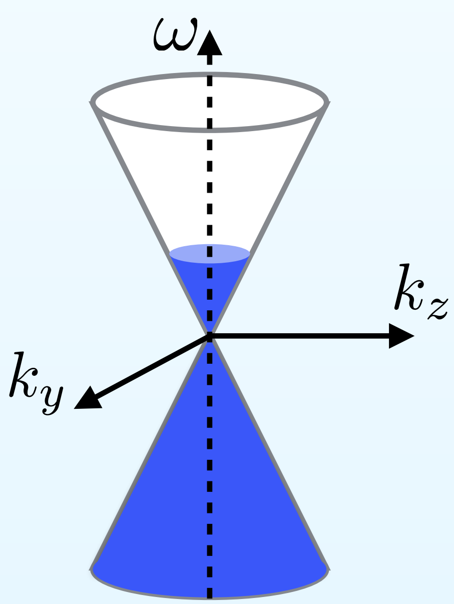
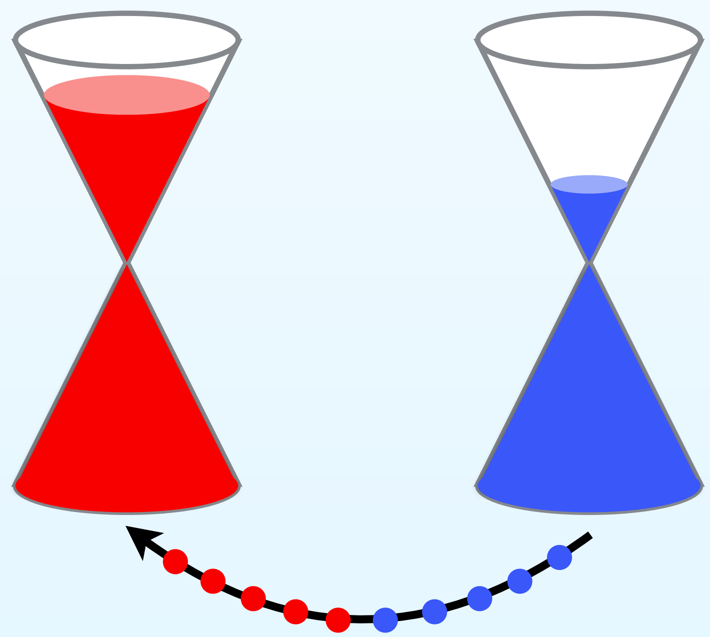
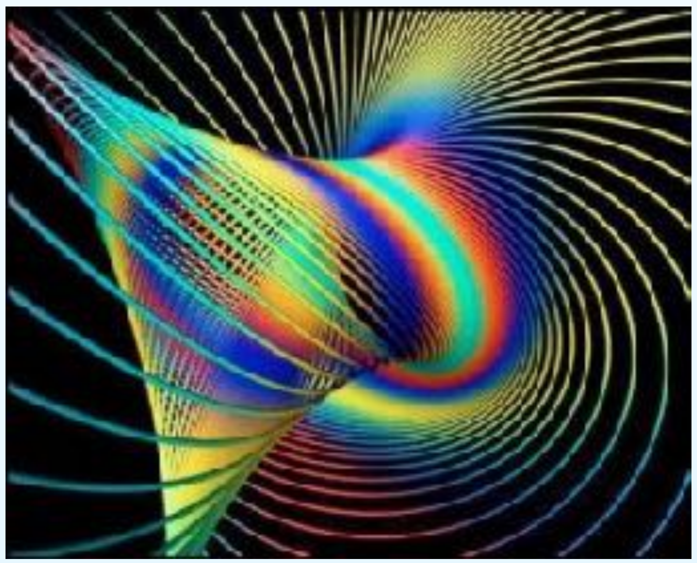
Mixed axial-torsional anomaly in Weyl semimetals







In collaboration with:

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Emil Bergholtz, SU

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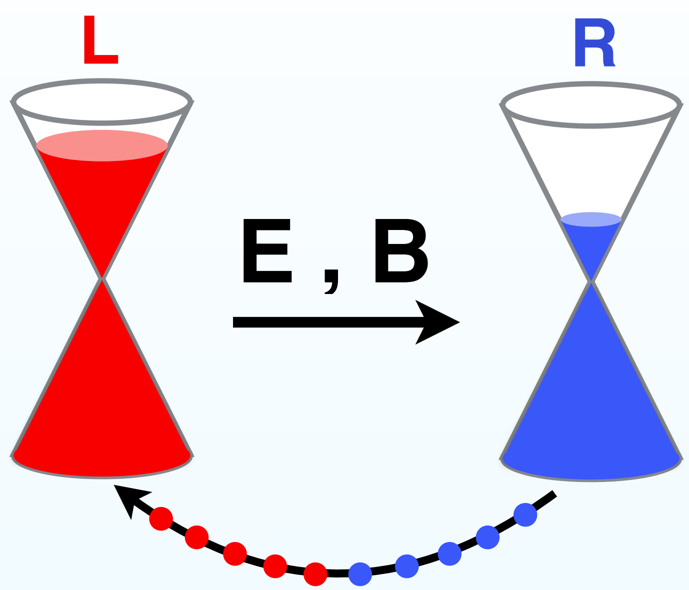
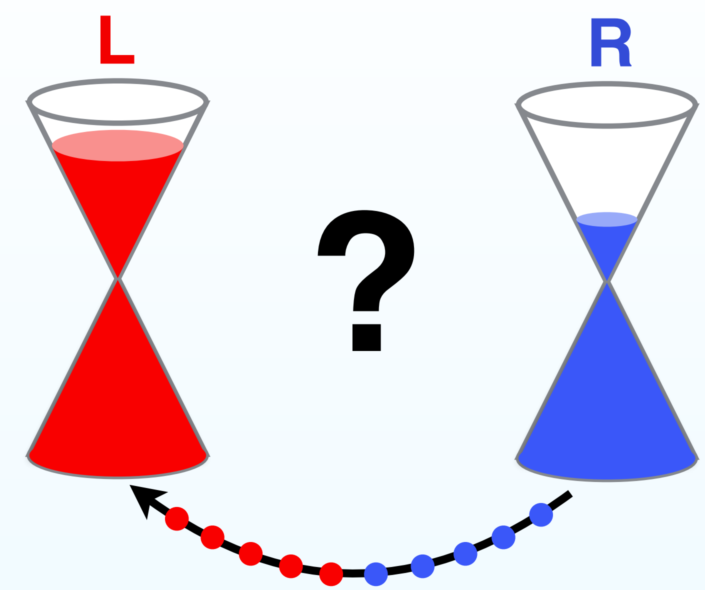
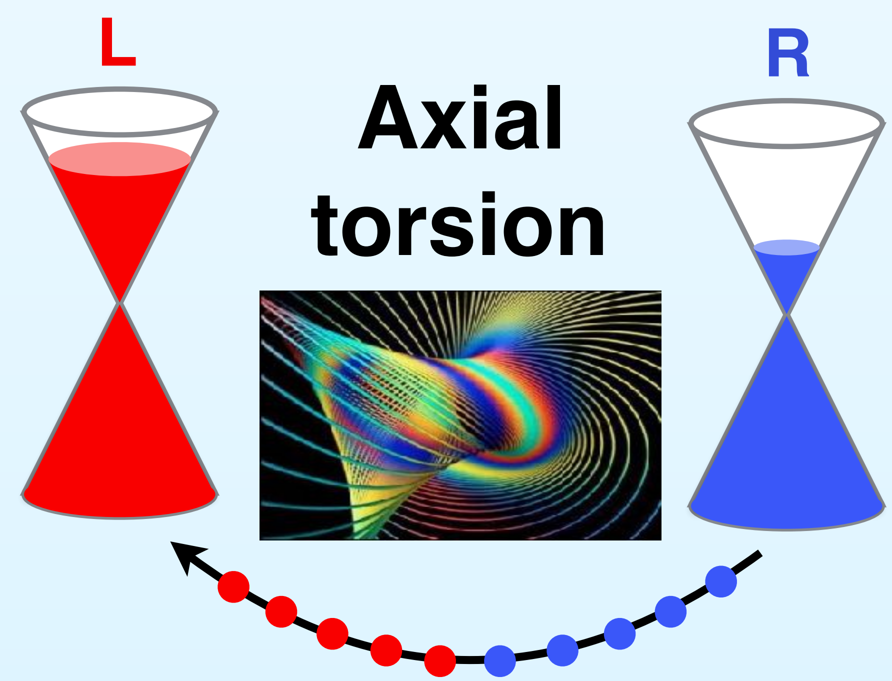
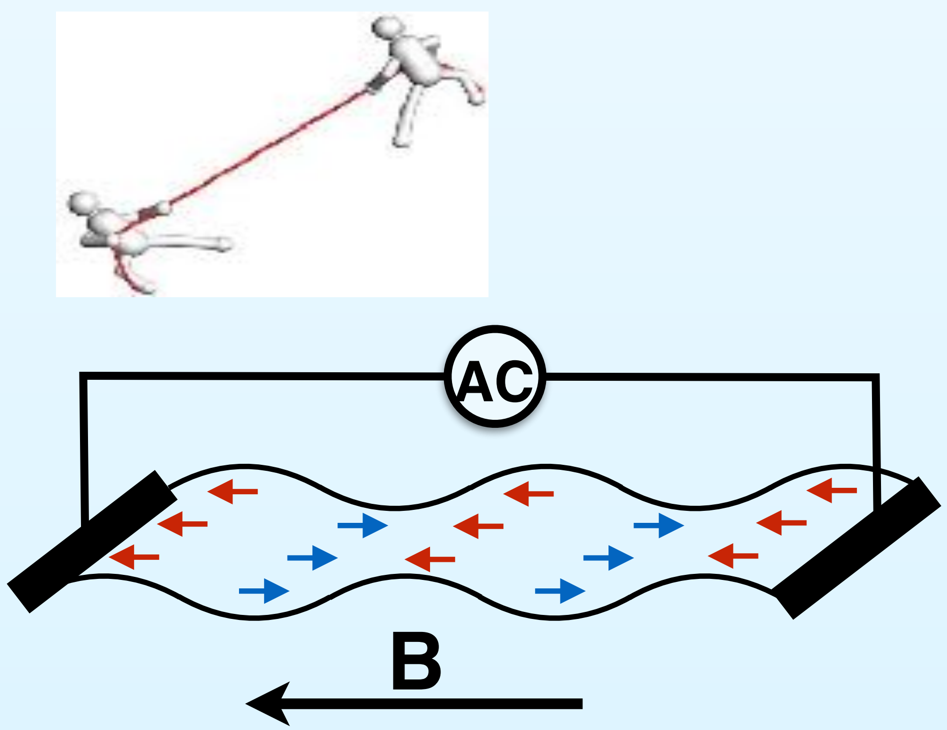
Yago Ferreiros

Motivation

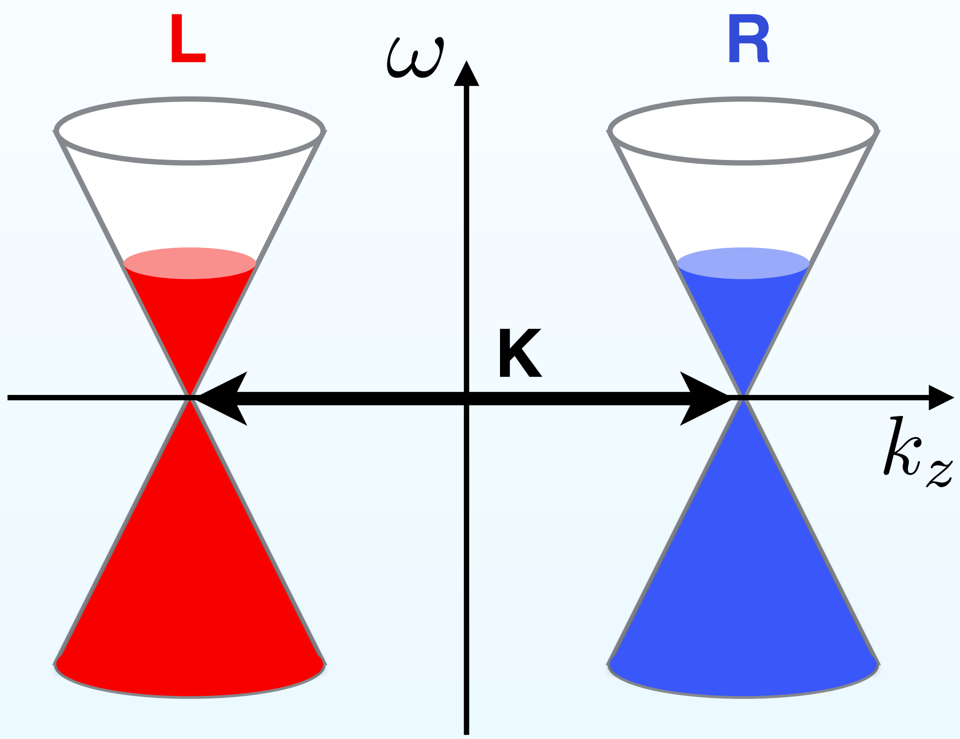




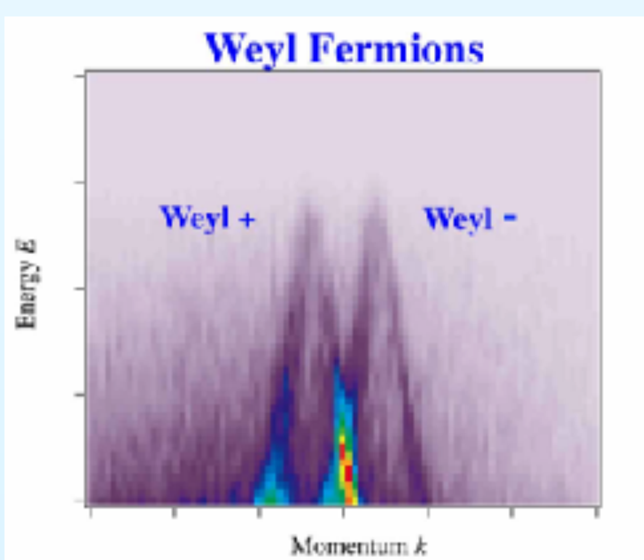




Weyl semimetals and the axial anomaly



$$\mathcal{H}_{L,R} = \pm \overline{\Psi}_{L,R}\, i\, \vec{\sigma} \cdot \vec{\partial}\, \Psi_{L,R}$$

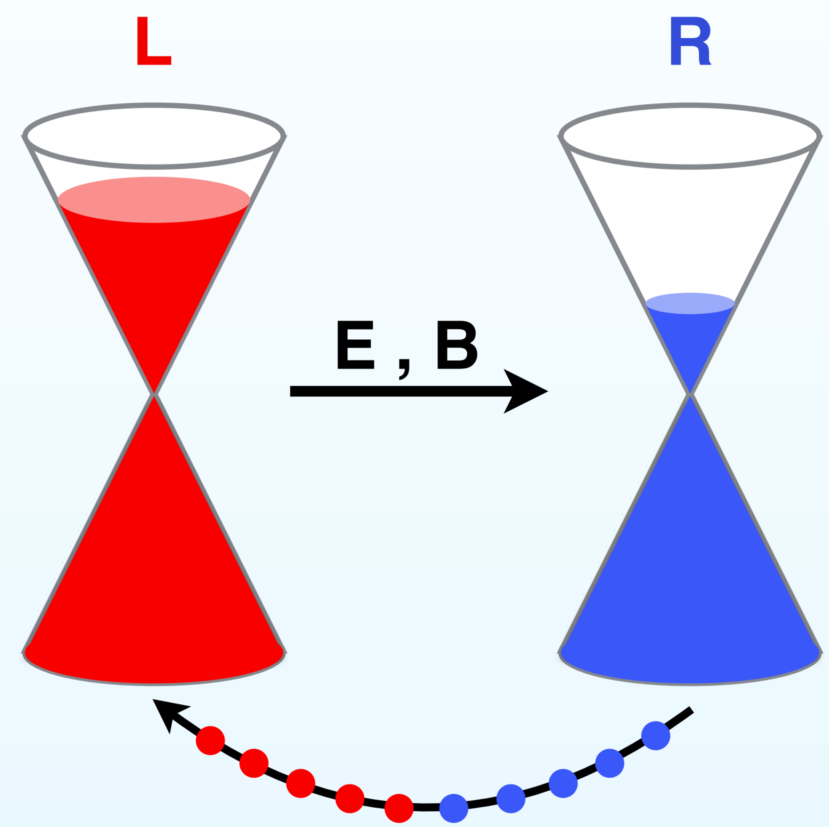


TaAs

NbAs

NbP

TaP



$$\dot{\rho}_5 + \vec{\partial} \cdot \vec{J}_5 \propto \vec{E} \cdot \vec{B}$$

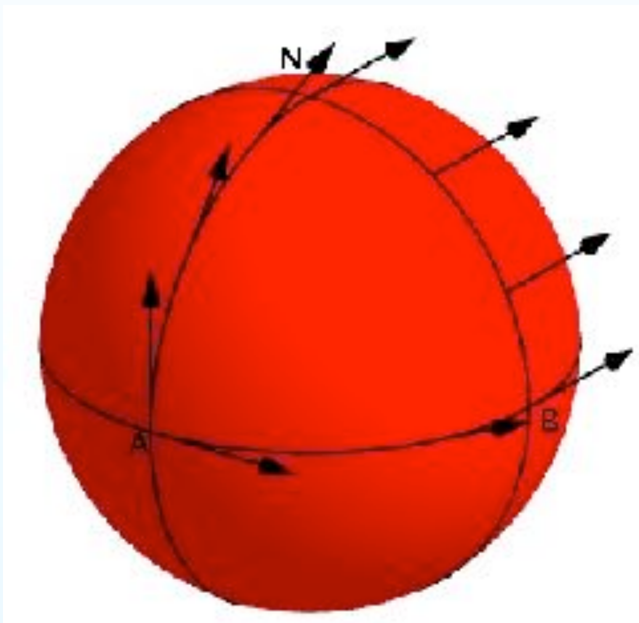
$\rho_5 = (\rho_L - \rho_R)/2$

$\vec{J}_5 = (\vec{J}_L - \vec{J}_R)/2$

$$\boxed{\dot{\rho}_5 \propto \vec{E} \cdot \vec{B}}$$

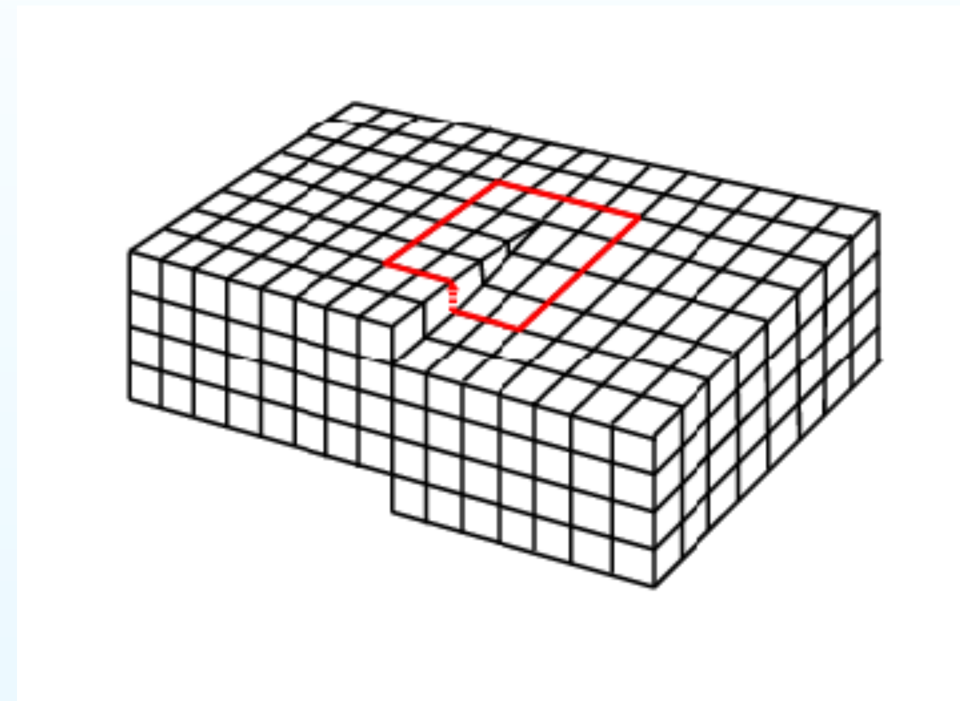
Torsion

Curvature:

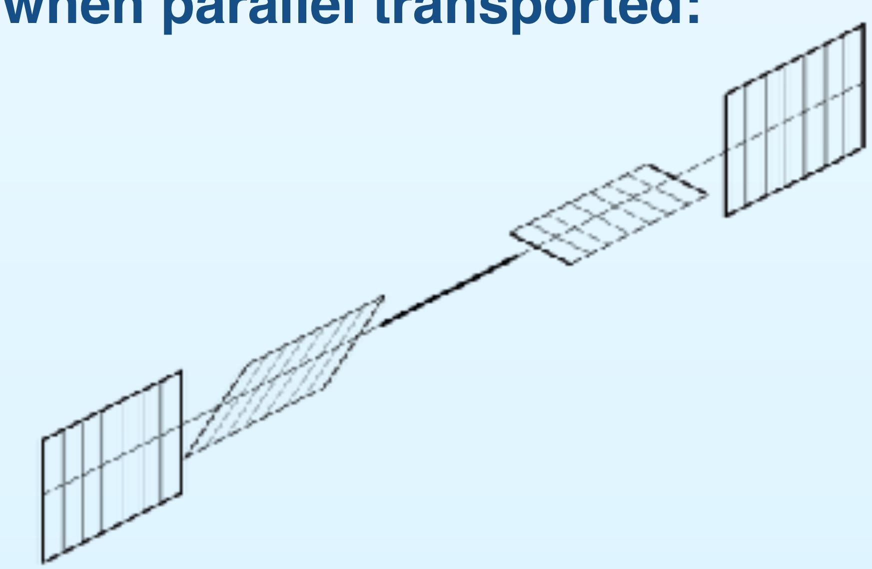


Describes how tangent spaces roll when parallel transported:

Torsion:



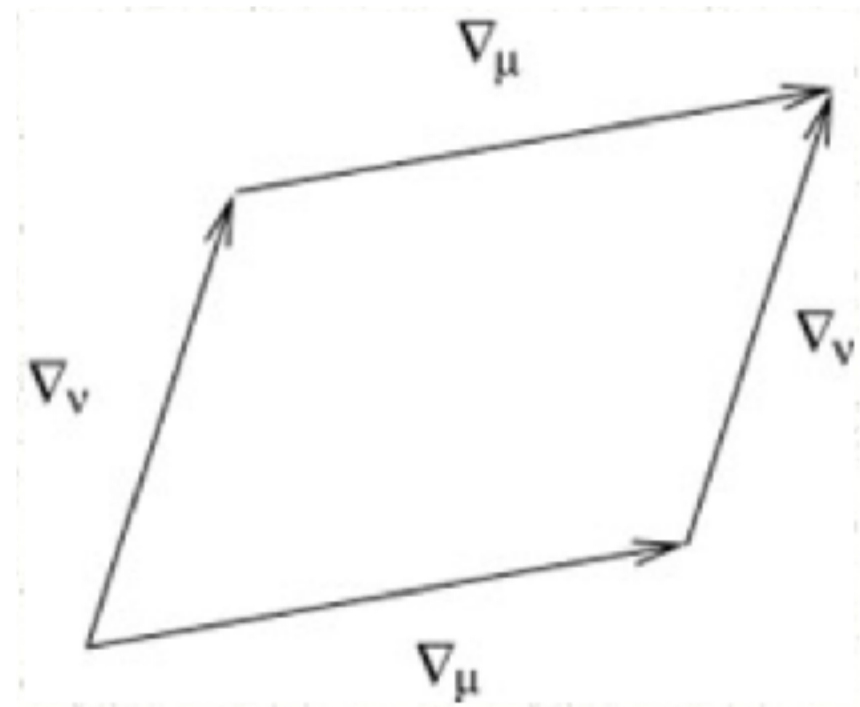
Describes how tangent spaces twist when parallel transported:



A bit more formal

Parallel transport of a vector $\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda$

connection



Commutator of covariant derivatives

$$[\nabla_\mu, \nabla_\nu] V^\rho = R^\rho_{\sigma\mu\nu} V^\sigma + T_{\mu\nu}{}^\lambda \nabla_\lambda V^\rho$$

Curvature (Riemann) tensor

Rotation

Torsion tensor

Displacement

Coupling of spinors to geometry: going beyond the metric

- **Deformations of a crystal with one orbital per site**

$$u_i \rightarrow u_{ij} = \partial_i u_j + \partial_j u_i \rightarrow g_{ij} \quad \text{metric}$$

- **Deformations of a crystal with more than one orbital per site**

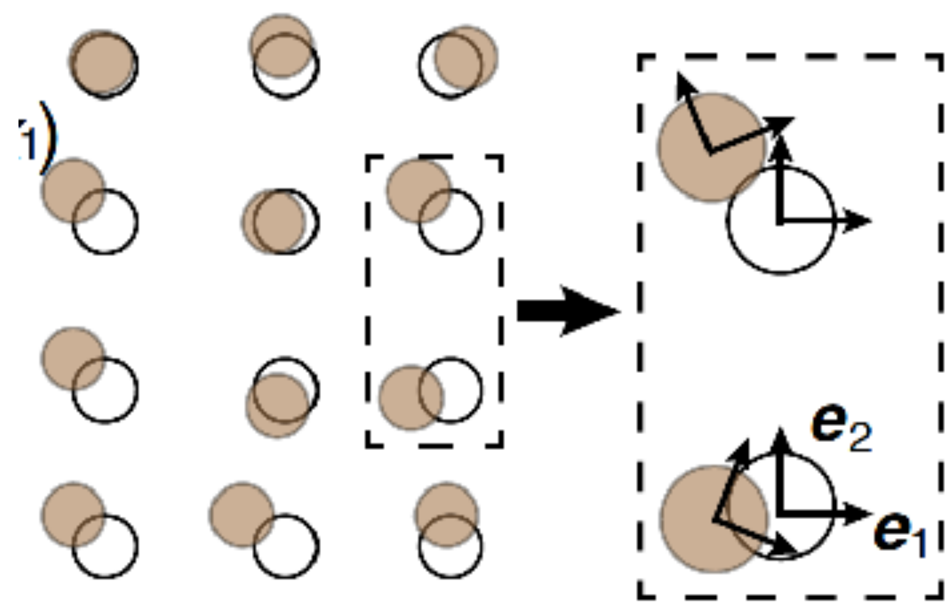
Set of 4 orthonormal basis vectors at each point $\underline{e}_a^{\mu}$

and their inverse e_{μ}^{a}

(co)frame field

Orthonormal basis $g_{\mu\nu}\underline{e}_a^{\mu}\underline{e}_a^{\nu} = \eta_{ab}$

The metric $g_{\mu\nu} = \eta_{ab} e_{\mu}^{a} e_{\nu}^{b}$



Local rotations, or local orbital deformations, are not captured by the metric

Metric not enough to capture changes in the background geometry in the presence of spinors, the frame is needed

The torsion tensor is just the field strength of the frame field (in the absence of curvature)

information not registered in the metric

Torsion tensor: $T^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu$

Analogy with electromagnetic field →

$$\vec{\mathcal{E}}^a = \partial_t \vec{e}^{\,a} - \vec{\partial} e^a_t$$

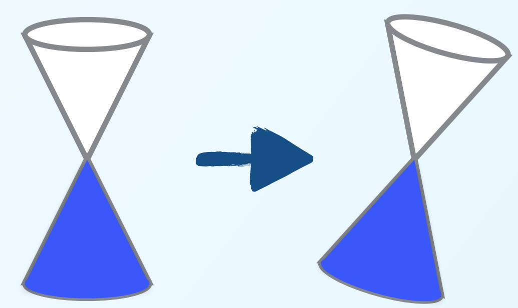
$$\vec{\mathcal{B}}^a = \vec{\partial} \times \vec{e}^{\,a}$$

Torsion in a Weyl semimetal

Coupling of the background geometry to Weyl fermions (vanishing curvature):

$$\mathcal{H}_{L,R} = \bar{\Psi} i \left(\underline{e}_t^{\,i} \pm \sigma^j \underline{e}_j^{\,i} \right) \hbar v \partial_i \Psi$$

$\underline{e}_t^{\,i} v$ → **Tilt:**



$\underline{e}_j^{\,i} v$ → **Anisotropic Fermi velocity**

Strain induced tilt generates axial torsion

Deformations of a continuous medium: $\underline{e}_i^j = \delta_i^j - \partial^j u_i$

tilt $\underline{e}_t^j = \partial^j u_t$ **tilt = 0**

Tight-binding: strain effects beyond the above

Elastic vector fields: $A_i^{(L,R)} \sim \pm K_j u_i^j$ **(proven t.b.)** A. Cortijo, YF, K. Landsteiner and M. A. H. Vozmediano, *PRL* 2015

Tilt: $e_i^{t(L,R)} \sim \pm K_j u_i^j$ **(expected t.b.)** V. Arjona and M. A. H. Vozmediano, *PRB* 2018

Torsion from strain

$$\vec{e}^{\,t} = (\vec{e}^{\,t(L)} + \vec{e}^{\,t(R)})/2 = 0 \qquad \vec{e}_5^{\,t} = (\vec{e}^{\,t(L)} - \vec{e}^{\,t(R)})/2 \sim K_j u_i^j$$

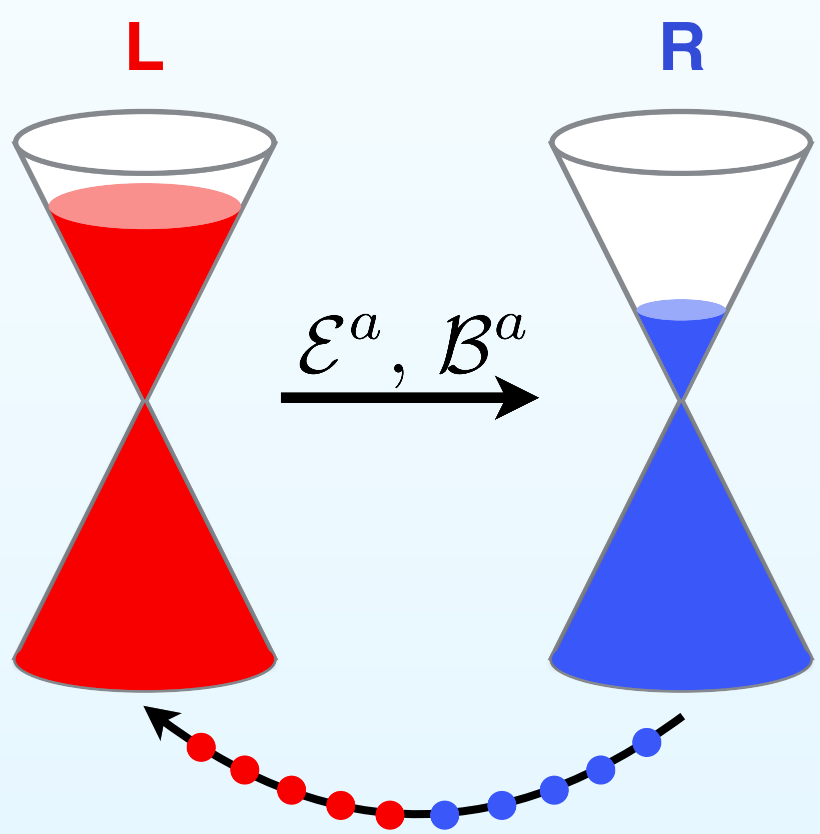
$$\vec{\mathcal{E}}_5^t = \partial_t \vec{e}_5^{\,t} \qquad \vec{\mathcal{B}}_5^t = \vec{\partial} \times \vec{e}_5^{\,t}$$

Axial torsion!!

No counterpart at a fundamental level

Mixed axial-torsional anomaly

With torsion

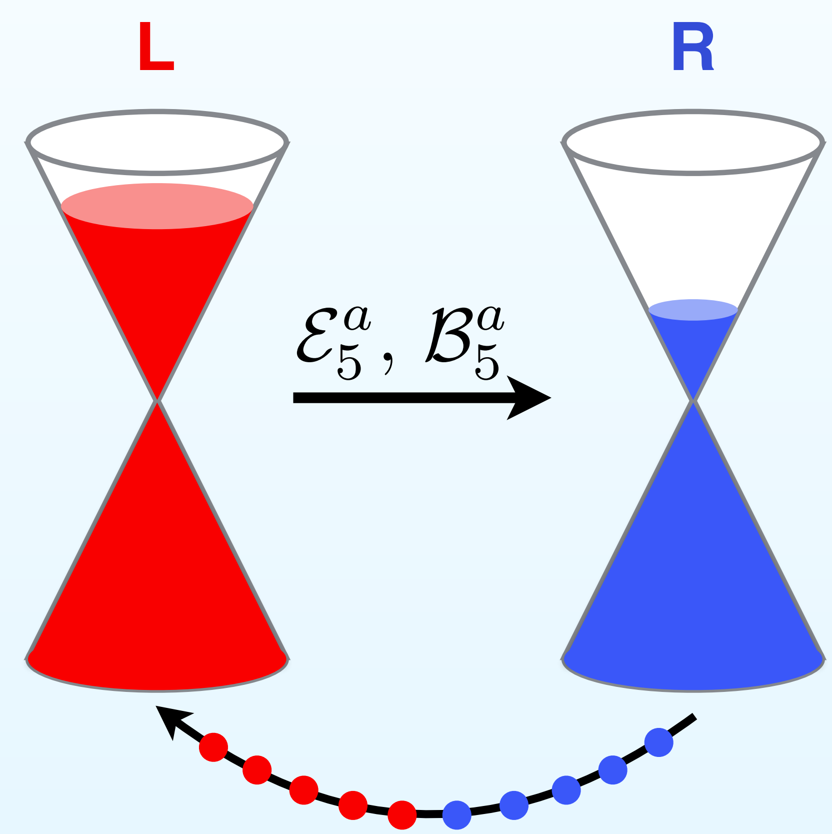


$$\dot{\rho}_5 \propto \frac{1}{l^2}\,\vec{\mathcal{E}}^a \cdot \vec{\mathcal{B}}^b\,\eta_{ab}$$

O. Chandía, J. Zanelli, *PRD 1997*

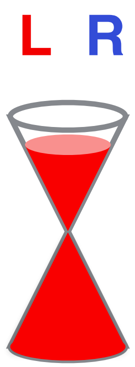
H. T. Nieh, M. L. Yan, J. Math. Phys. *1982*

With axial torsion



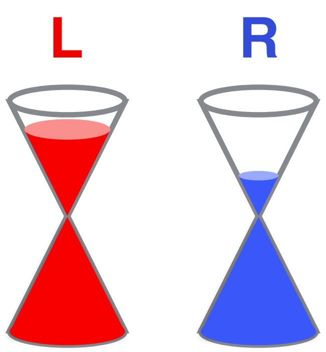
$$\dot{\rho}_5 \propto \frac{1}{l_5^2}\,\vec{\mathcal{E}}^a_5 \cdot \vec{\mathcal{B}}^b_5\,\eta_{ab}$$

YF, Y. Kedem, E. Bergholtz, J. H. Bardarson, *PRL 2019*



$$\vec{\mathcal{E}} = (\vec{\mathcal{E}}_L + \vec{\mathcal{E}}_R)/2$$
$$\vec{\mathcal{E}}_5 = (\vec{\mathcal{E}}_L - \vec{\mathcal{E}}_R)/2$$

$$\partial_\mu J^\mu_{L,R} \propto \pm \frac{1}{2l^2}\, \vec{\mathcal{E}}^a_{L,R} \cdot \vec{\mathcal{B}}^b_{L,R}\, \eta_{ab}$$



$$\partial_\mu J^\mu_5 = \partial_\mu (J^\mu_L - J^\mu_R) \propto \frac{1}{l^2}\, (\vec{\mathcal{E}}^a \cdot \vec{\mathcal{B}}^b + \vec{\mathcal{E}}^a_5 \cdot \vec{\mathcal{B}}^b_5)\eta_{ab}$$

$$\partial_\mu J^\mu = \partial_\mu (J^\mu_L + J^\mu_R) \propto \frac{1}{l^2}\, (\vec{\mathcal{E}}^a \cdot \vec{\mathcal{B}}^b_5 + \vec{\mathcal{E}}^a_5 \cdot \vec{\mathcal{B}}^b)\eta_{ab}$$

We impose charge conservation → there has to be an extra term in the effective action:

$$-\frac{1}{2l^2} \int dt d^3x\, \epsilon^{\mu\nu\rho\lambda} A_\mu e^{a,5}_\nu T^b_{\rho\lambda}\, \eta_{ab}$$

Symmetries allow yet another term

$$\frac{1}{\tilde{l}^2} \int dt d^3x\, \epsilon^{\mu\nu\rho\lambda} A^5_\mu e^{a,5}_\nu T^{b,5}_{\rho\lambda}\, \eta_{ab}$$

We have

$$\boxed{\partial_\mu J^\mu_5 \propto (\frac{1}{l^2} \vec{\mathcal{E}}^a \cdot \vec{\mathcal{B}}^b + \frac{1}{l_5^2} \vec{\mathcal{E}}^a_5 \cdot \vec{\mathcal{B}}^b_5)\eta_{ab}}$$

Regularization and relation to Hall viscosity

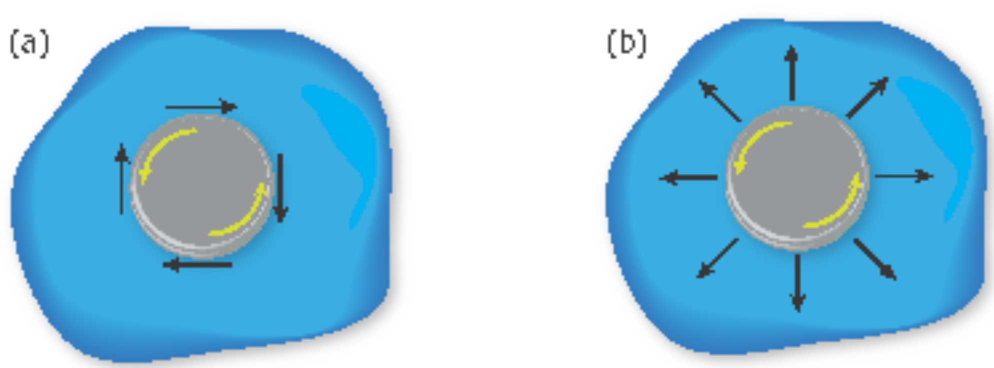
$$\dot{\rho}_5 \propto \frac{1}{l^2}\,\vec{\mathcal{E}}^a \cdot \vec{\mathcal{B}}^b\,\eta_{ab} \quad \longrightarrow \quad S_{axion} \propto \frac{1}{l^2}\int d^4x\,\vec{K}\cdot\vec{x}\;\vec{\mathcal{E}}^a\cdot\vec{\mathcal{B}}^b\,\eta_{ab}$$

I integrate by parts: $$S_{CS} \propto \frac{K_i}{l^2}\,\epsilon^{i\mu\nu\gamma}e^a_\mu\partial_\nu e^b_\gamma\,\eta_{ab}$$

$$\Big\downarrow \; e^i_j = u_{ij}$$ **conventional elasticity theory**

$$\eta_H \hat{K}_i \epsilon^{ij0k} u_{jl}\dot{u}_{kl}$$ **Hall viscosity**

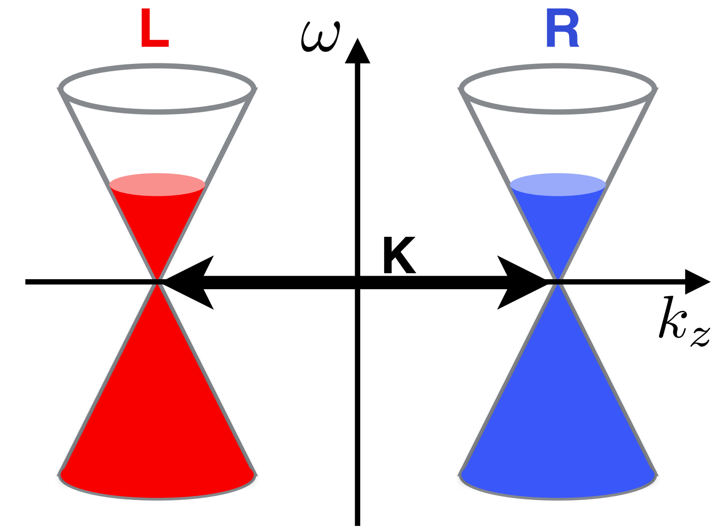
Shear viscosity **Hall viscosity**



Dissipationless: $\dot{s} = 0$

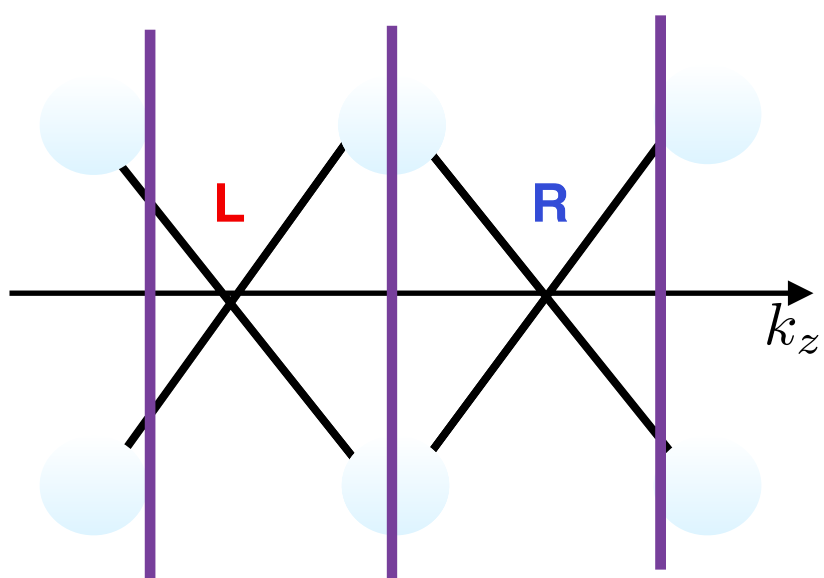
Avron, Seiler and Zograf, *Phys. Rev. Lett. 75, 697* (1995)

In our time-reversal breaking Weyl semimetal



Anomalous Hall conductivity σ_H → **Transport coefficient fixed by** $\dot{\rho}_5 \propto \vec{E} \cdot \vec{B}$

Anomalous Hall viscosity η_H → **Transport coefficient fixed by** $\dot{\rho}_5 \propto \frac{1}{l^2} \vec{\mathcal{E}}^a \cdot \vec{\mathcal{B}}^b \eta_{ab}$



$sign(m_1) = -sign(m_2)$

2 CS with
$sign(m_1) = sign(m_2)$

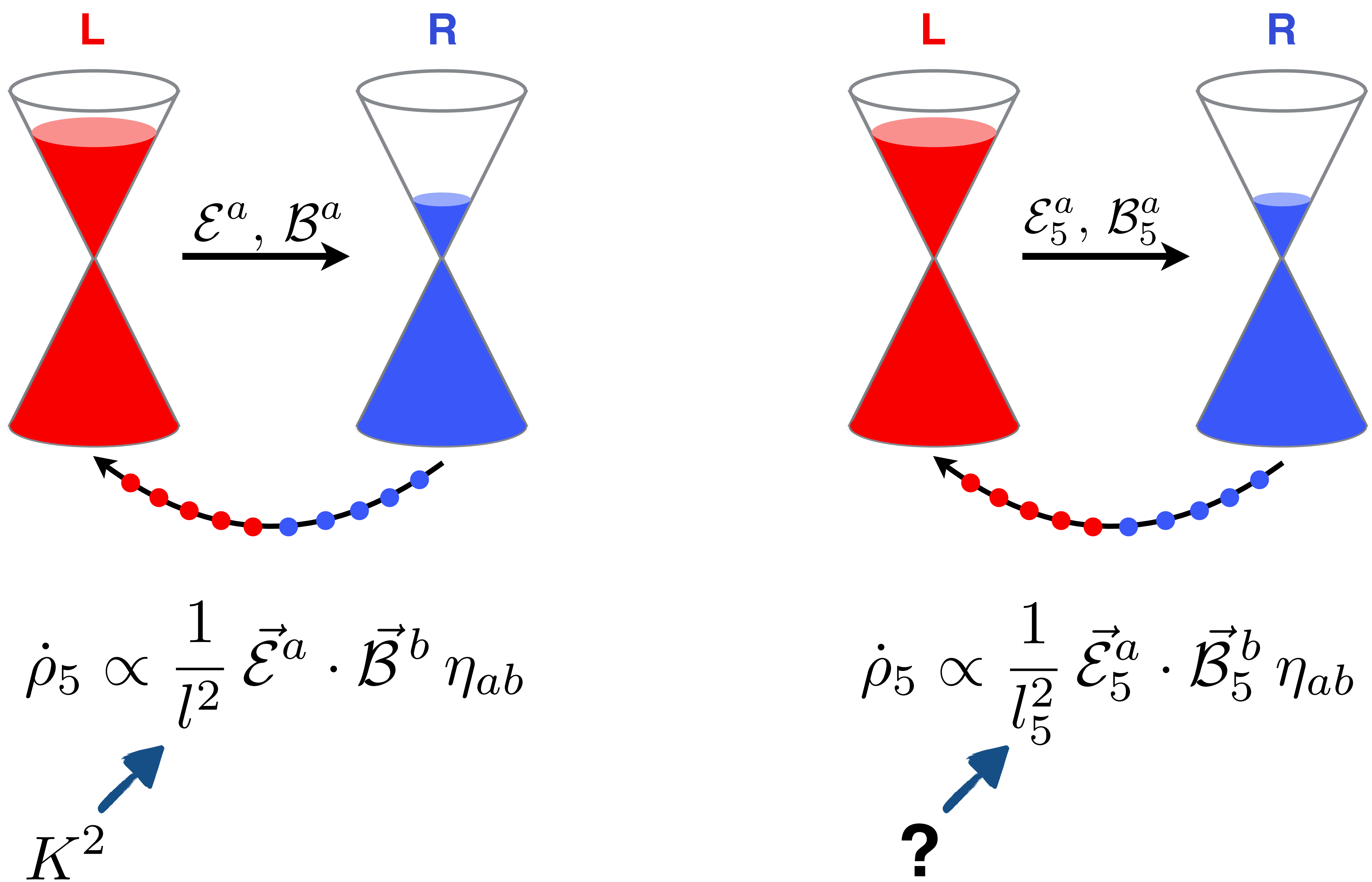
$sign(m_1) = -sign(m_2)$

$$\eta_H = \int dk_z \, \eta_H^{2D}(k_z)$$

↓

$$\boxed{\frac{1}{l^2} \sim K^2 + ...}$$

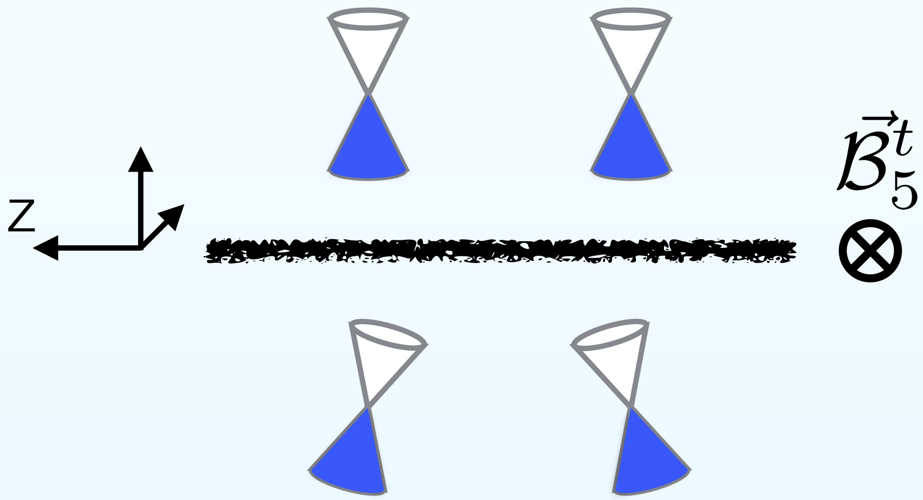
Hughes, Leigh and Fradkin, *PRL 2011*



L
R
$\mathcal{E}^a,\, \mathcal{B}^a$
$\dot{\rho}_5 \propto \frac{1}{l^2}\, \vec{\mathcal{E}}^a \cdot \vec{\mathcal{B}}^b\, \eta_{ab}$
K^2
L
R
$\mathcal{E}^a_5,\, \mathcal{B}^a_5$
$\dot{\rho}_5 \propto \frac{1}{l_5^2}\, \vec{\mathcal{E}}^a_5 \cdot \vec{\mathcal{B}}^b_5\, \eta_{ab}$
?

Torsional anomaly: activation

Tilted interface



+

Phonons

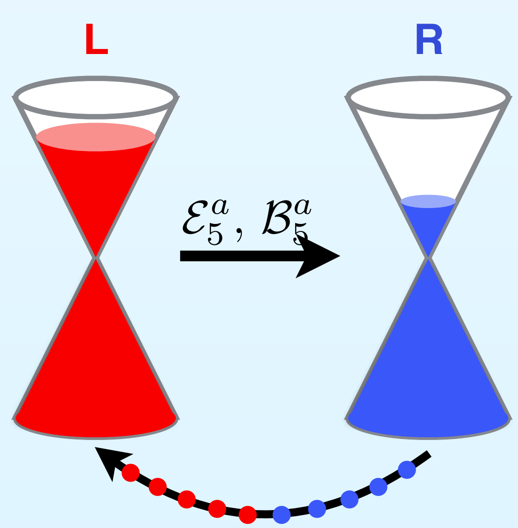
⊗ $\vec{u} = u_0 \sin(k_s z - \omega t)$

↓

⊗ $\vec{\mathcal{E}}_5^t \sim \sin(k_s z - \omega t)$

Axial chemical potential

=



$\dot{\rho}_5 \propto \vec{\mathcal{E}}_5^a \cdot \vec{\mathcal{B}}_5^b \, \eta_{ab}$ → $\mu_5 \sim \dfrac{\vec{\mathcal{E}}_5^t \cdot \vec{\mathcal{B}}_5^t}{T^2 + \mu^2}$

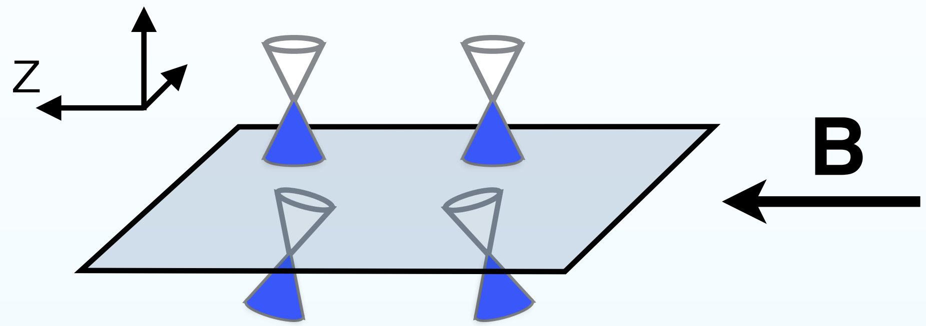
K. Fukushima, D. E. Kharzeev, and H. J. Warringa, *PRD* 2008

Torsional anomaly: observation

Chiral magnetic effect at the interface

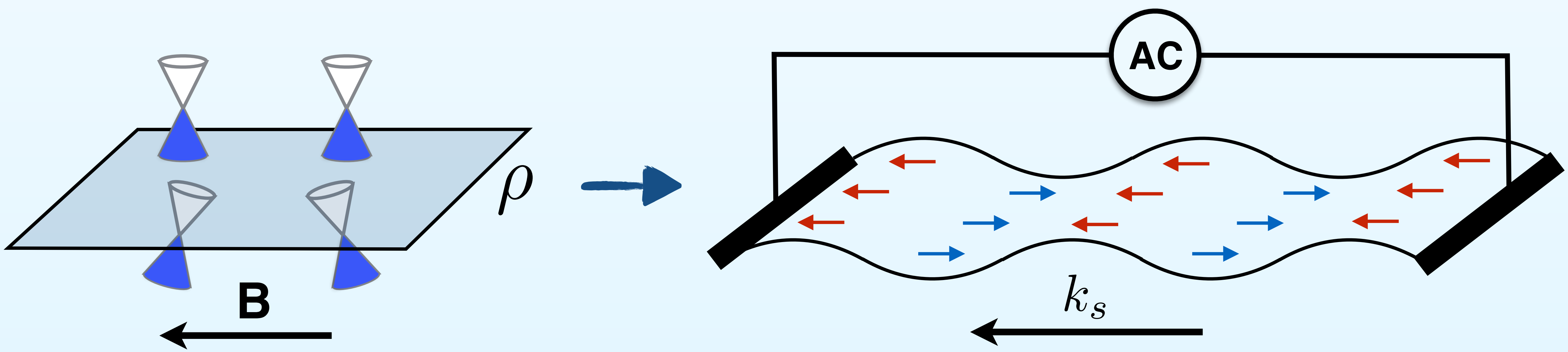
$$\vec{J} \sim \mu_5 \vec{B} \longrightarrow J^z \sim \cos(k_s z - \omega t)\, B$$

K. Fukushima, D. E. Kharzeev, and H. J. Warringa, *PRD* 2008



Propagating charge density wave at the interface

$$\dot{\rho} + \vec{\partial} \cdot \vec{J} = 0 \longrightarrow \rho \sim \cos(k_s z - \omega t)\, B$$



Amplitude of AC current

$$B \sim 10\, mT \longrightarrow J \sim 40\, nA$$

YF, Y. Kedem, E. Bergholtz, J. H. Bardarson, *PRL 2019*

Conclusions

Main result: previously overlooked contribution to the axial anomaly under axial torsion. Only exists in Weyl semimetals, concept with no counterpart at a fundamental level

Implementation by sending transverse sound waves through a tilted Weyl semimetal interface

Observation through the generated AC current when applying a magnetic field