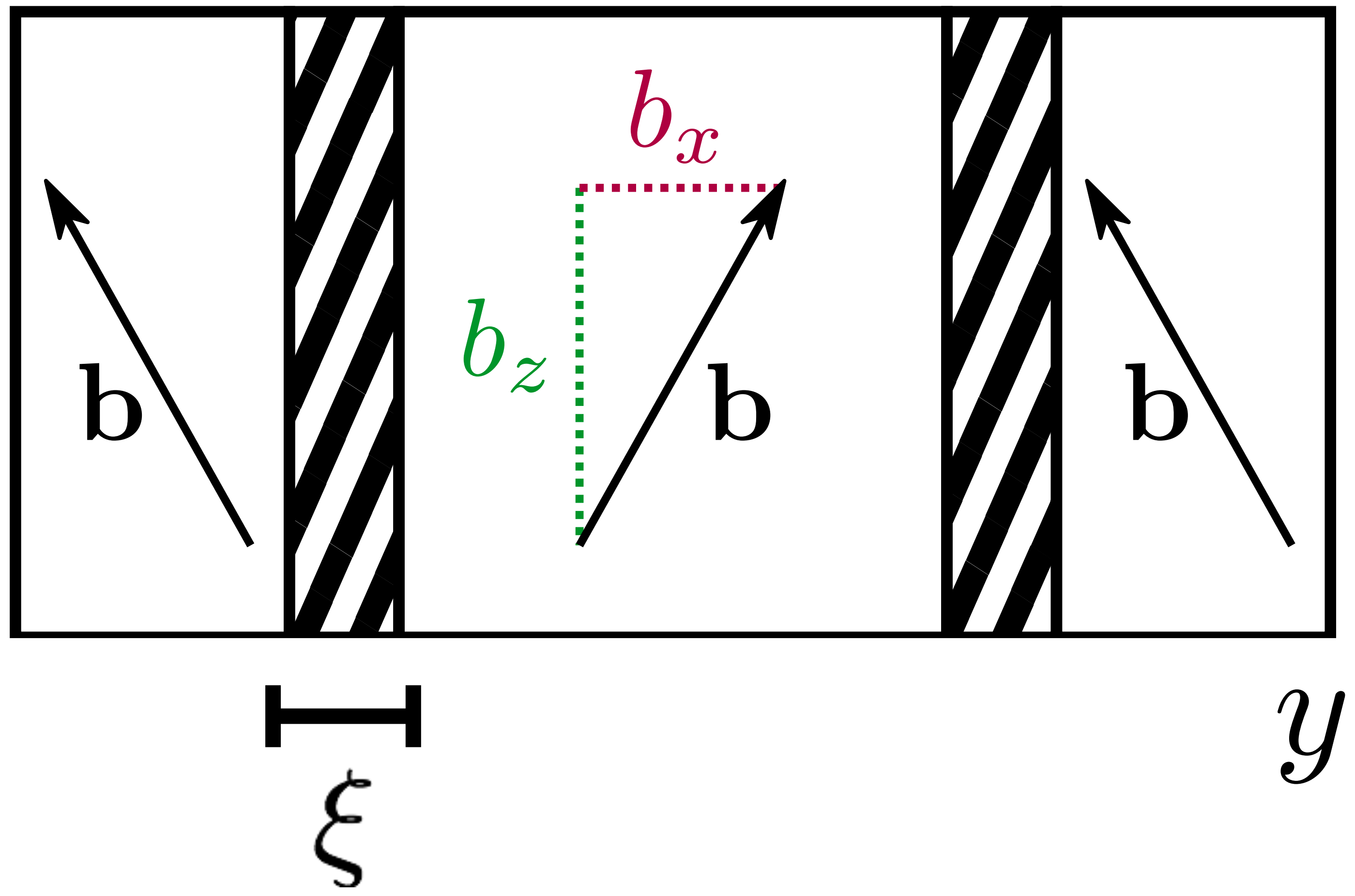
$$\partial_\mu J^\mu_5 = \frac{e^2}{4\pi^2\hbar}\left(\mathbf{E}\cdot\mathbf{B} + \mathbf{E}_5\cdot\mathbf{B}_5\right)$$

Consistent anomaly

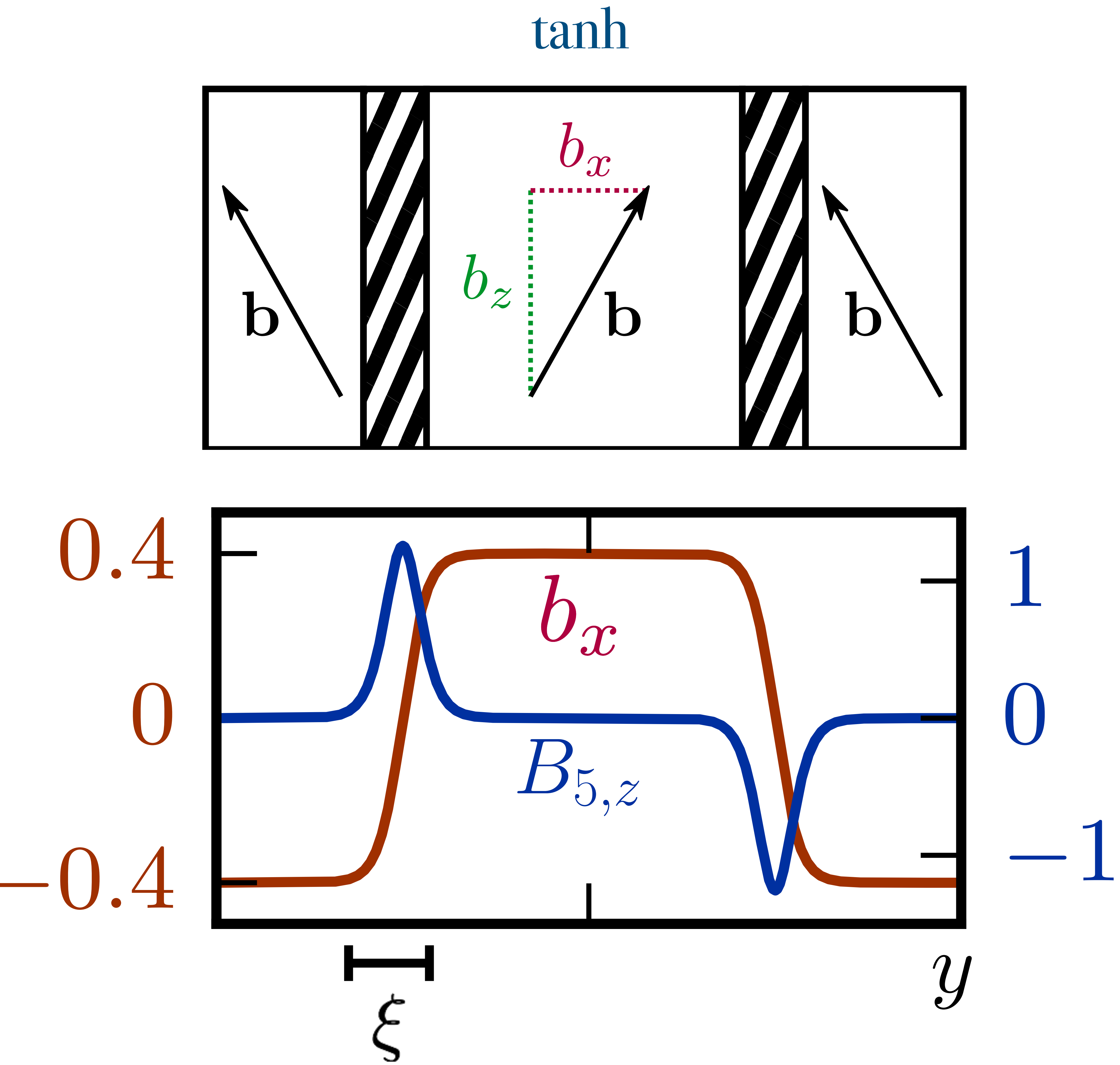
$$\partial_\mu J^\mu_{5,\mathrm{cons}} = \frac{e^2}{4\pi^2\hbar}\left(\mathbf{E}\cdot\mathbf{B} + \frac{1}{3}\mathbf{E}_5\cdot\mathbf{B}_5\right)$$

Fermi surface contribution = 3 Total band contribution

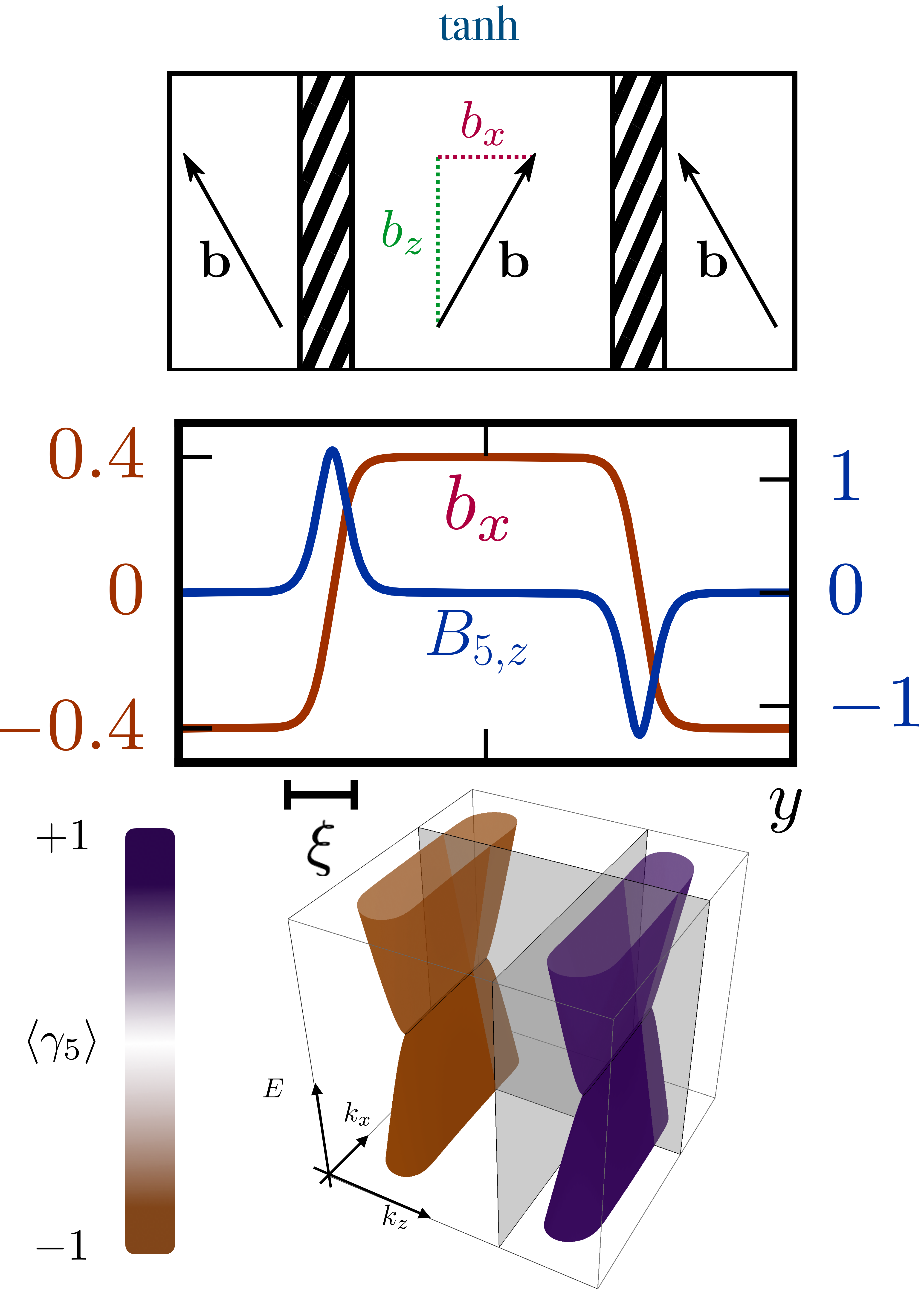
tanh

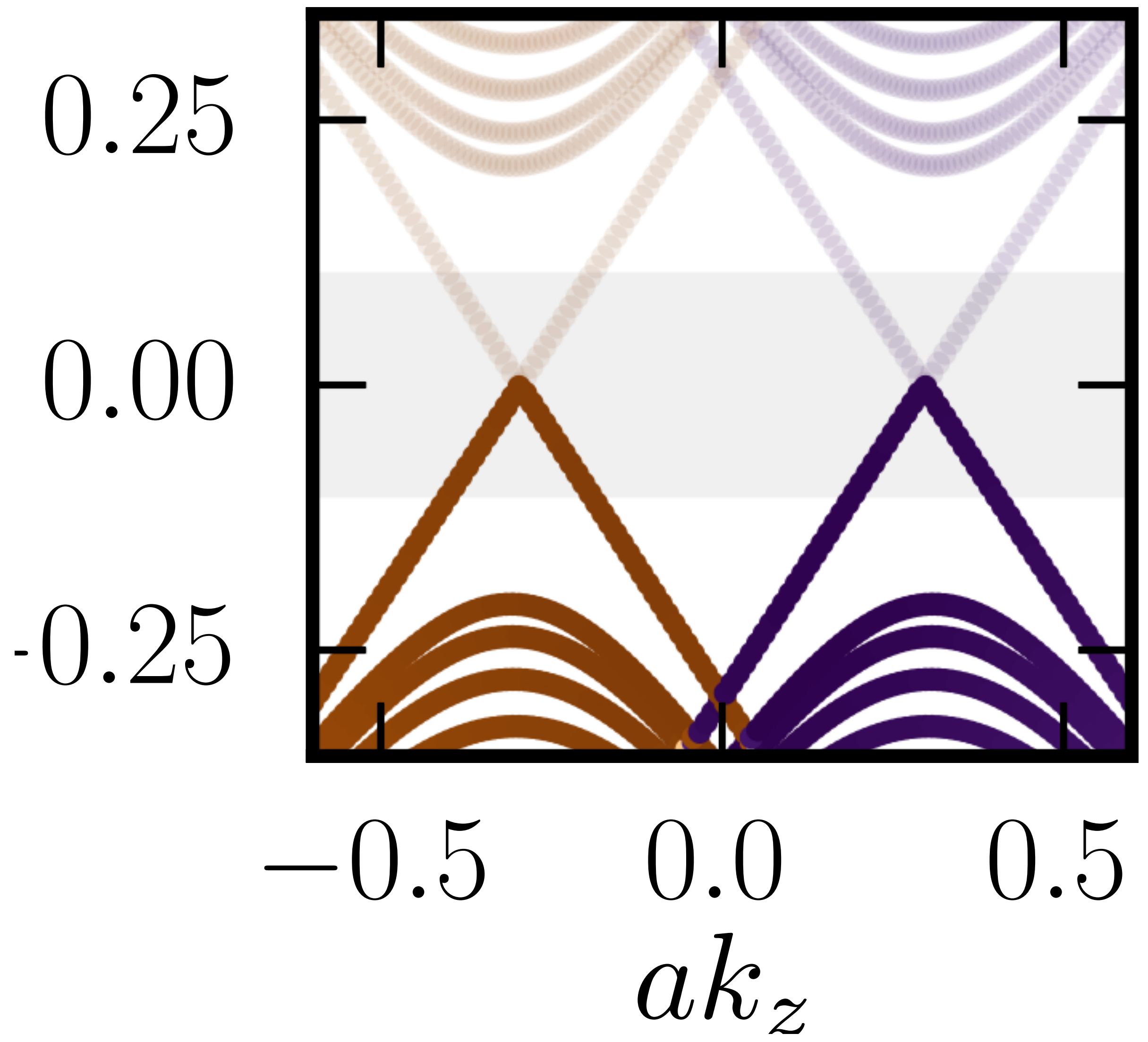


1) Choose a pseudo-field profile

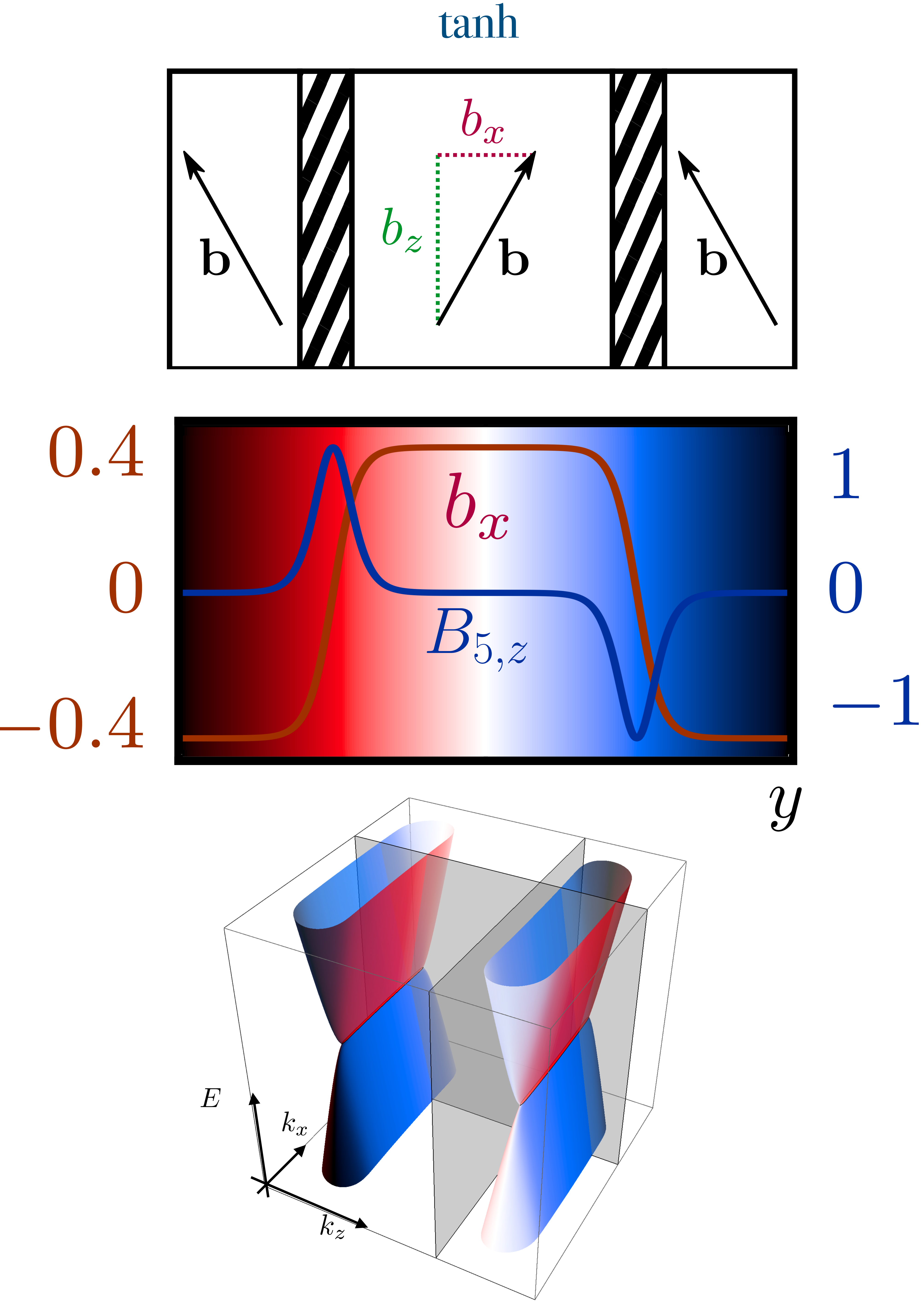


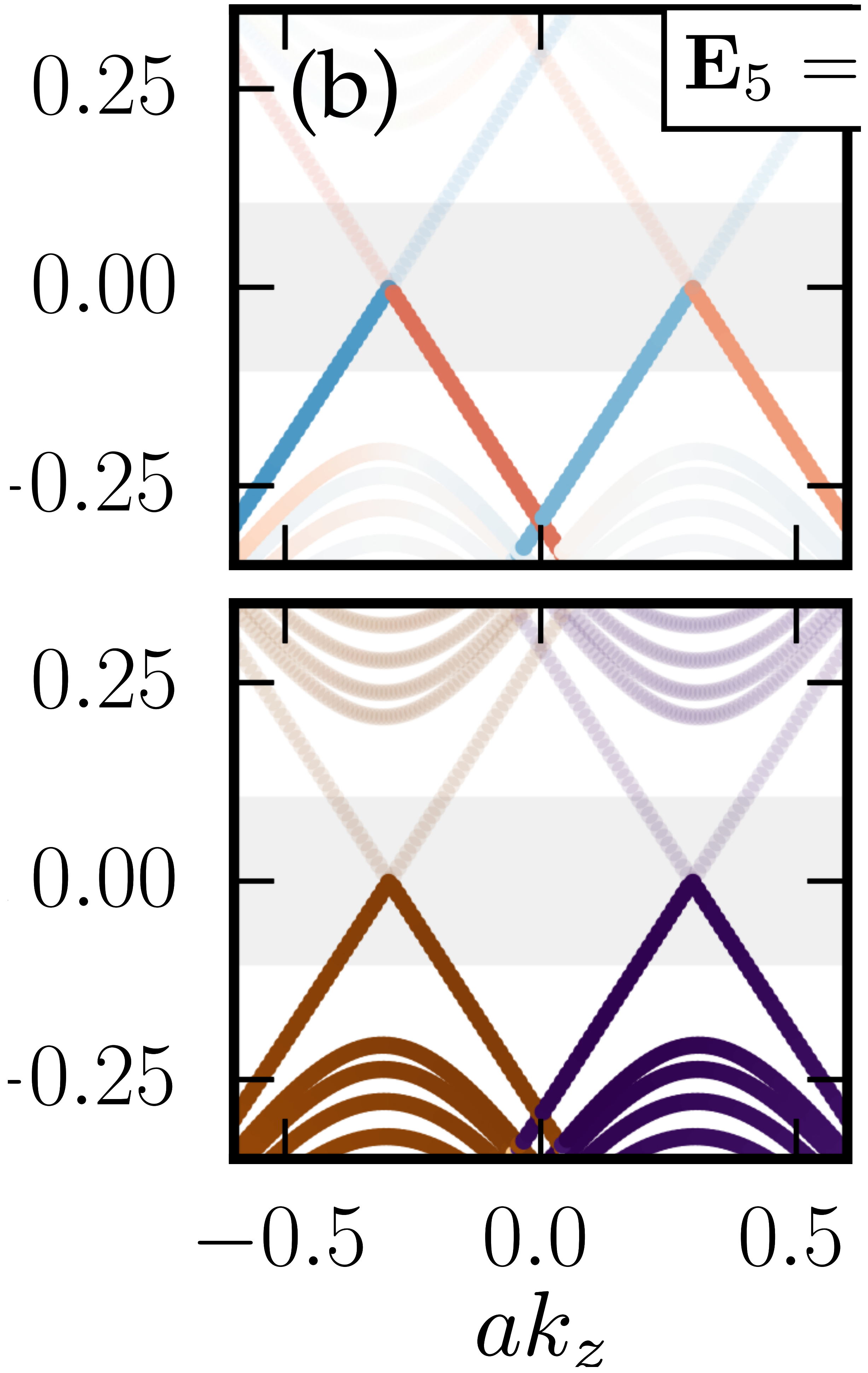
1) Choose a pseudo-field profile



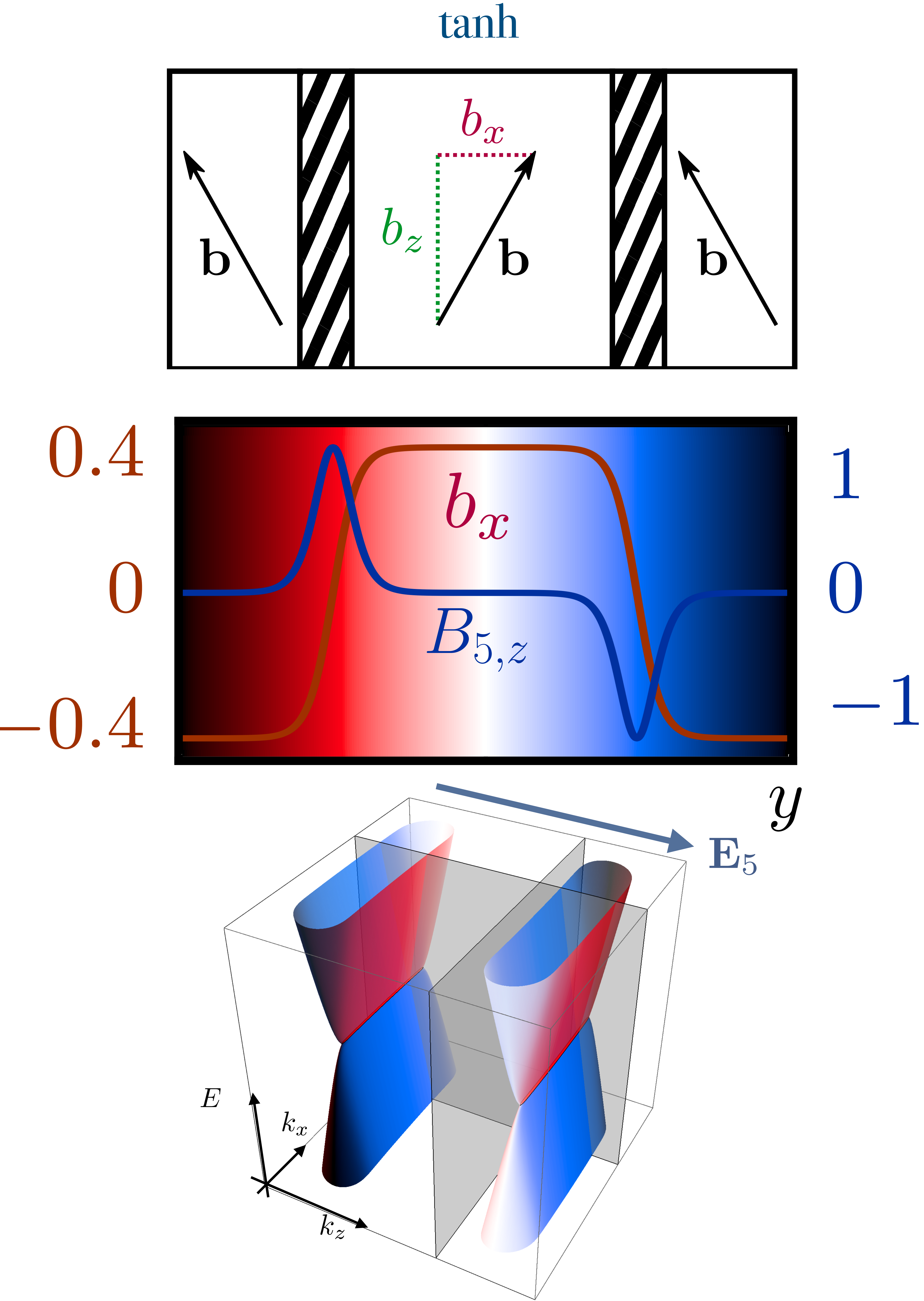


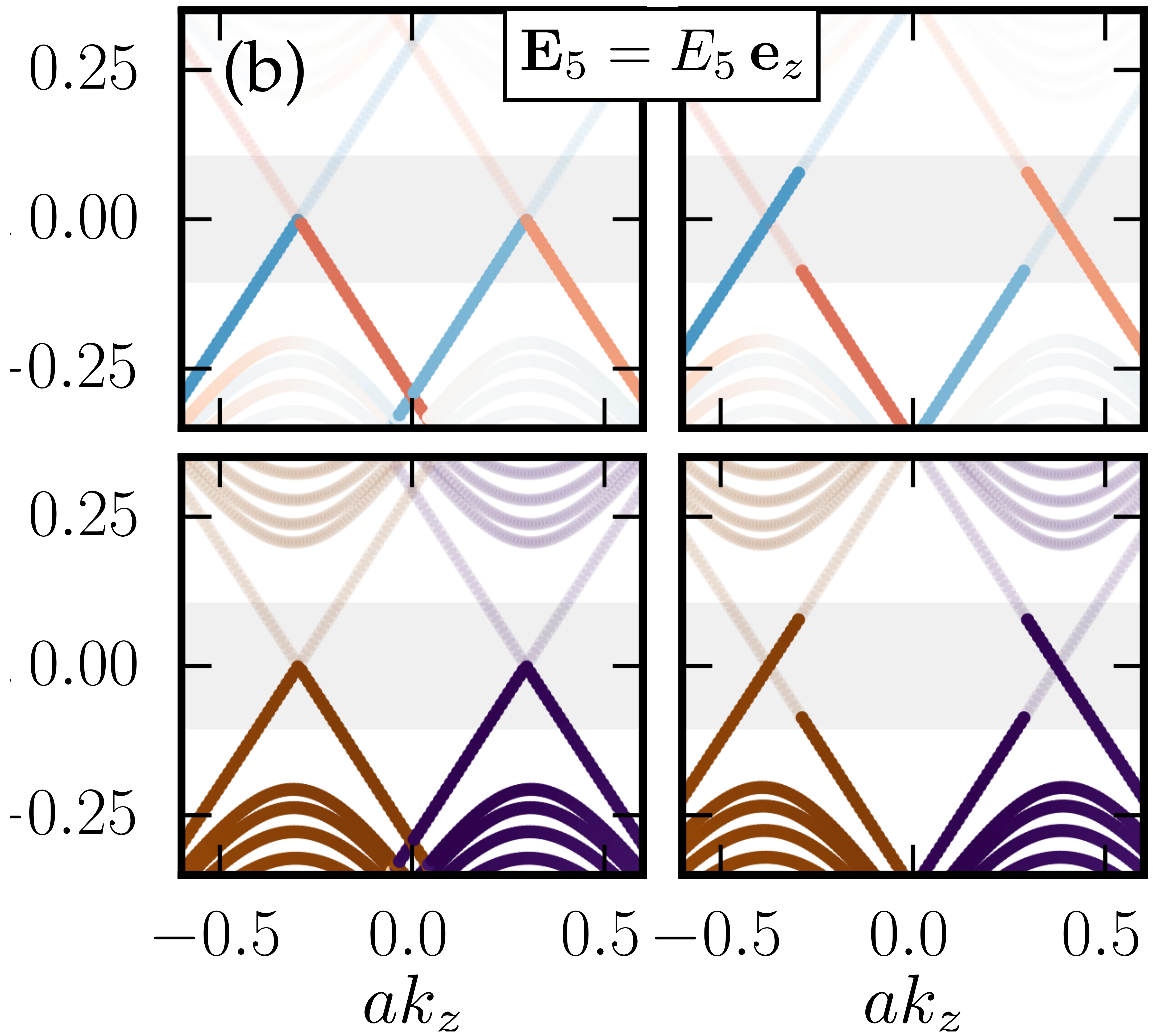
1) Choose a pseudo-field profile with well defined low energy chirality





1) Choose a pseudo-field profile with well defined low energy chirality





1) Choose a pseudo-field profile with well defined low energy chirality

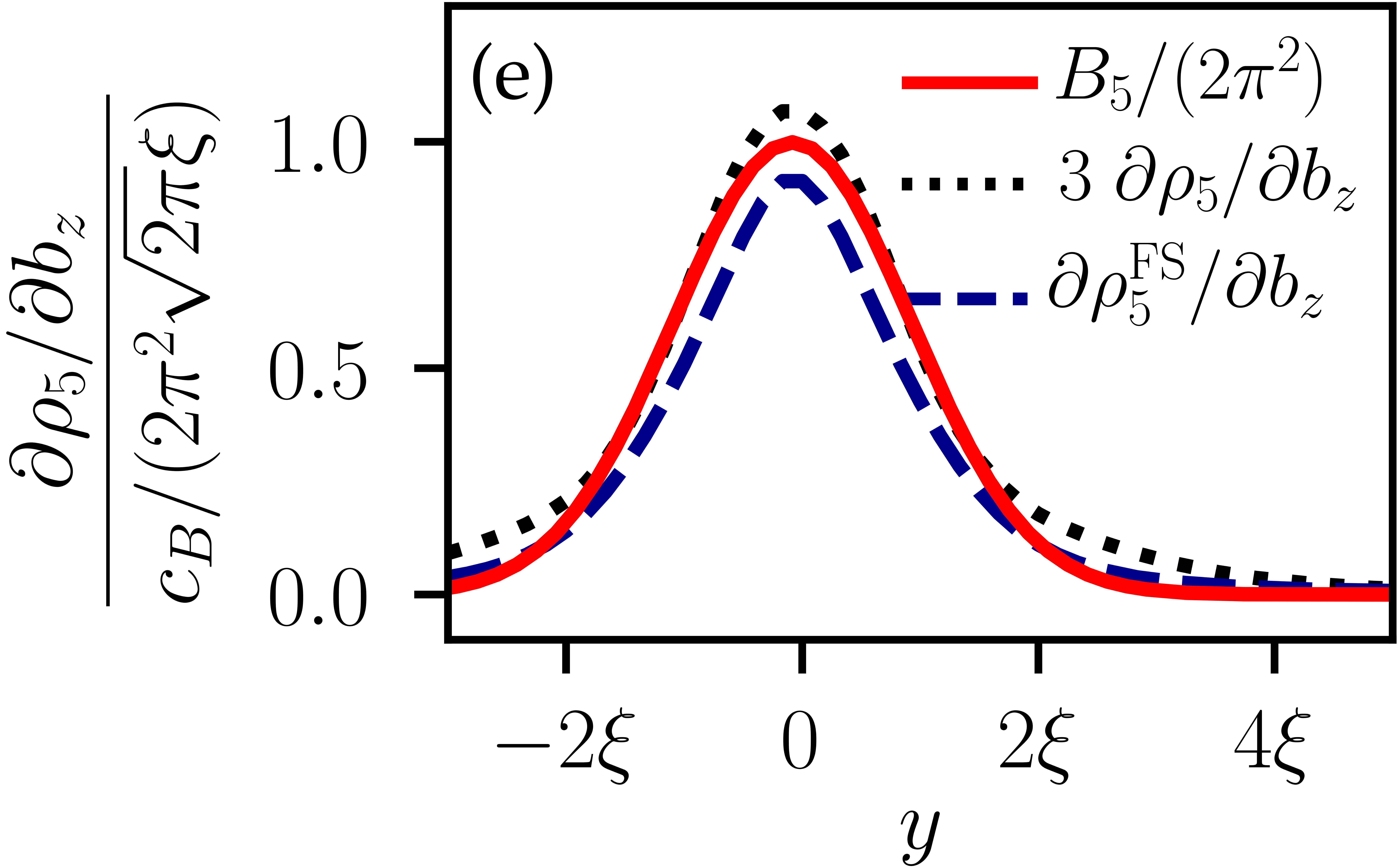
2) Calculate the Fermi surface contribution

Count charge transversing Fermi surface

3) Calculate full band contribution

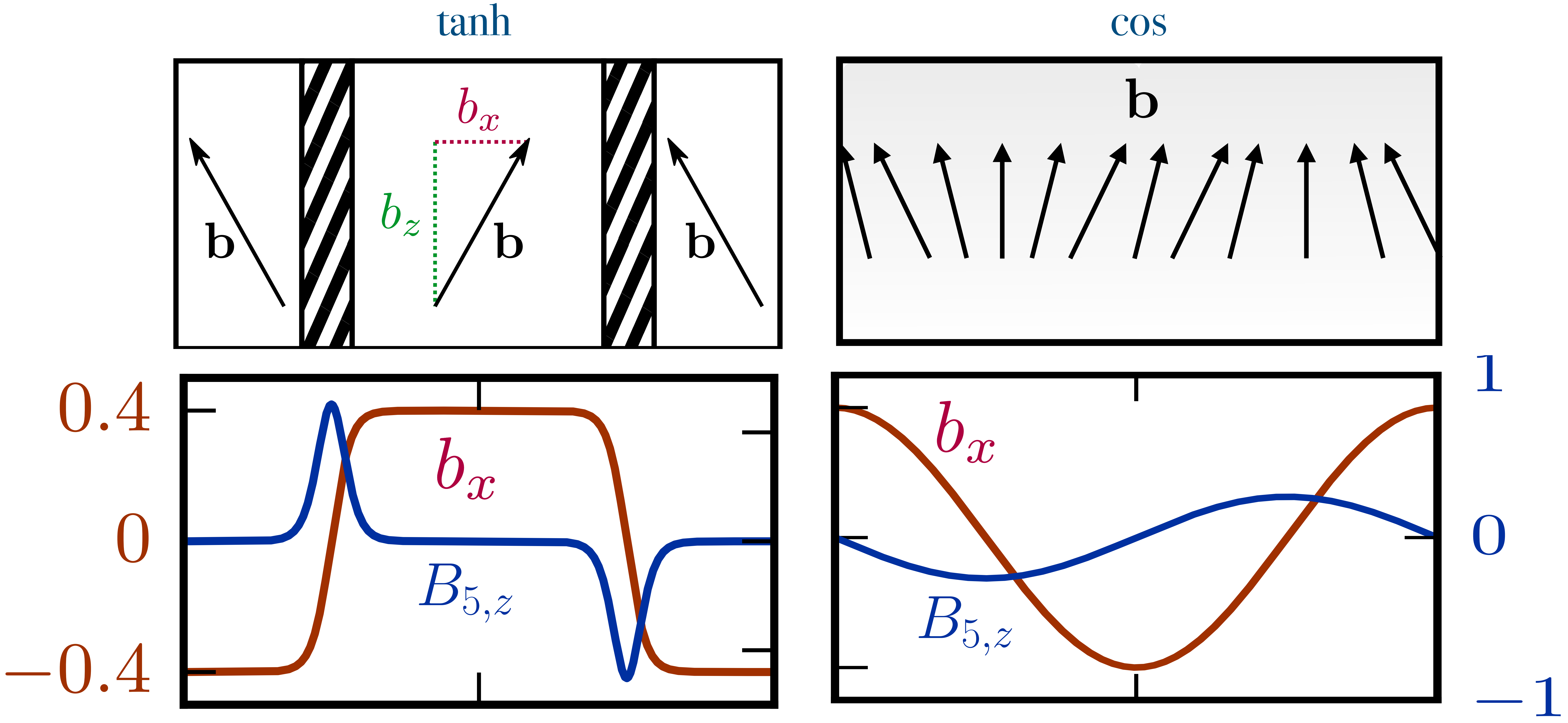
$$\rho_5(y) = \sum_{n \in \text{occ.}} \langle \psi_n(y) | \gamma^5 | \psi_n(y) \rangle$$

Fermi surface contribution = 3 Total band contribution



Corrections to the field theory

Length scales matter



Consistent vs covariant anomaly is well defined if

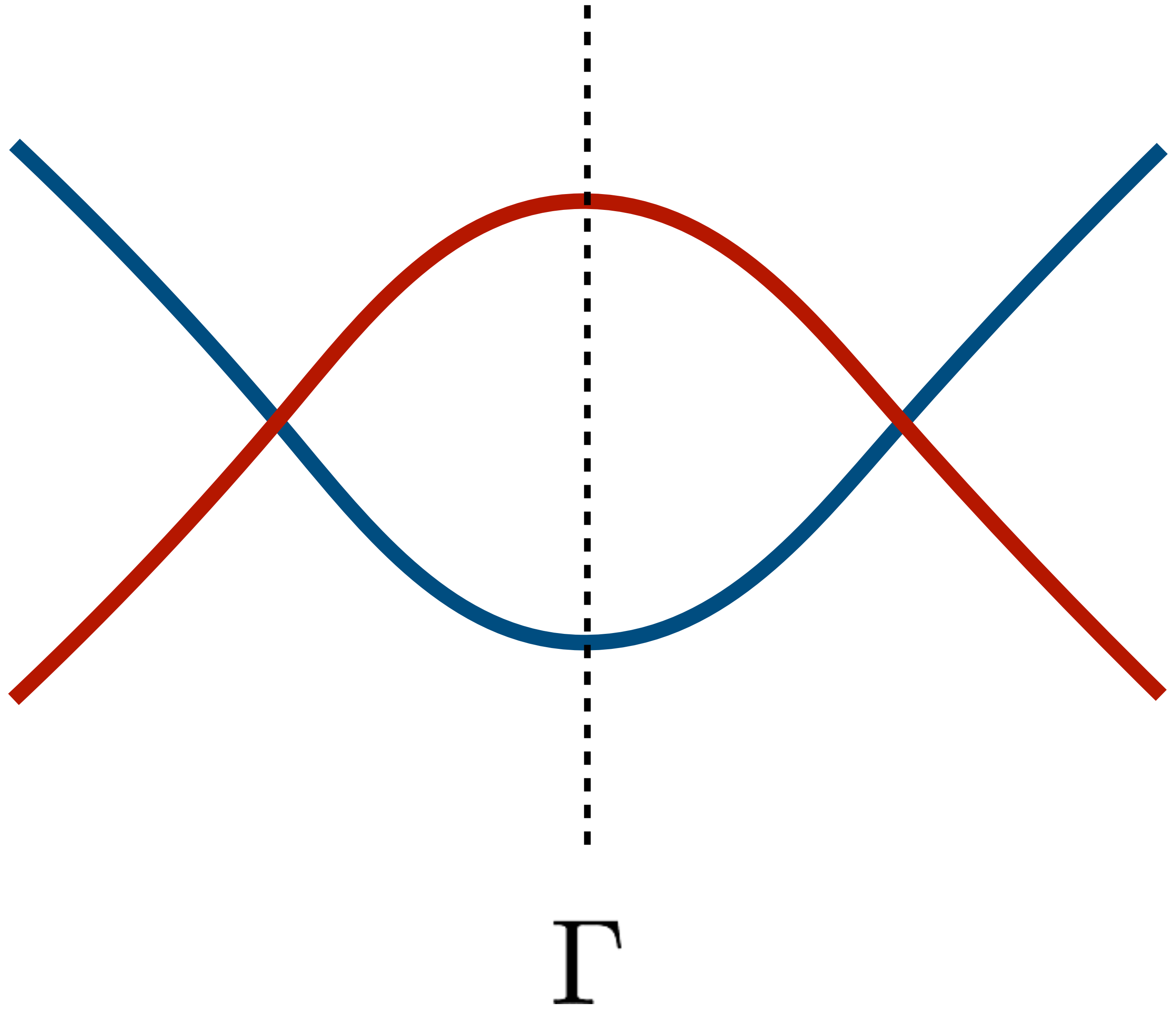
$$L \gg \xi \gg a$$

✓

✗

Lattice scales matter: a question for field theorists

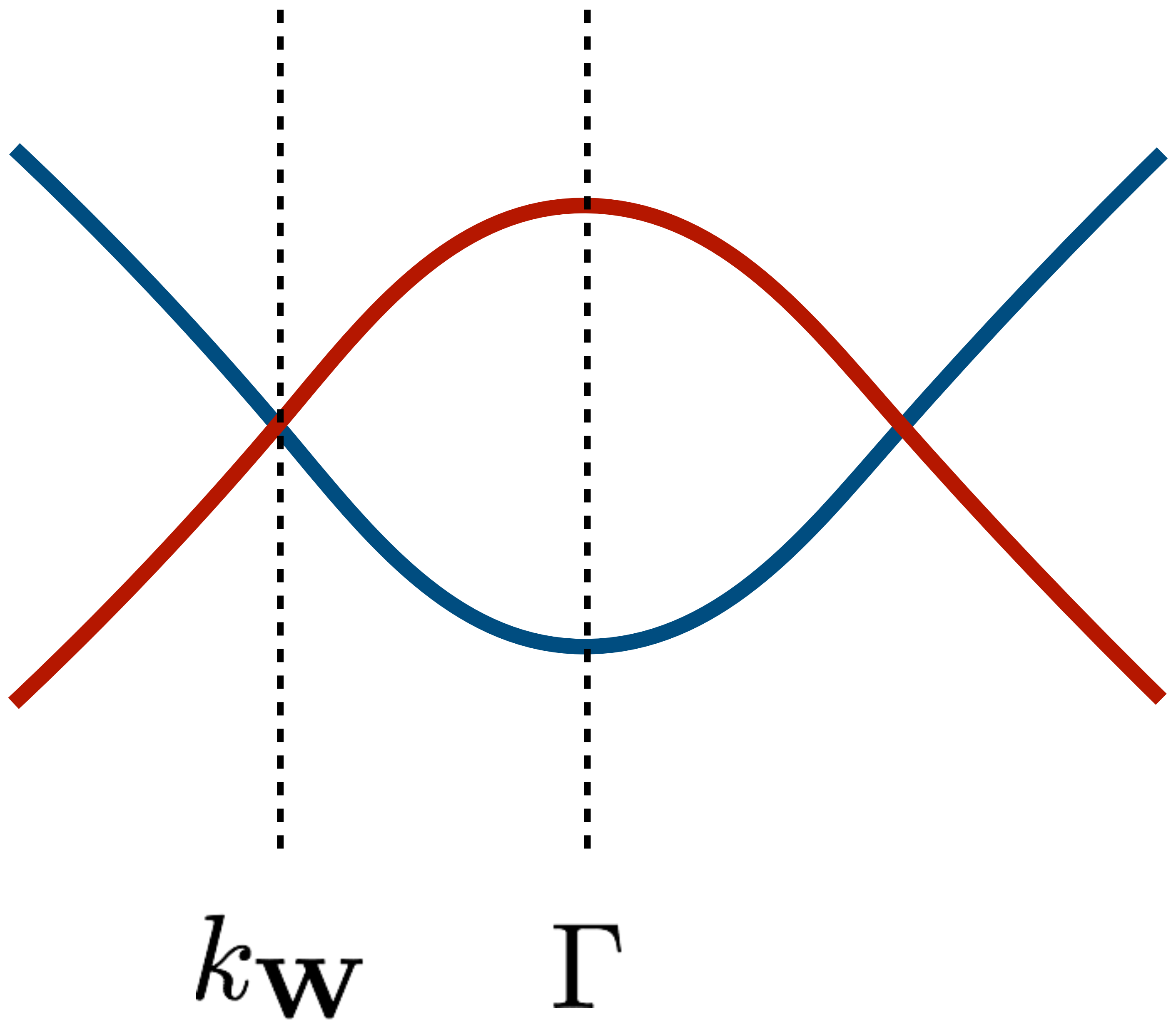
$$\partial_\mu j_5^\mu = \frac{1}{2\pi^2}\sqrt{1-\frac{m^2}{b^2}}\,\mathbf{E}\cdot\mathbf{B}.$$



$$M_{\mathbf{k}} = m + t\sum_i (1-\cos k_i),$$

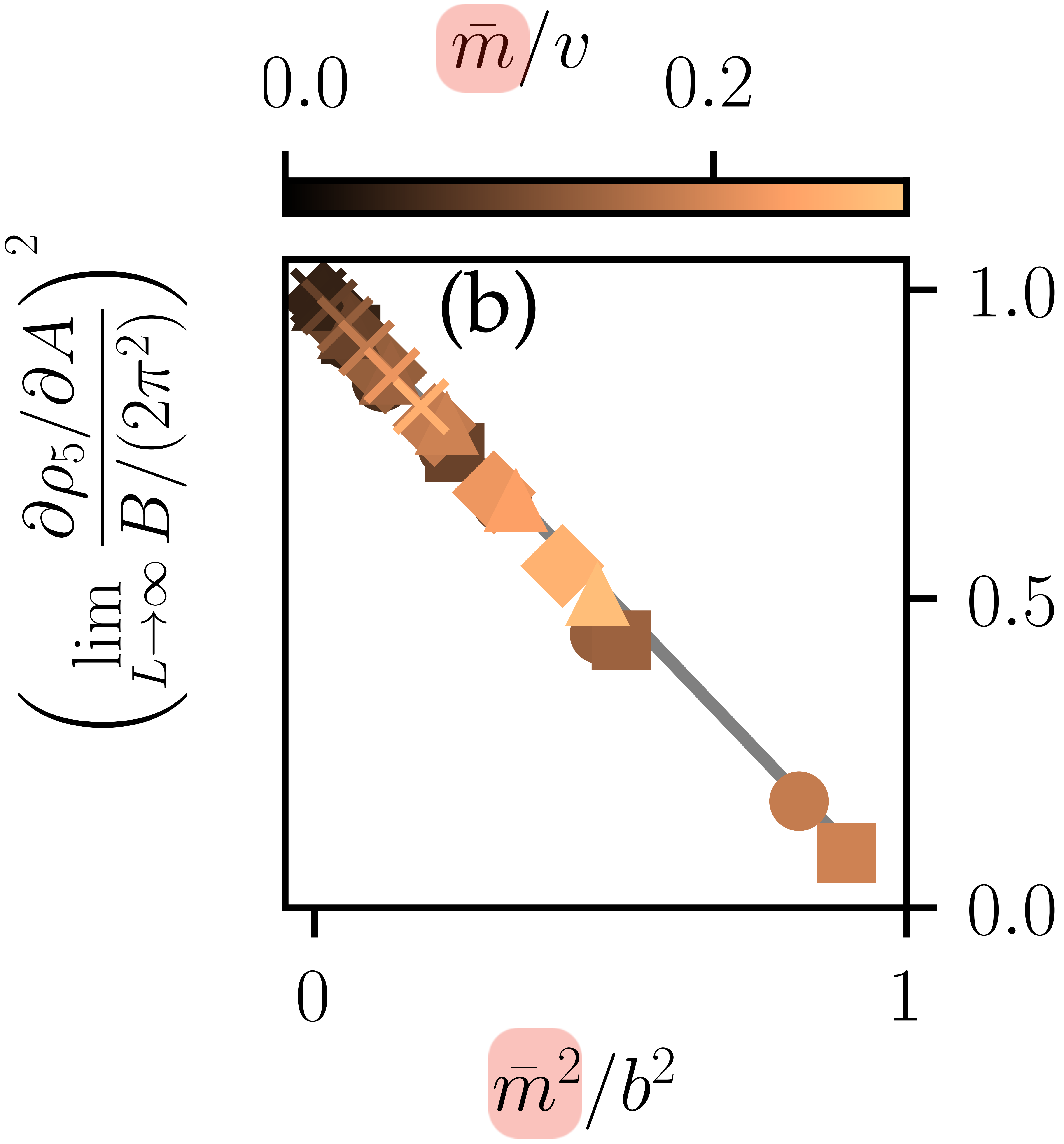
Lattice scales matter: a question for field theorists

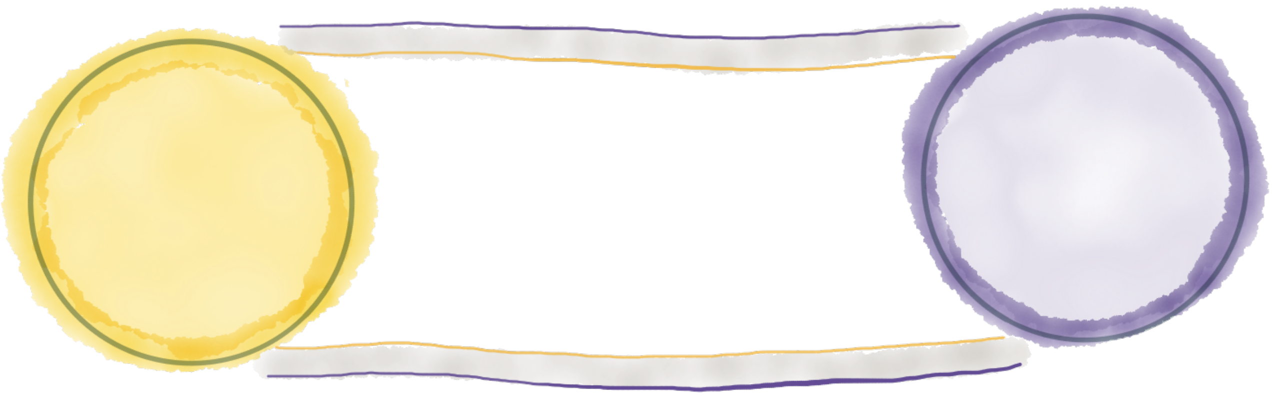
$$\partial_\mu j_5^\mu = \frac{1}{2\pi^2}\sqrt{1-\frac{\bar{m}^2}{b^2}}\,\mathbf{E}\cdot\mathbf{B}$$



$$\bar{m} = M_{\mathbf{k_W}}$$

$$M_{\mathbf{k}} = m + t\sum_i (1-\cos k_i),$$



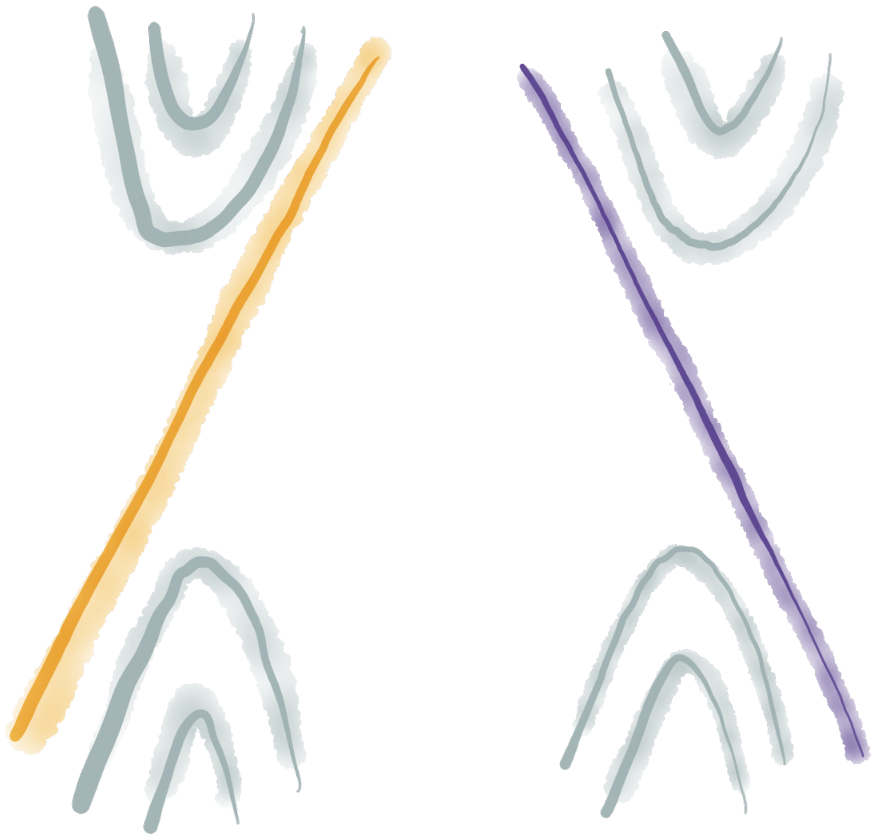


0th pseudo-Landau levels

Fermi arcs

vehicles of **covariant** anomaly

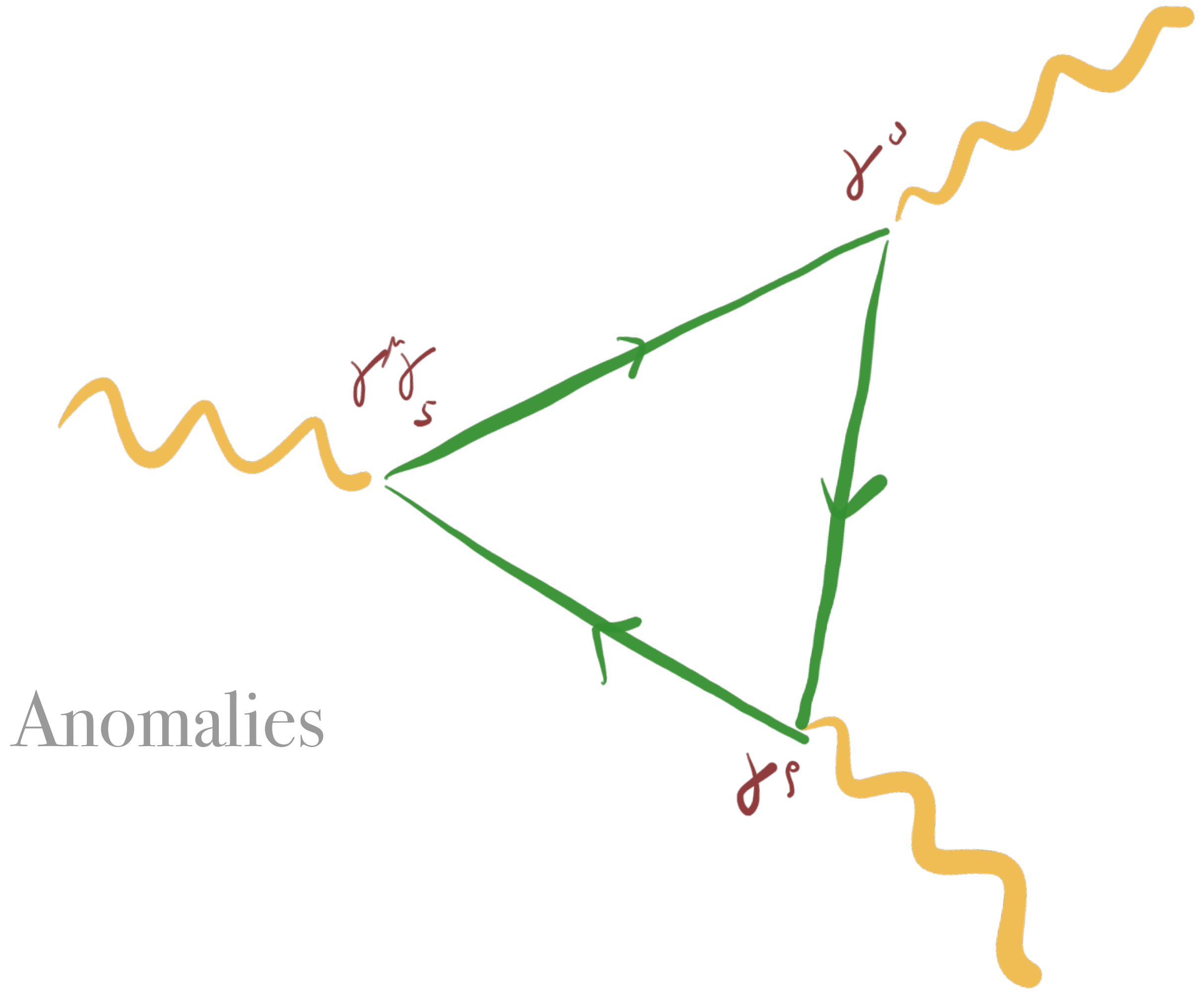
🟨 🟩



Landau levels

■ Consistent = 1/3 covariant

$$L \gg \xi \gg a$$



Anomalies

Cond-mat /

AMO systems /

Acoustic Weyls

B_5

Strain Fields

E_5

Anomalous conservation laws

From lattice to field theory

Topological insulator

model

$$\mathcal{H} = v\left(\sin k_y\sigma_x - \sin k_x\sigma_y\right)\tau_z + v\sin k_z\tau_y + m\tau_x \; + t\sum_i\left(1 - \cos k_i\right)\tau_x$$

$$+ v\sum_\mu u^\mu b_\mu,$$

Weyl node separation at low energies

Vazifeh, Franz PRL (2014)

chiral density

$$\langle\gamma_5\rangle = \sum_{n\in\text{occ.}}\langle\psi_n(y)|\gamma^5|\psi_n(y)\rangle \qquad \rho_5(y) = \sum_{n\in\text{occ.}}\langle\psi_n(y)|\gamma^5|\psi_n(y)\rangle$$

low energy field theory

$$\mathcal{S} = \int d^4x\,\bar{\psi}\left[\gamma^\mu\left(i\partial_\mu - eA_\mu + b_\mu\gamma^5\right) - m\right]\psi$$