Topics on cosmological structure formation

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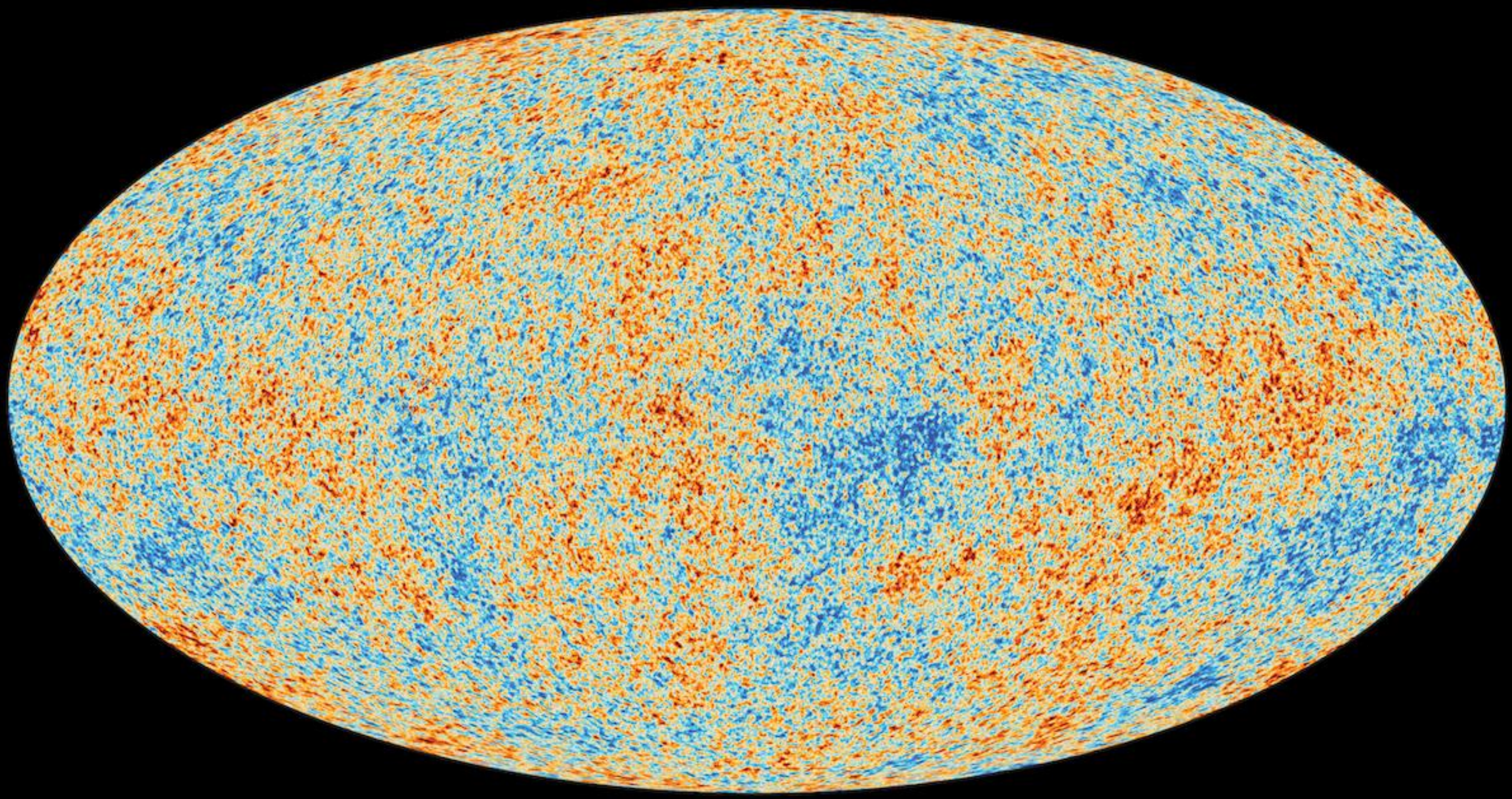
Linear theory of cosmological perturbations

Structure formation theory: cosmological perturbations



Credit: ESA/Planck collaboration 2018

The Cosmic Microwave Background Radiation

→ From the CMB we obtain the major properties that define the standard cosmological model (i.e. that govern the physics of the background Universe)

→ From the CMB as well, we know that the early Universe (~400K years from the Big Bang) contained tiny temperature fluctuations ($\sim 10^{-5}$)

→ The slightly overdense (hot) regions are the <u>seeds</u> of all cosmic structures

The collisionless Boltzmann equation with self-gravity

The Boltzmann equation is the key equation needed in structure formation theory

→ full Boltzmann equation:

$$\frac{df_\chi}{dt} = C_\chi[f_{all}] \qquad \text{with} \quad f_\chi = f_\chi(\vec{x}, \vec{p}, t)$$

→ locally, at non-cosmological scales, the cosmological principle is not valid

→ If the particle species is fully decoupled from the others $\Rightarrow C_\chi = 0$

$$\boxed{\frac{df_\chi}{dt} = 0}$$

Collisionless Boltzman Equation → related to Liouville's theorem in Hamiltonian Mechanics

→ Liouville's operator: $\displaystyle \frac{d}{dt} = \frac{\partial}{\partial t} + \sum_{i=1}^{3} \left(\frac{dx_i}{dt} \frac{\partial}{\partial x_i} + \frac{dp_i}{dt} \frac{\partial}{\partial p_i} \right) = \frac{\partial}{\partial t} + \frac{d\vec{r}}{dt} \cdot \underbrace{\vec{\nabla}_x}_{\text{gradient in real space}} + \frac{d\vec{p}}{dt} \underbrace{\vec{\nabla}_p}_{\text{gradient in momentum space}}$

The collisionless Boltzmann equation with self-gravity

$$\boxed{\frac{\partial f_x}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_x f_x - m_x \vec{\nabla}_x \phi_x \cdot \vec{\nabla}_p f_x = 0}$$

$$\boxed{\nabla^2 \phi_x = 4\pi G \rho_x}$$

Vlasov-Poisson equation

$\rho_x = m_x \int f \, d^3\vec{p} \longrightarrow$ self-gravity

gravitational field generated by the distribution of particle species x

$\downarrow$

self-gravity

→ This equation gives the evolution of f_x due to the self-gravity of the distribution of particle species x

→ In general, it is a very difficult equation to solve; instead, we concentrate on "macroscopic" average quantities that characterize the evolution of the system in a "thermodynamic" (statistical) way.

Moment equations of the Vlasov-Poisson equation

$$\boxed{\frac{\partial f_x}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_x f_x - m_x \vec{\nabla}_x \phi_x \cdot \vec{\nabla}_p f_x = 0} \qquad \boxed{\nabla^2 \phi_x = 4\pi G \rho_x}$$

→ An average, moment or expectation value is defined as:

$$\langle Q \rangle \equiv \langle Q \rangle(\vec{x}, t) = \frac{\int Q(\vec{p})\, f_x(\vec{x}, \vec{p}, t)\, d^3\vec{p}}{\int f_x(\vec{x}, \vec{p}, t)\, d^3\vec{p}} \equiv \text{avg. value of } Q(\vec{p}) \text{ at } (\vec{x}, t)$$

Recall that number density $\equiv n_x(\vec{x}, t) = \int f_x(\vec{x}, \vec{p}, t)\, d^3 t$

$$\Rightarrow \boxed{n \langle Q \rangle = \int Q(\vec{p})\, f_x(\vec{x}, \vec{p}, t)\, d^3\vec{p}}$$

→ moment or expectation value of Q over f_x

Moment equations of the Vlasov-Poisson equation

$$\int Q(\vec{p})\frac{\partial f_x}{\partial t}d^3\vec{p} + \int Q(\vec{p})\left(\frac{\vec{p}}{m_x}\cdot\vec{\nabla}_x f_x\right)d^3\vec{p} - m_x\int Q(\vec{p})\vec{\nabla}_x\phi_x\cdot\vec{\nabla}_p f_x\, d^3\vec{p} = 0$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t}\left(n\langle Q\rangle\right) + \frac{1}{m_x}\vec{\nabla}_x\cdot\left[n\langle\vec{p}Q\rangle\right] + n m_x\vec{\nabla}_x\phi_x\cdot\langle\vec{\nabla}_p Q\rangle = 0}$$

master-moment equation

Collisional Boltzmann equation in the fluid regime

→ full Boltzmann equation: $\frac{df_\chi}{dt} = C_\chi[f_{all}]$ with $f_\chi = f_\chi(\vec{x}, \vec{p}, t)$

→ In the previous (collisionless) case we dealt with the LHS, what about the RHS?

$C_\chi[f_{all}] \equiv \left(\frac{\partial f_\chi}{\partial t}\right)_{coll}$ → characterizes changes in f_χ due to interactions

⇒ Despite having a collisional system, under the assumptions: contact, elastic collisions, molecular chaos, the zeroth, first and second moment equations are identical to the collisionless case!

* These assumptions/conditions are Known as the fluid regime of the Boltzmann equation

Moments of the Boltzmann equation

$$\boxed{\frac{\partial}{\partial t}(n\langle Q\rangle) + \frac{1}{m_x}\vec{\nabla}_x\cdot[n\langle \vec{p}\,Q\rangle] + n m_x \vec{\nabla}_x \phi_x \cdot \langle \vec{\nabla}_p Q\rangle = 0}$$

master-moment equation

1) Collisionless systems: $\frac{df_x}{dt} = 0$

$$\frac{\partial \rho_x}{\partial t} + \vec{\nabla}_x\cdot(\rho_x \vec{u}) = 0$$ Continuity

$$\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i} = -\frac{\partial \phi_x}{\partial x_j} - \frac{1}{\rho_x}\frac{\partial}{\partial x_i}(\rho_x \sigma_{ij}^2)$$ Jeans (3 equations)

$$\nabla^2 \phi_x = 4\pi G \rho_x$$ Poisson

$\Rightarrow$ 5 equations with 11 variables (ρ_x, ϕ_x, σ_{ij} (6), u_j(3))

σ_{ij} $\downarrow$ symmetric

$\Rightarrow$ We need additional assumption/equations to solve the system

Dark Matter

2) Collisional system in the fluid regime (elastic, contact interactions + molecular chaos)

$\rightarrow$ only inviscid fluids (no viscosity)

$$\frac{\partial \rho_x}{\partial t} + \vec{\nabla}_x\cdot(\rho_x \vec{u}) = 0$$ Continuity

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla}_x)\vec{u} = -\vec{\nabla}_x \phi_x - \frac{1}{\rho_x}\vec{\nabla} P$$ Euler equation

$$\nabla^2 \phi_x = 4\pi G \rho_x$$ Poisson

$\Rightarrow$ 5 equations with 6 variables (ρ_x, ϕ_x, P, u_j(3))

$\Rightarrow$ We need additional assumption/equations to solve the system

Baryons

Newtonian perturbation theory: comoving frame in an expanding Universe

→ Before using the equations we derive for fluids and collisionless systems, it is convenient to introduce <u>comoving coordinates</u> to separate the effect of the Hubble expansion from the peculiar motion

$$\vec{x}_{phys} = a\,\vec{x}_{com} \quad \Rightarrow \quad \vec{p}_{phys} = m\left(\dot{a}\,\vec{x}_{com} + a\,\dot{\vec{x}}_{com}\right) \quad ; \; H = \frac{\dot{a}}{a} \qquad \vec{v}_{pec} = a\,\dot{\vec{x}}_{com}$$

→ Since the system is still not closed, we need another equation

$$P \equiv P(\rho, T) \equiv P(\rho, S) \quad \rightarrow \text{Equation of State (EoS)}$$

→ Using this form for the EoS is convenient because we can write the pressure gradient as a term depending on spatial density variations and another one on entropy variations (heat exchange)

Newtonian perturbation theory: fluid regime

Continuity
$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\left([1+\delta]\vec{v}_{pec}\right) = 0$$

Euler
$$\frac{\partial \vec{v}_{pec}}{\partial t} + H\vec{v}_{pec} + \frac{1}{a}(\vec{v}_{pec}\cdot\vec{\nabla})\vec{v}_{pec} = -\frac{1}{a}\vec{\nabla}\Phi - \frac{c_s^2}{a}\frac{\vec{\nabla}\delta}{(1+\delta)} - \frac{2T}{3a}\vec{\nabla}s^*$$

Poisson
$$\nabla^2\Phi = 4\pi G a^2\bar{\rho}\delta$$

EoS ideal monoatomic gas

$$\boxed{P(\rho, s^*) \propto \rho^{5/3}\exp\left[\frac{2}{3}\frac{m_x}{k_B}s^*\right]}$$

$c_s^2 \equiv$ adiabatic sound speed

→ In this form of the Euler equation, it is clear that the pressure gradient term has two contributions:

i) $\vec{\nabla}\delta$ → related to spatial variations in the density field $\rho(\vec{x},t) = \bar{\rho}(t)(1+\delta(\vec{x},t))$

ii) $\vec{\nabla}s^*$ → related to spatial variations in entropy, i.e. heat exchange

From now on, we will not consider heat exchange (adiabatic or isentropic case): $\vec{\nabla}s^* = 0$

Newtonian perturbation theory: fluid regime

Continuity
$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\left([1+\delta]\vec{v}_{pec}\right) = 0$$

Euler
$$\frac{\partial \vec{v}_{pec}}{\partial t} + H\vec{v}_{pec} + \frac{1}{a}(\vec{v}_{pec}\cdot\vec{\nabla})\vec{v}_{pec} = -\frac{1}{a}\vec{\nabla}\Phi - \frac{c_s^2}{a}\frac{\vec{\nabla}\delta}{(1+\delta)}$$

Poisson
$$\nabla^2\Phi = 4\pi G a^2\bar{\rho}\delta$$

EoS ideal monoatomic gas

$$\boxed{P(\rho, s^*) \propto \rho^{5/3}\exp\left[\frac{2}{3}\frac{m_x}{k_B}s^*\right]}$$

adiabatic/isentropic case

→ Now that the system of equations is closed, we can finally use the fact that we are dealing with small perturbations over the background

$$\boxed{\delta \ll 1 \quad \text{and} \quad \vec{v}_{pec} \ll \dot{a}\,\vec{x}_{com}}$$

Newtonian perturbation theory: fluid regime

Continuity
$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{v}_{pec} = 0$$

Euler
$$\frac{\partial \vec{v}_{pec}}{\partial t} + H\vec{v}_{pec} = -\frac{1}{a}\vec{\nabla}\Phi - \frac{c_s^2}{a}\vec{\nabla}\delta$$

Poisson
$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta$$

Linearized fluid equations for isentropic perturbations (ideal monoatomic gas)

→ comoving frame

→ We can combine these equations to write a 2nd order differential equation for the density fluctuations/perturbations:

$$\boxed{\ddot{\delta} + \underbrace{2H\dot{\delta}}_{\text{Hubble drag}} = \underbrace{4\pi G\bar{\rho}\delta}_{\text{perturbation growth due to gravity}} + \underbrace{\frac{c_s^2}{a^2}\nabla^2\delta}_{\text{pressure gradient due to spatial variations in density}}}$$

perturbation eq. (non-relativistic fluid)

For the collisionless case replace:

$$\sigma^2 \leftrightarrow c_s^2$$

Newtonian perturbation theory: Fourier space

$$\boxed{\ddot{\delta} + 2H\dot{\delta} = 4\pi G\bar{\rho}\,\delta + \frac{c_s^2}{a^2}\nabla^2\delta}$$

for collisionless case replace c_s^2 for σ^2

→ Taking the Fourier transform and noting that $\nabla^2 \rightarrow -k^2$ (under Fourier transform):

$$\boxed{\ddot{\delta}_{\vec{k}} + 2H\dot{\delta}_{\vec{k}} = \left[4\pi G\bar{\rho} - \frac{k^2 c_s^2}{a^2}\right]\delta_{\vec{k}}}$$

for collisionless case replace c_s^2 for σ^2

We then have an evolution equation for the density perturbation of each mode (wavenumber $\vec{k}$) evolving independently

→ The modes are decoupled from each other (this comes from the equation being linear in $\delta(\vec{x}, t)$)

The fluid case: baryonic perturbations

Jeans length

$$\boxed{\ddot{\delta}_{\vec{k}} + \cancelto{0}{2H\dot{\delta}_{\vec{k}}} = \left[4\pi G\bar{\rho} - \frac{k^2 c_s^2}{a^2}\right]\delta_{\vec{k}}}$$

→ If we ignore for the time being the expansion of the Universe ($H=0$), then this equation reduces to the wave equation

$$\Rightarrow \underbrace{\ddot{\delta}_k = -\omega^2 \delta_k}_{\text{wave equation}} \quad \text{with} \quad \omega^2 = \frac{k^2 c_s^2}{a^2} - 4\pi G\bar{\rho}$$

→ The special case $\omega = 0$ divides the behaviour of the solution into two regimes, and defines a characteristic scale:

$$\boxed{\lambda_{J,phys} = a\lambda_{J,com} = c_s\sqrt{\frac{\pi}{G\bar{\rho}}}}$$

Jeans length

Jeans length: fluid

→ Wave equation:

$$\ddot{\delta}_K = -\omega^2 \delta_K \quad \text{with} \quad \omega^2 = \frac{K^2 c_s^2}{a^2} - 4\pi G\bar{\rho}$$

$$\boxed{\lambda_{J,phys} = a\lambda_{J,com} = c_s\sqrt{\frac{\pi}{G\bar{\rho}}}}$$

Jeans length (defined for $\omega = 0$)

→ To understand the meaning of the Jeans length, let's look at the solution to the wave equation:

$$\delta_K \propto e^{\pm i\omega t} \left\{ \begin{array}{l} \text{i) if } \omega^2 > 0 \Rightarrow K > K_J \quad (\lambda < \lambda_J) \\ \Rightarrow \omega \text{ is a real number and } \delta_K(t) \text{ has an oscillatory behaviour} \end{array} \right.$$

Fluid Case i) acoustic oscillations : gravity and pressure support balance each other creating a stable oscillatory behaviour. As the perturbation tries to grow, the pressure increases halting the growth and then expanding the overdense region until pressure drops to the point where gravity wins over again; this cycle then repeats.

If $\lambda \ll \lambda_J$ the modes oscillate with a frequency $\omega_K \sim K_{phys} c_s$ (sound waves with speed c_s)

Jeans length: fluid

→ Wave equation:

$$\ddot{\delta}_K = -\omega^2 \delta_K \quad \text{with} \quad \omega^2 = \frac{K^2 c_s^2}{a^2} - 4\pi G \bar{\rho}$$

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Jeans length (defined for $\omega = 0$)

→ To understand the meaning of the Jeans length, let's look at the solution to the wave equation:

$$\delta_K \propto e^{\pm i\omega t} \begin{cases} ii) \text{ if } \omega^2 < 0 \Rightarrow K < K_J \quad (\lambda > \lambda_J) \\ \Rightarrow \omega \text{ is an imaginary number and the solution is exponentially} \\ \text{unstable} \end{cases}$$

Fluid Case ii) unstable modes: the amplitude of the perturbation either increases (growing mode) or decreases (decaying mode). The former leads to **gravitational collapse (exponential growth of the perturbation): pressure support cannot stop gravity**

Jeans length: fluid

→ Wave equation:

$$\ddot{\delta}_K = -\omega^2 \delta_K \quad \text{with} \quad \omega^2 = \frac{K^2 c_s^2}{a^2} - 4\pi G \bar{\rho}$$

$$\boxed{\lambda_{J,phys} = a\lambda_{J,com} = c_s \sqrt{\frac{\pi}{G\bar{\rho}}}}$$

Jeans length (defined for $\omega = 0$)

→ To understand the meaning of the Jeans length, let's look at the solution to the wave equation:

$$\delta_K \propto e^{\pm i\omega t} \begin{cases} \text{i) } \lambda < \lambda_J \\ \text{ii) } \lambda > \lambda_J \end{cases}$$

i) acoustic oscilations (Fluid)

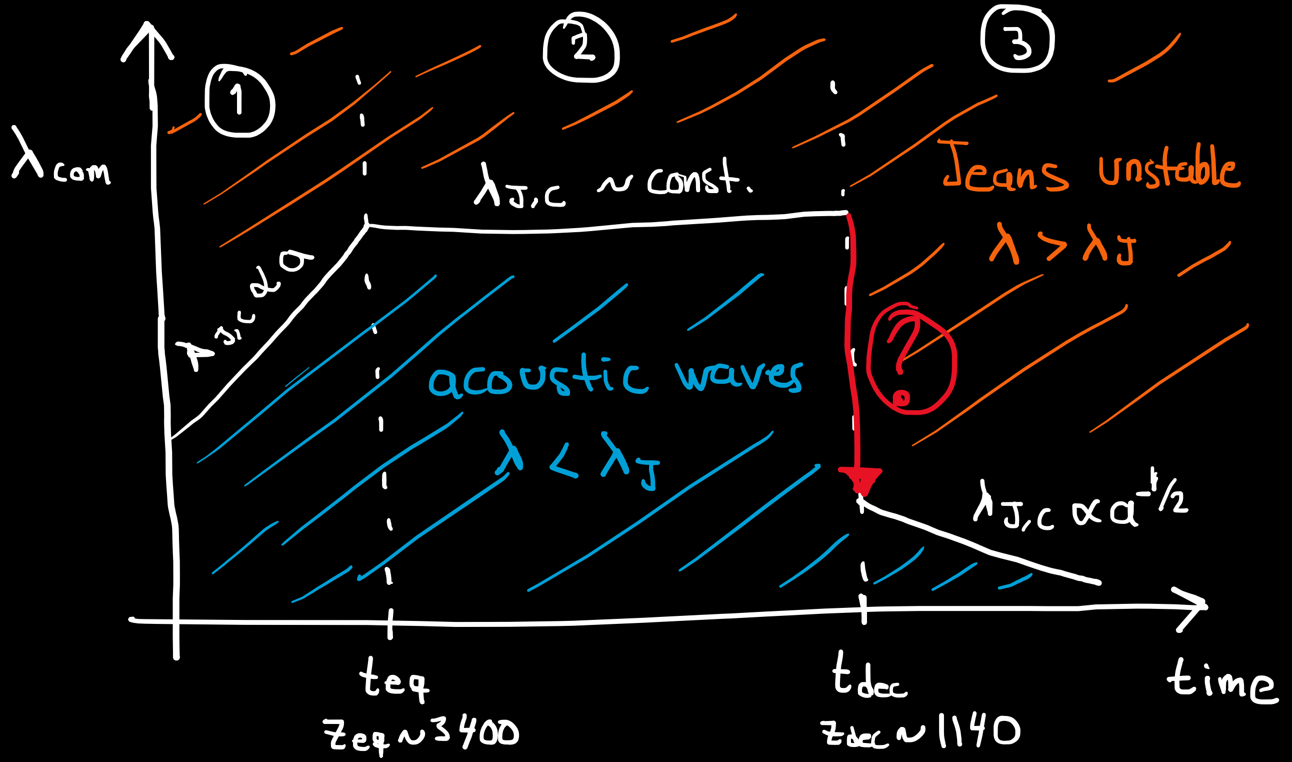
ii) unstable modes (+ sign leads to exponential growth)

⇒ The Jeans length is the maximum distance that sound waves can travel to exert pressure support on a perturbation to prevent its gravitational collapse

baryonic perturbations in the Early-Universe

The Jeans scale depends on whether radiation or matter dominates the background density, in addition to the sound speed, which depends on the photon-baryon coupling

$$\boxed{\lambda_{J,\mathrm{com}} = \frac{c_s}{a}\sqrt{\frac{\pi}{G\bar{\rho}}}}$$



Why the big drop after decoupling?

→ just before decoupling $M_J \sim 10^{15}\, M_\odot$

→ just after decoupling $M_J \sim 10^{5}\, M_\odot$

⇒ 10 orders of magnitude drop in Jeans mass

This is because at decoupling, photons decouple from baryons (very rapidly), which dramatically reduces the pressure from $\frac{1}{3}\bar{\rho}_r c^2$ to $\frac{\bar{\rho}_b}{m_x} k_B T_x$

Silk (collisional) damping

→ Although we are considering an evolution equation that <u>does not</u> have interactions between the relevant species and other species, we can nevertheless get an approximation for the scales that are affected strongly by interactions

- The coupling between baryons/ordinary matter and photons is not perfect. It occurs mainly through Compton scattering (between free electrons and photons), which has a mean free path that is not zero:

photon mean free path due to Compton scattering

$$\boxed{\lambda_\gamma = \frac{1}{n_e \sigma_T}}$$

$n_e \equiv$ number density of free electrons

$\sigma_T \equiv$ Thomson cross section (Compton → Thomson at low energies)

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The larger the mean free path, the larger photons can diffuse from high-density to low-density regions, decreasing photon pressure, and thus reducing the amplitude of the acoustic waves.

$\rightarrow$ This diffusion/damping mechanism is more efficient at the decoupling/recombination epoch when photons are about to decouple (λ_γ is larger)

$\rightarrow$ The Silk damping scale λ_d is defined as the typical distance can diffuse from time $t=0$ to $t=t_{dec}$

Silk (collisional) damping

photon mean free path due to Compton scattering

$$\boxed{\lambda_\gamma = \frac{1}{n_e \sigma_T}}$$

$n_e \equiv$ number density of free electrons

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$\rightarrow$ The Silk damping scale λ_d is defined as the typical distance can diffuse from time $t=0$ to $t=t_{dec}$

$\rightarrow$ This scale is estimated using a random walk approximation (diffusion originates from random collisions between photons and electrons)

$$\lambda^2_{d,com}(t_{dec}) \sim c\, t_{dec} \left(\frac{\lambda_\gamma(t_{dec})}{a^2(t_{dec})} \right) \Rightarrow \boxed{\lambda^2_{d,phys} \sim c\, t_{dec}\, \lambda_\gamma(t_{dec}) = \frac{c\, t_{dec}}{n_e(t_{dec})\, \sigma_T}}$$

Silk damping scale

Silk (collisional) damping

$$\boxed{\lambda^2_{d,phys} \sim c\, t_{dec}\, \lambda_\gamma(t_{dec}) = \frac{c\, t_{dec}}{n_e(t_{dec})\, \sigma_T}}$$

Silk damping scale

Silk damping mass

$$M_d = \frac{\pi}{6} \bar{\rho}_b(t_{dec})\, \lambda^3_{d,phys}(t_{dec})$$

$$\boxed{M_d(z_{dec}) \sim 10^{13} M_\odot}$$

assuming $\Omega_{b,0} = \Omega_{m,0} = 1$

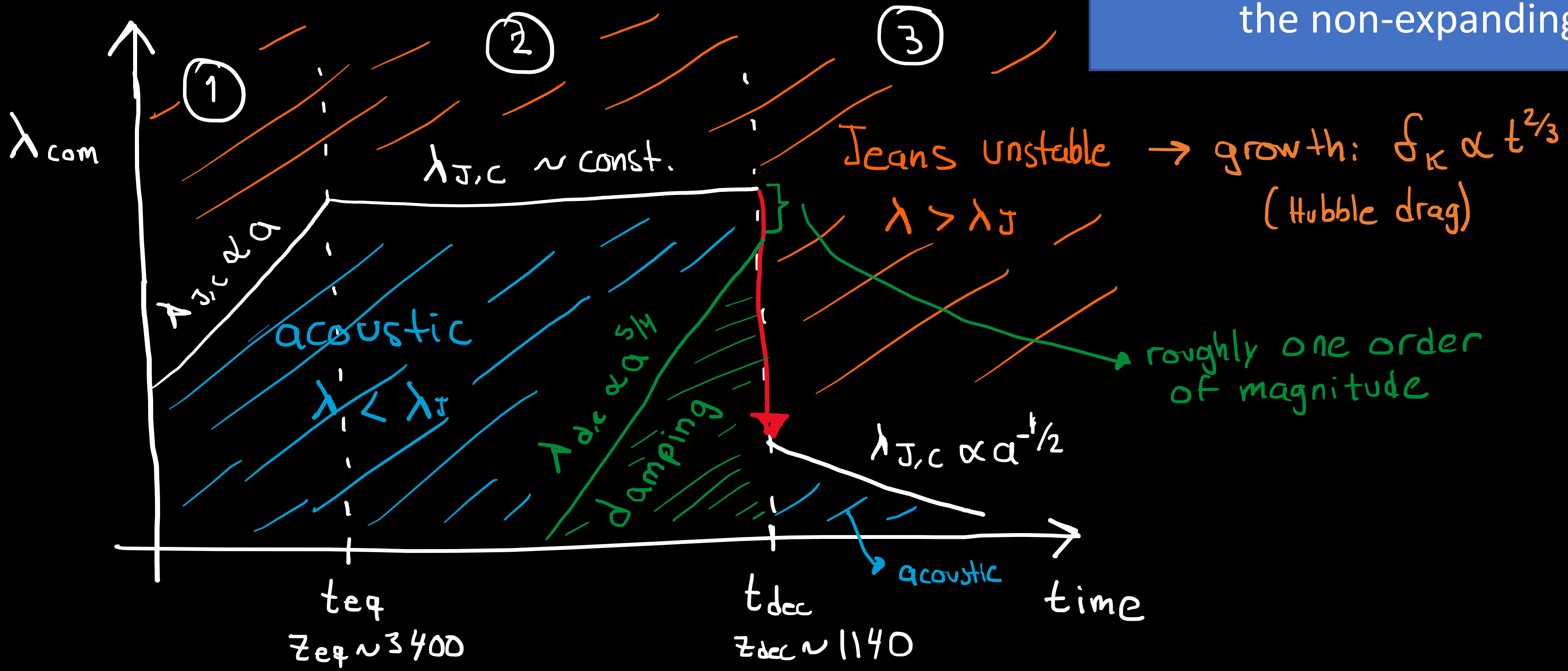
If there is no DM!

→ Perturbations of a scale $M < M_d(z_{dec})$ are expected to be damped (essentially erased) by the epoch of decoupling/recombination (time of the CMB emission)

→ Since $M_d \sim 10^{13} M_\odot$, then the only way in which galaxies could form would be through the fragmentation of more massive systems

Baryonic perturbations: summary

The effect of the expansion of the Universe (Hubble drag) is to slowdown the growth of perturbation relative to the non-expanding case

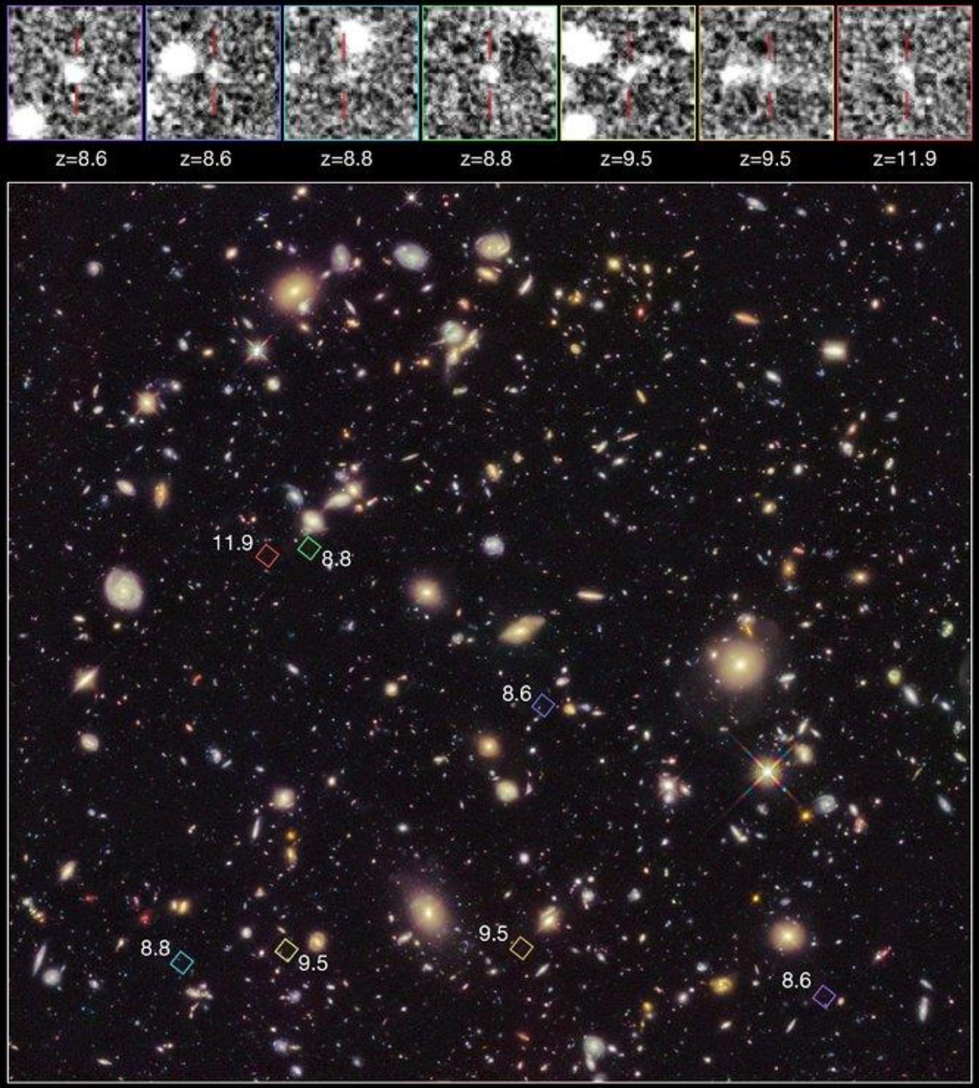


By the time of decoupling, all perturbations with a scale $\lambda < \lambda_d(t_{dec})$ $(M < 10^{13} M_\odot)$ have been erased due to Silk damping (structure formaton has to proceed "top-down")

Baryonic perturbation are not enough for structure formation: the need for dark matter

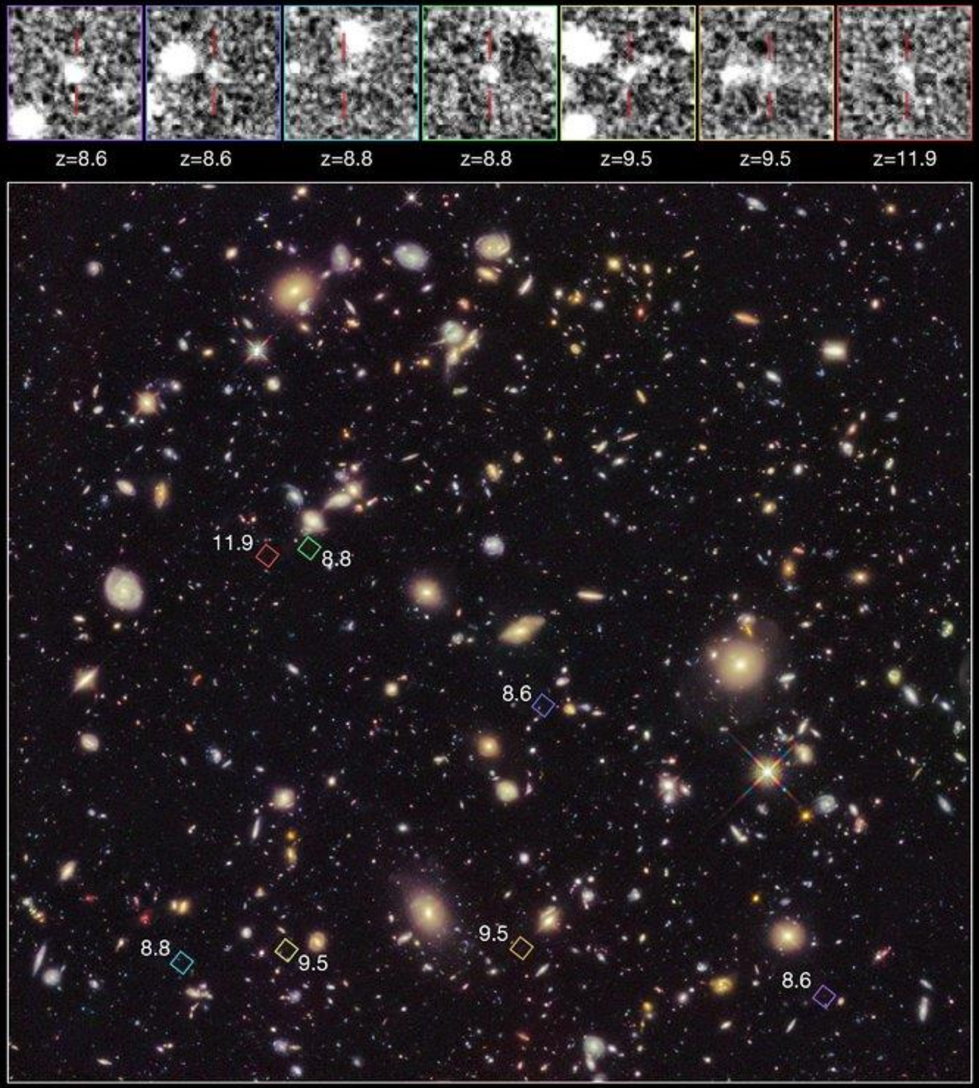
Baryonic perturbation are not enough for structure formation: the need for dark matter

Hubble Ultra Deep Field 2012, Credit:NASA/ESA



→ We Know that galaxies exist at high redshifts ($z \sim 10$), i.e galaxies with a scale below the Silk mass were already present at a time when the Universe was ~ 0.5 Gyr

Baryonic perturbation are not enough for structure formation: the need for dark matter



Hubble Ultra Deep Field 2012, Credit:NASA/ESA

→ We know that galaxies exist at high redshifts ($z \sim 10$), i.e galaxies with a scale below the Silk mass were already present at a time when the Universe was ~ 0.5 Gyr

→ If galaxies exist already then, we need:

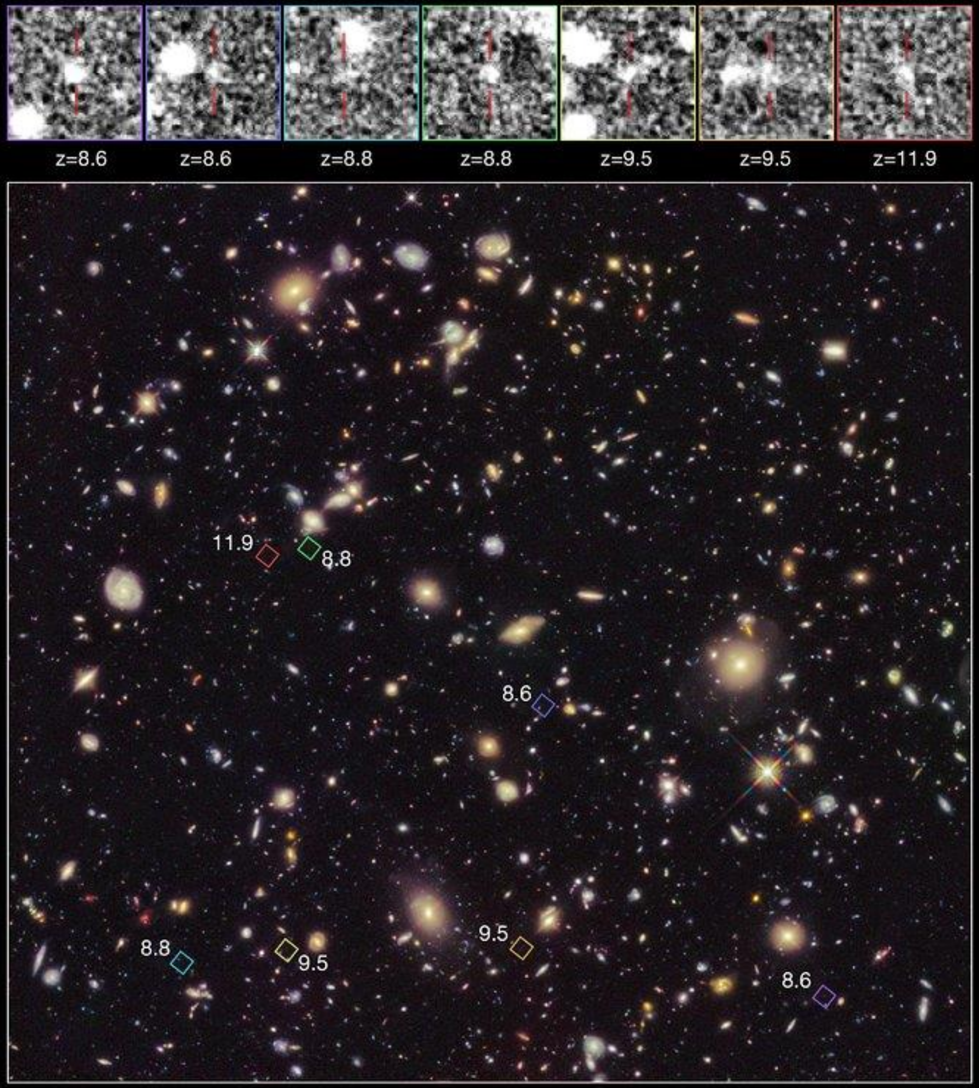
$$\delta_{M_{gal}}(z_{gal}) > 1$$

→ average overdensity at scales of order M_{gal}

$$\Rightarrow \frac{\delta_{M_{gal}}(z_{dec})}{\delta_{M_{gal}}(z_{gal})} \sim \frac{a_{dec}}{a_{gal}} = \frac{(1+z_{gal})}{(1+z_{dec})}$$

EdS Universe

Baryonic perturbation are not enough for structure formation: the need for dark matter



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$$\delta_{M_{gal}}(z_{gal}) > 1$$

→ average overdensity at scales of order M_{gal}

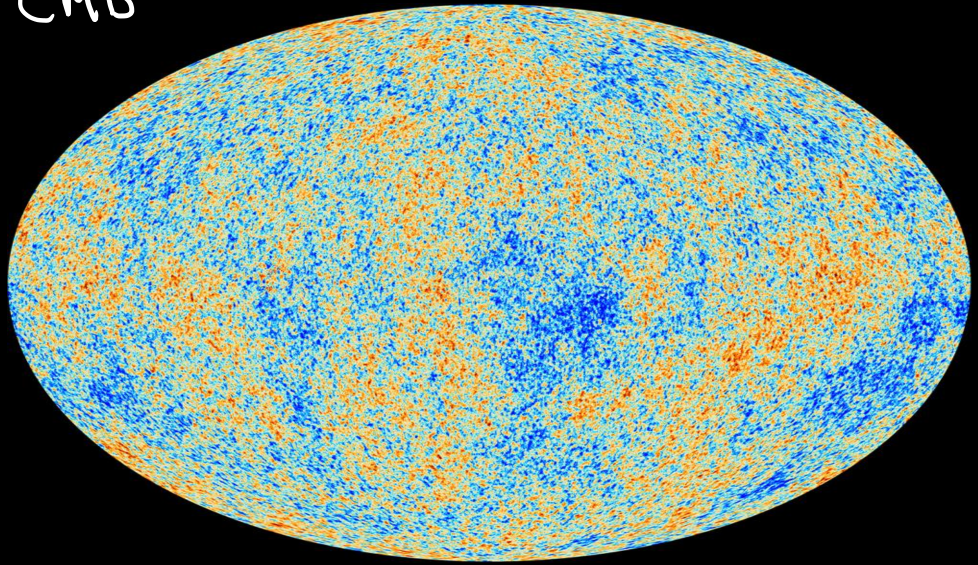
$$\Longrightarrow \frac{\delta_{M_{gal}}(z_{dec})}{\delta_{M_{gal}}(z_{gal})} \sim \frac{a_{dec}}{a_{gal}} = \frac{(1+z_{gal})}{(1+z_{dec})}$$

EdS Universe

$$\Longrightarrow \boxed{\delta_{M_{gal}}(z_{dec}) > \frac{11}{1141} \sim 10^{-2}}$$

Baryonic perturbation are not enough for structure formation: the need for dark matter

CMB



→ On the other hand, from CMB observations we know that in average, temperature fluctuations at the time of decoupling are of order 1 in 100,000

$$\Rightarrow \boxed{\delta\rho_m \big|_{dec} \sim 3\times 10^{-5}}$$

From galaxies observed at high-z

$$\boxed{\delta_{M_{gal}}(z_{dec}) > \frac{11}{1141} \sim 10^{-2}}$$

⇒ Required amplitude of baryonic perturbation is constrained by the CMB to be at least 3 orders of magnitude too small to explain galaxies at high redshift

⇒ additional form of matter with perturbations that can grow before decoupling

The collisionless case: DM perturbations

Dark matter perturbations: free streaming damping

Case 2: Dark matter (Universe without baryons; radiation as background only)

$$\boxed{\ddot{\delta}_{\vec{k}} + 2\cancelto{0}{H}\dot{\delta}_{\vec{k}} = \left[4\pi G\bar{\rho} - \frac{k^2\sigma^2}{a^2}\right]\delta_{\vec{k}}}$$

$$\boxed{\lambda_{J,phys} = a\lambda_{J,com} = \sigma\sqrt{\frac{\pi}{G\bar{\rho}}}}$$

Jeans length for DM perturbations

→ For $H=0$, this is the wave equation $\ddot{\delta}_k = -\omega^2\delta_k$ with $\omega^2 = \frac{k^2\sigma^2}{a^2} - 4\pi G\bar{\rho}$

$$\delta_k \propto e^{\pm i\omega t} \begin{cases} i)\ \lambda < \lambda_J \\ ii)\ \lambda > \lambda_J \end{cases}$$

i) No acoustic oscillations (collisionless, no sound waves)

ii) unstable modes (+ sign leads to exponential growth)

→ For $\lambda < \lambda_J$ we have a new phenomenon, only present for collisionless systems:

Free-streaming damping

Dark matter perturbations: free streaming damping

$$\boxed{\lambda_{J,phys} = a\lambda_{J,com} = \sigma\sqrt{\frac{\pi}{G\bar{\rho}}}}$$

Jeans length for DM perturbations

→ Let's consider the characteristic time for DM particles to disperse due to random motions around a perturbation of a characteristic size λ :

$$\boxed{\tau_{disp} = \frac{\lambda}{\sigma}} \quad ; \text{ if } \lambda < \lambda_J \Rightarrow \boxed{\tau_{disp} < \sqrt{\frac{\pi}{G\bar{\rho}}}}$$

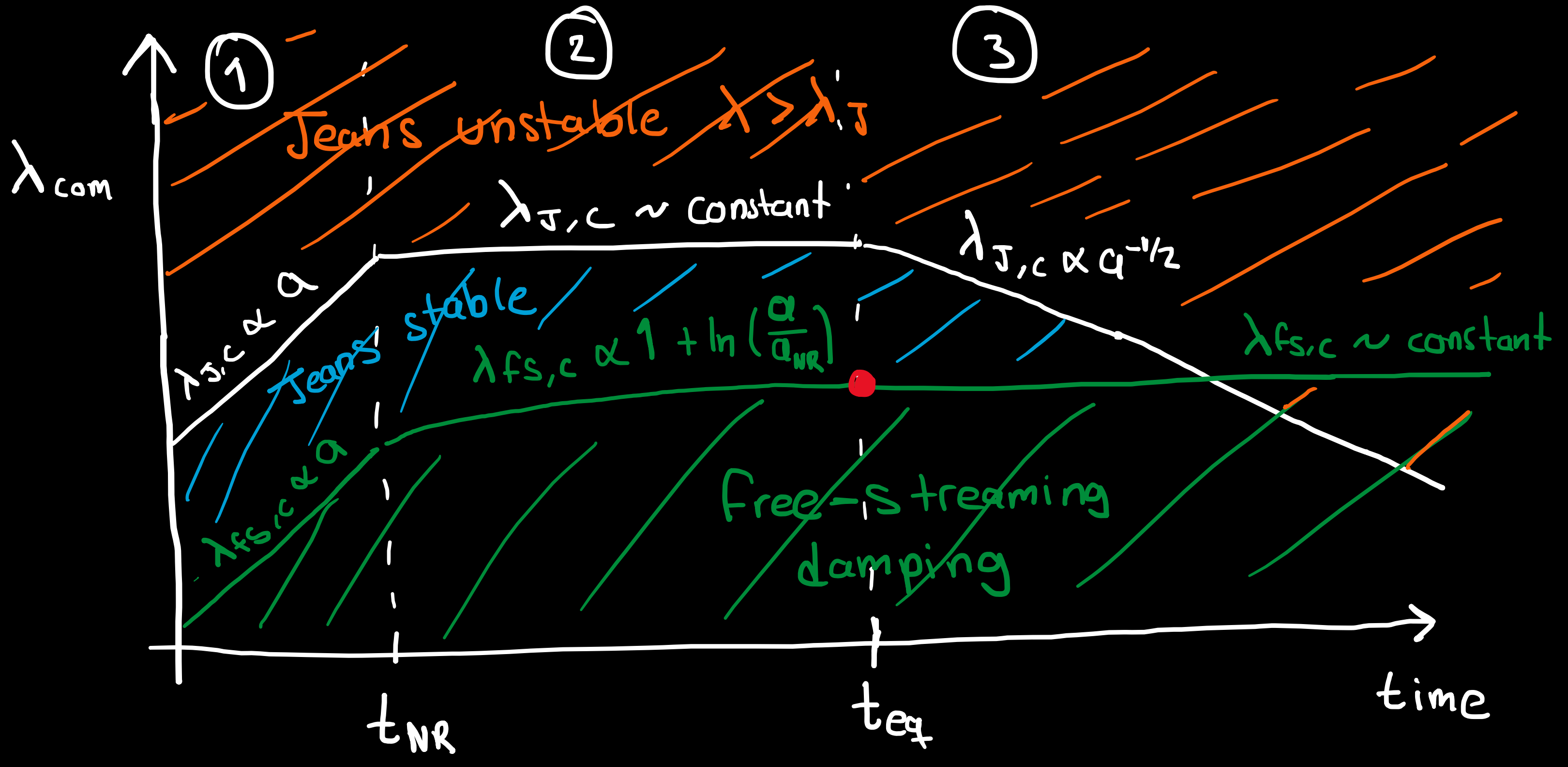
For DM perturbations with a scale less than the Jeans scale

→ Now, from the Friedmann eq.: $H^2 = \frac{8\pi G\bar{\rho}}{3} \Rightarrow \boxed{\tau_H = \frac{1}{H} = \sqrt{\frac{3}{8\pi G\bar{\rho}}}}$

Hubble time $\sim$ Age of the Universe

⇒ If $\lambda \ll \lambda_J$, $\tau_{disp} \ll \tau_H$: the dispersion time is shorter than the age of the Universe at a given time, which means that DM particles have had enough time to disperse/erase perturbations of size λ (free-streaming or collisionless damping)

Dark matter perturbations: free streaming damping



All perturbations below the free-streaming scale are erased due to this collisionless damping mechanism

→ What is required for structure formation to explain the formation of galaxies by $z \sim 10$, is for the free-streaming length to be much smaller than the scale of galaxies by the time of equality

Dark matter perturbations: the Hubble drag term

→ We will consider the impact of the Hubble expansion for the evolution of non-relativistic DM perturbations

→ Let's concentrate also only on the modes that are Jeans unstable and where the pressure term can be neglected ($\lambda >> \lambda_J$). This is for simplicity to see the impact of the Hubble drag term cleanly

$$\Rightarrow \boxed{\ddot{\delta}_{\vec{k}} + 2H\dot{\delta}_{\vec{k}} = \left[4\pi G\bar{\rho} - \cancelto{0}{\frac{k^2\sigma^2}{a^2}}\right]\delta_{\vec{k}}}$$

Dark matter perturbations: the Hubble drag term

→ We will consider the impact of the Hubble expansion for the evolution of non-relativistic DM perturbations

$$\boxed{\ddot{\delta}_{\vec{k}} + 2H\dot{\delta}_{\vec{k}} = 4\pi G\bar{\rho}\,\delta_{\vec{k}}}$$

$\lambda \gg \lambda_J$

matter-dominated era

$$\boxed{\text{Growing mode: } \delta_k^+ \propto t^{2/3} \propto a}$$

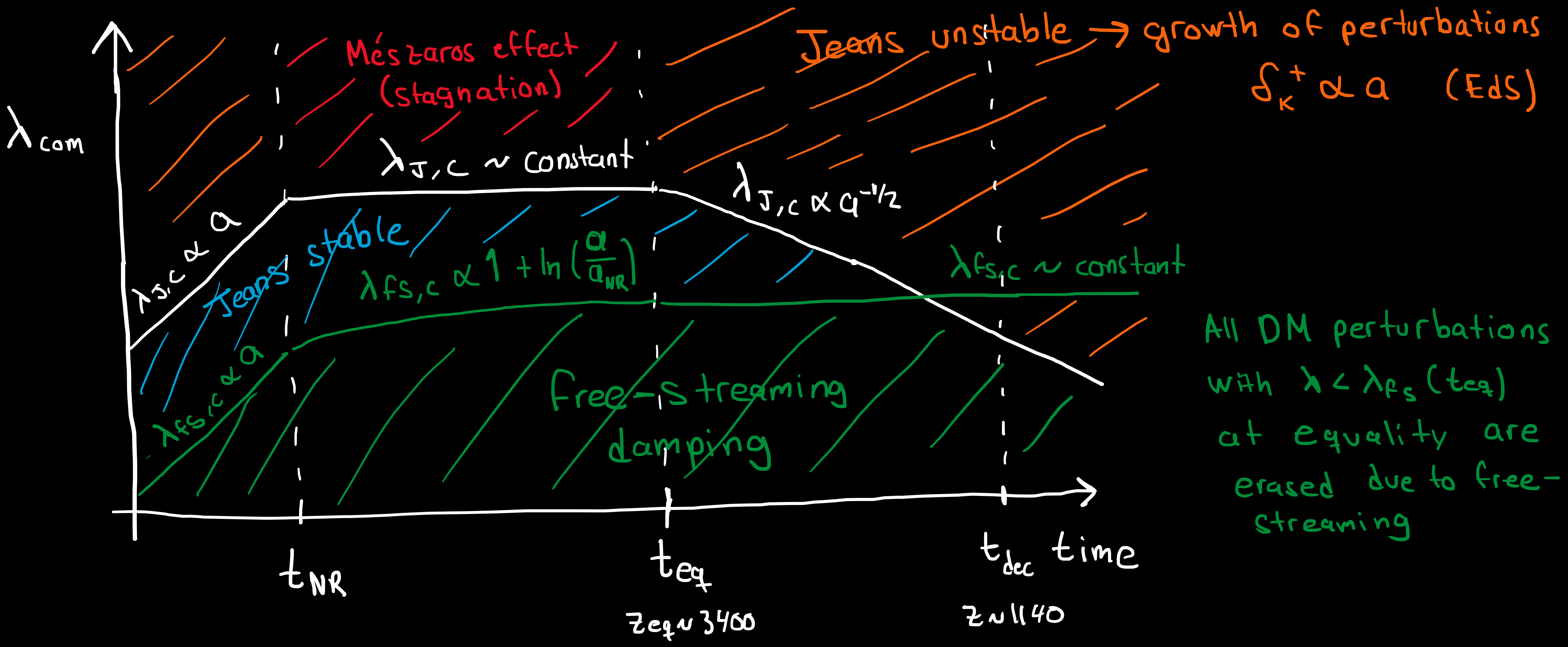
radiation-dominated era

Growing mode

$$\boxed{\delta_k^+ \propto 1 + \frac{3}{2}\left(\frac{a}{a_{eq}}\right)}$$

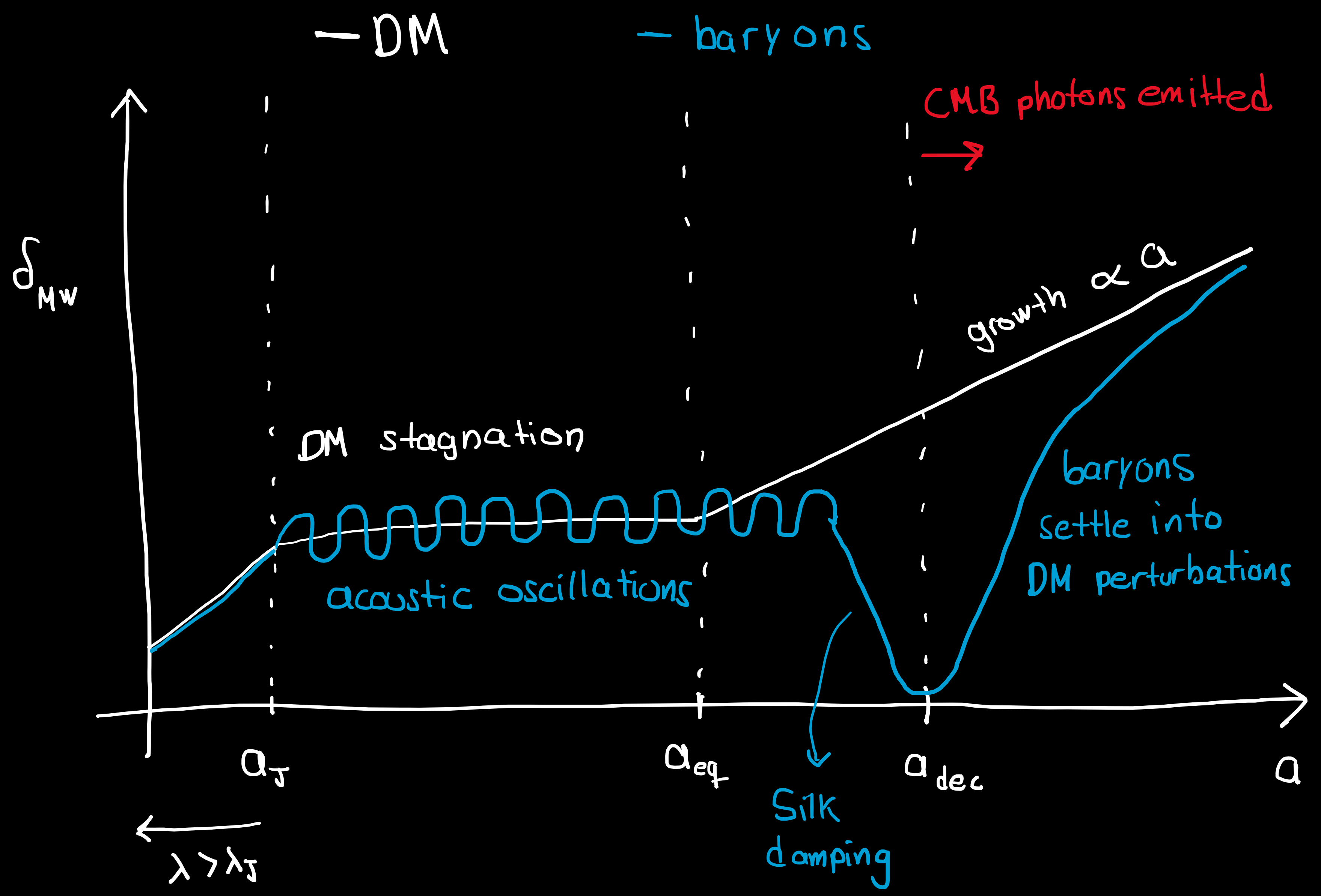
Mészáros effect: in the radiation-dominated era, radiation drives the Universe to expand so fast that DM has no time to respond keeping perturbations frozen

Dark matter perturbations: summary



* For CDM, λ_{fs} is typically less than a parsec, and thus all galactic scales can grow

cosmological perturbations: summary



$\delta_{MW} \equiv$ amplitude of a matter (baryons and CDM) perturbation with a mass $M_w = \frac{4}{3}\pi\bar{\rho}\left(\frac{\lambda}{2}\right)^3$

$\sim 10^{12}\, M_\odot$ (Milky-Way)

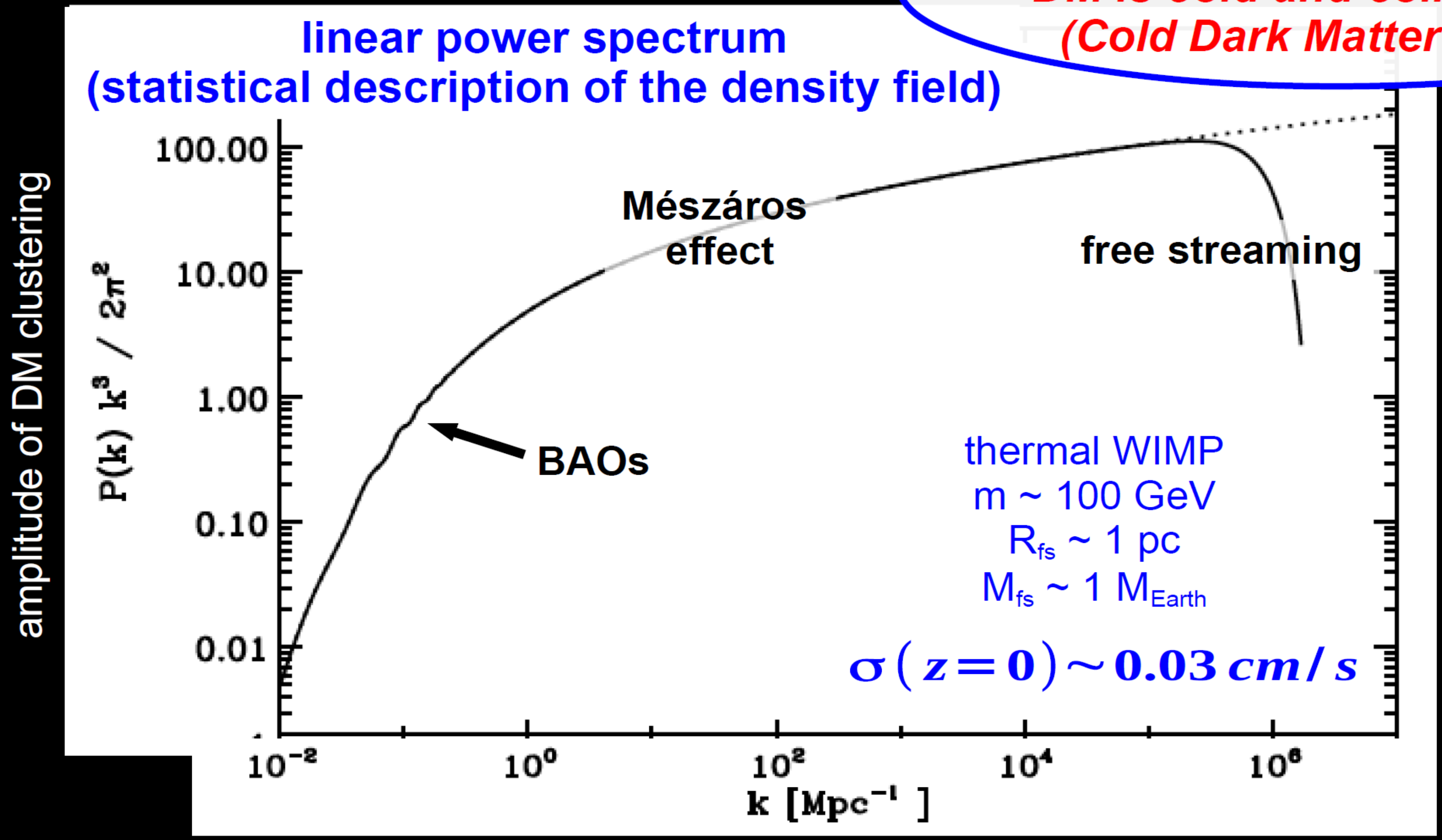
→ Since CDM ⇒ no significant free-streaming

→ Since $M_w < M_d \sim 10^{13}\, M_\odot$ perturbations suffers from Silk damping

LINEAR REGIME (cosmological perturbation theory)

Angulo & White, 2010

Standard hypotheses:
DM is cold and collisionless
(Cold Dark Matter model)



Non-linear structure formation theory: gravitational collapse of collisionless systems

Non-linear structure formation theory: gravitational collapse of collisionless systems

$$\rho(\vec{x},t) = \bar{\rho}(t)[1+\delta(\vec{x},t)]$$

→ Linear regime : $\delta \ll 1$ ⇒ linear perturbation theory

→ Non-linear regime : $\delta \gtrsim 1$ ⇒ perturbation theory breaks down!

⇒ Without linearization, the evolution equation for a given Fourier mode $\delta_{\vec{k}}$ depends on other modes $(\vec{k}', \vec{k}'', \ldots)$. This is known as mode coupling. The coupled system of equations cannot be solved analytically. Numerically not feasible neither once many modes are coupled (infinite wave numbers in principle).

NON-LINEAR REGIME (N-body simulations)

If $\delta(x,t) \gtrsim 1$ *perturbation theory breaks down!!*

Standard hypotheses: DM is cold and collisionless (Cold Dark Matter model) → **the only DM interaction that matters is gravity!!**

In principle: solve Collisionless Boltzmann Equation (coupled with the Poisson equation) with the initial conditions given by linear perturbation theory

$$\frac{df}{dt} = 0 \qquad \nabla^2 \phi = 4\pi G \rho$$

i.e., find the local DM distribution in phase space at all points and at all times:

$$f(\vec{x},\vec{v},t)\,\mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{v} \longrightarrow \rho(\vec{x},t) = \int f(\vec{x},\vec{v},t)\,d^3\vec{v}$$

In practice however, we can only compute, measure, the DM distribution averaged over a certain macroscopic scale (coarse-grained distribution)

NON-LINEAR REGIME (N-body simulations)

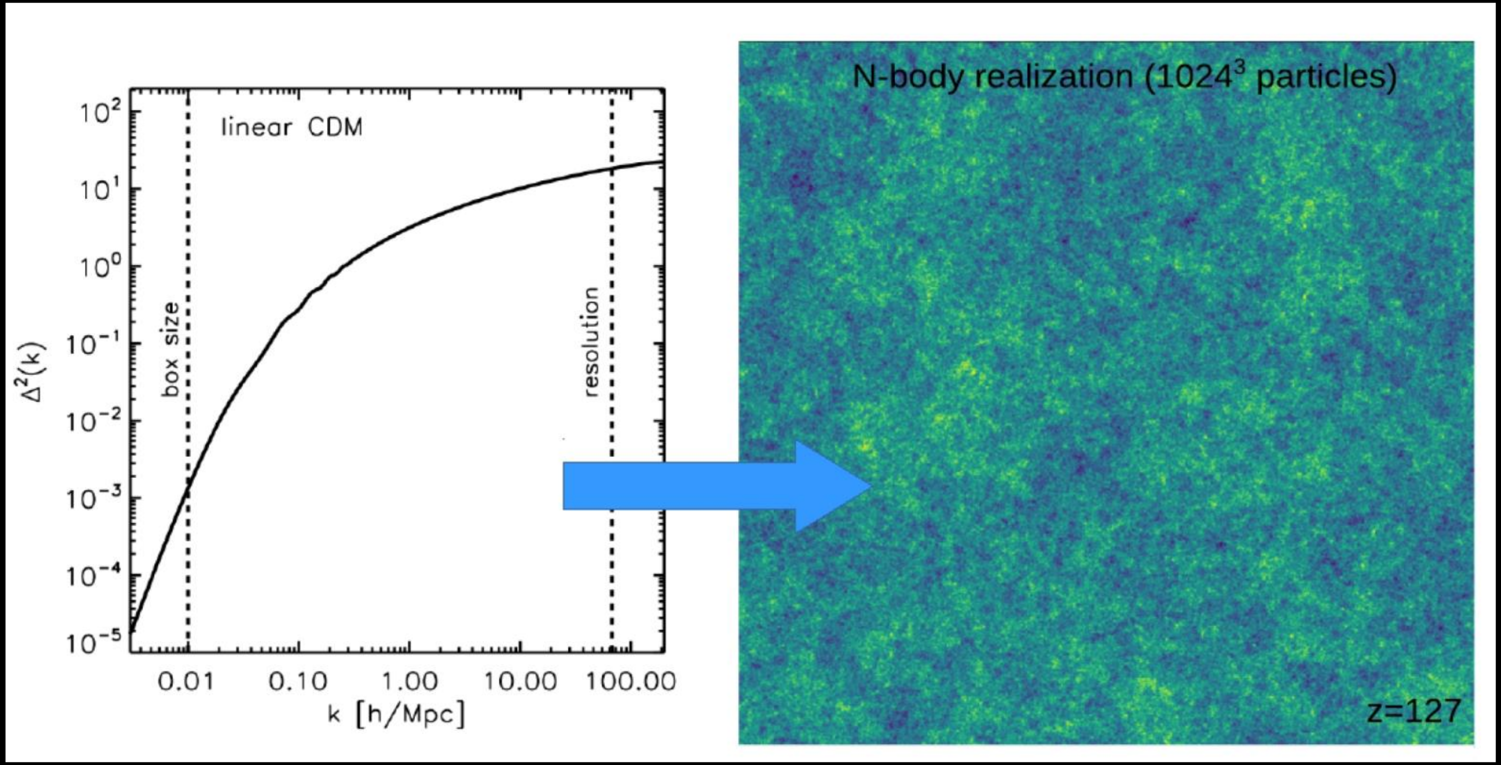
N-body sim: the coarse-grained distribution is given by a discrete representation of N particles:

macro-to-micro-particle mass ratio

each particle is smoothed in space to give a smooth local density

each macro-particle travels at one speed

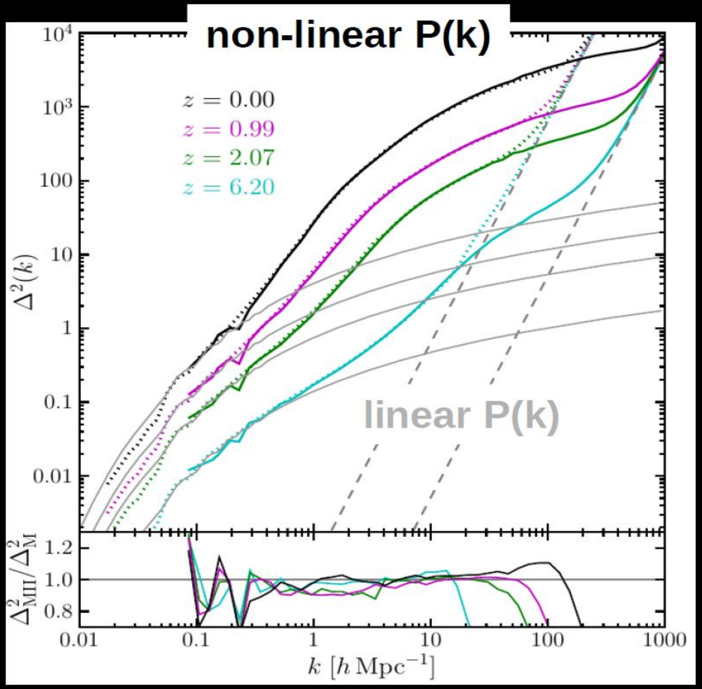
$$\hat{f}(\mathbf{x},\mathbf{v},t) = \sum_i (M_i/m) W(|\mathbf{x}-\mathbf{x}_i|; h_i)\delta^3(\mathbf{v}-\mathbf{v}_i)$$



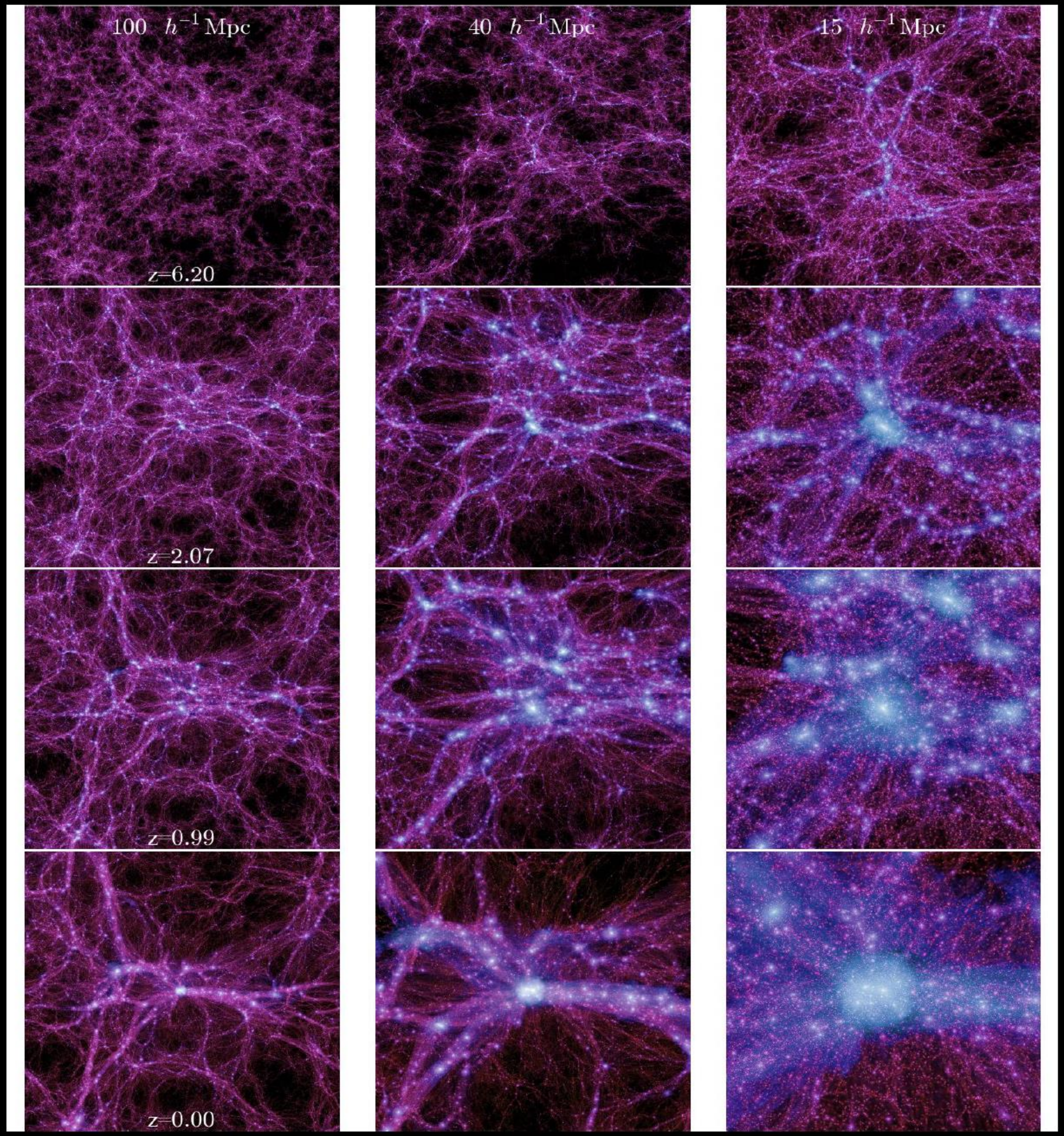
Initial Conditions for N-body simulations

NON-LINEAR REGIME (N-body simulations)

Boylan-Kolchin+09



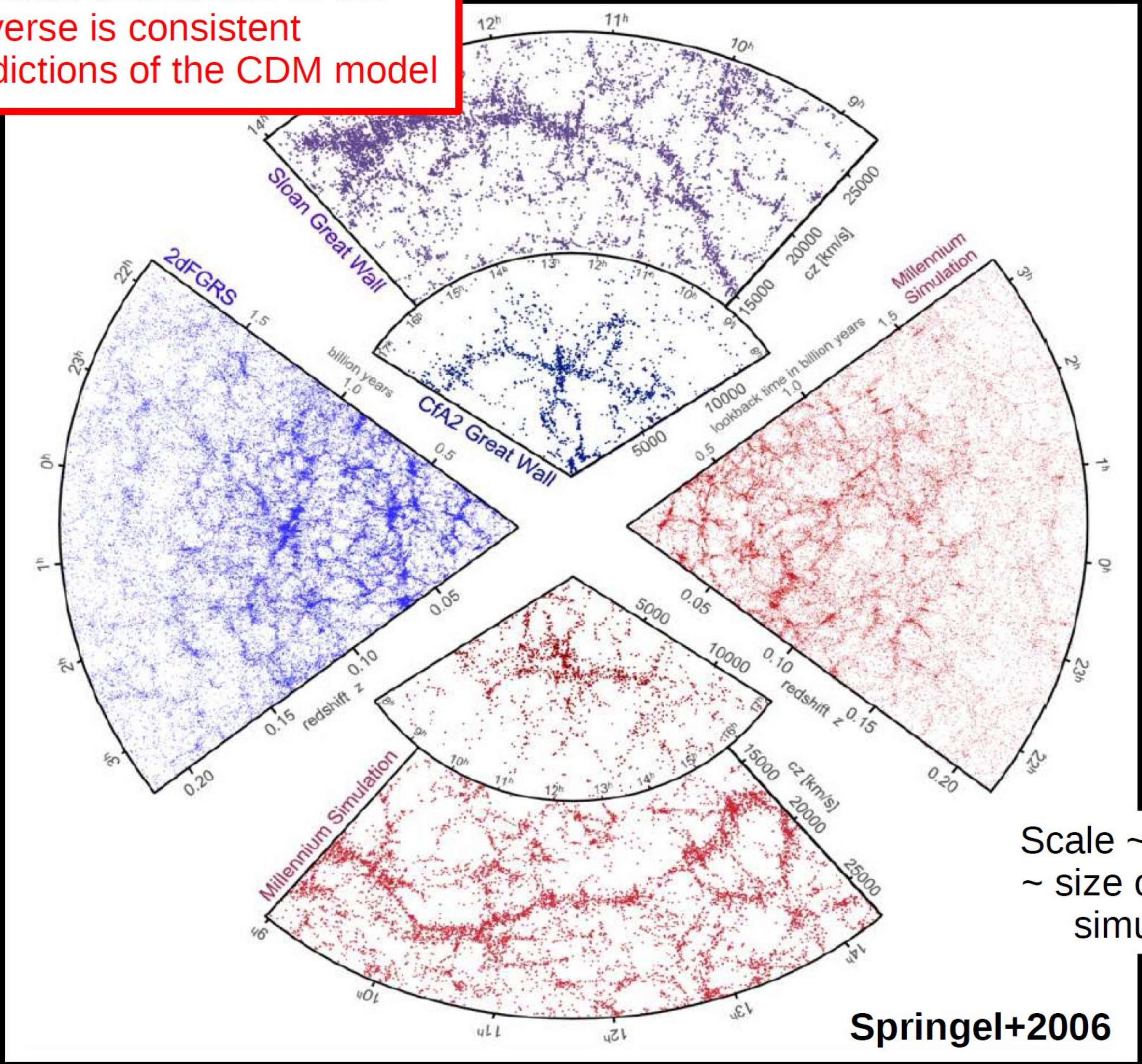
Millennium II simulation



Time

Large-scale structure

the large-scale distribution of the Universe is consistent
with the predictions of the CDM model

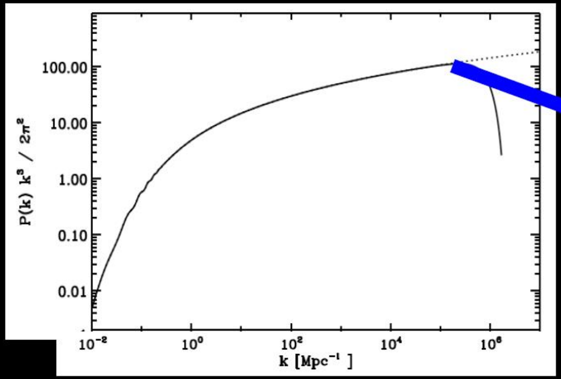


Scale ~400-600 Mpc
~ size of Millennium simulation box

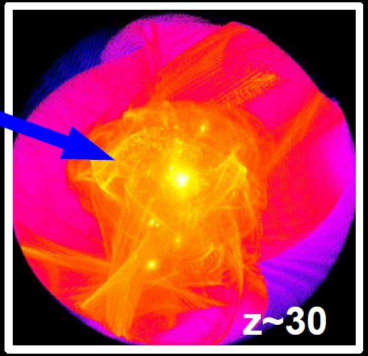
Springel+2006

Self-gravitating DM structures: haloes

CDM predicts a hierarchichal growth of structures



DM halo seeds



Anderhalden & Diemand 14

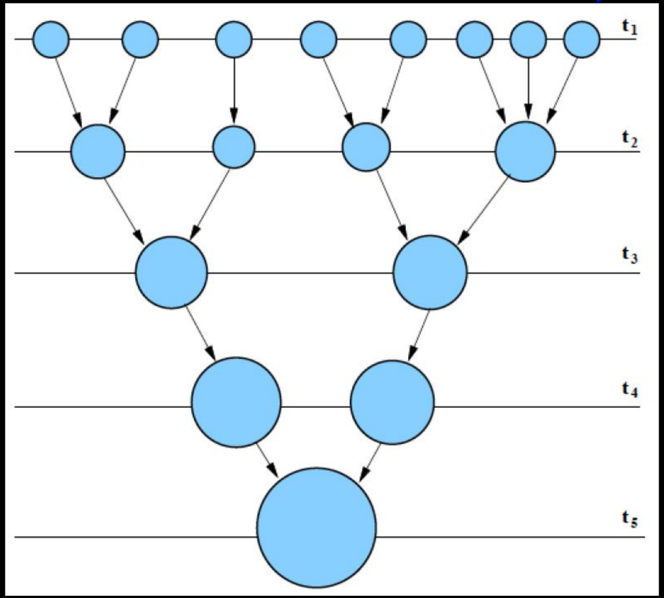
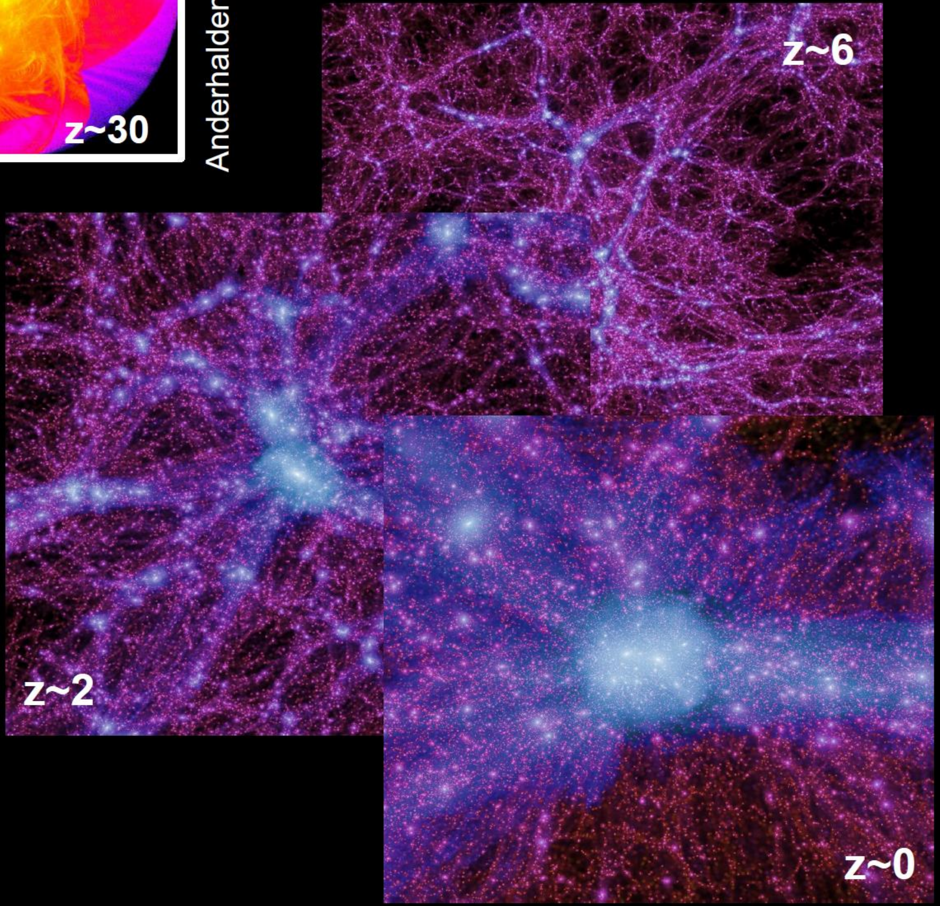


Fig. from Baugh 2006

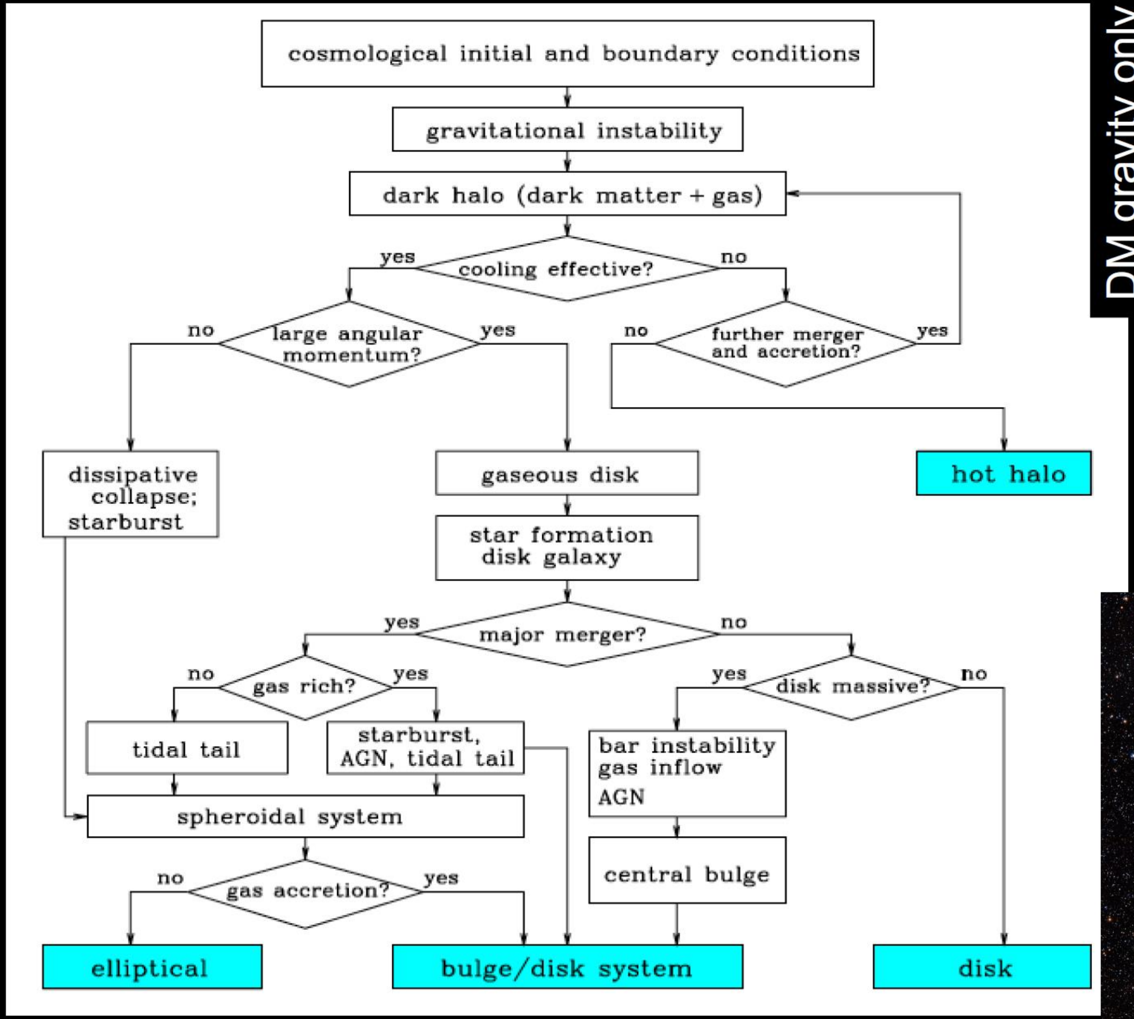
Boylan-Kolchin+2009



Standard structure formation theory

NON-LINEAR REGIME (gas and stellar physics)

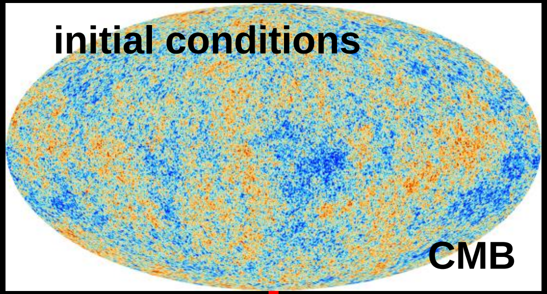
Fig. from Mo, Mao and White, 2010



DM gravity only

Aquarius project Springel+08

The Cold Dark Matter (CDM) hypothesis is the cornerstone of the current structure formation theory

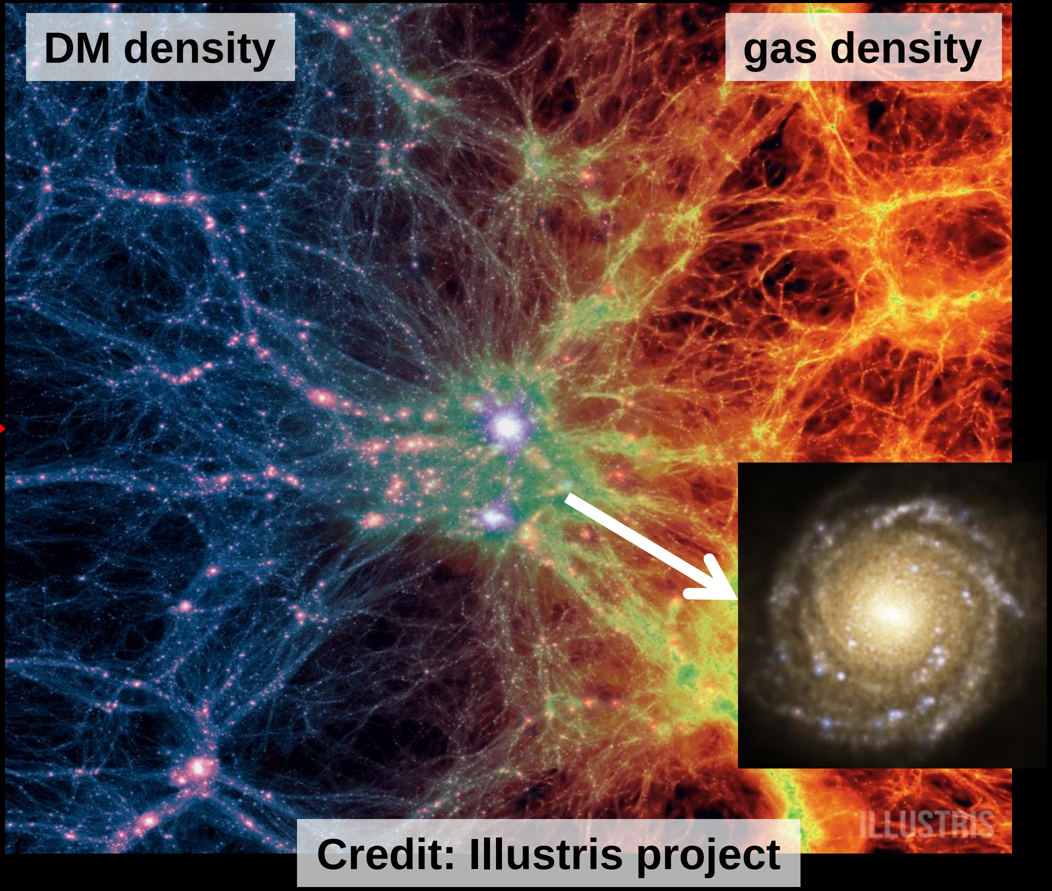


CDM assumes that the only DM interaction that matters is gravity!!

2000 CPU years!!

cosmological simulations

DM gravity only
+
"baryonic" physics (radiative cooling, gas hydrodynamics, star formation, supernova and AGN feedback,...)



100 Mpc (comoving)

Is gravity the only dark matter interaction that matters in the physics of galaxies?

despite the spectacular progress in developing a galaxy formation/evolution theory, it remains incomplete since we still don't know:

what is the nature of dark matter?

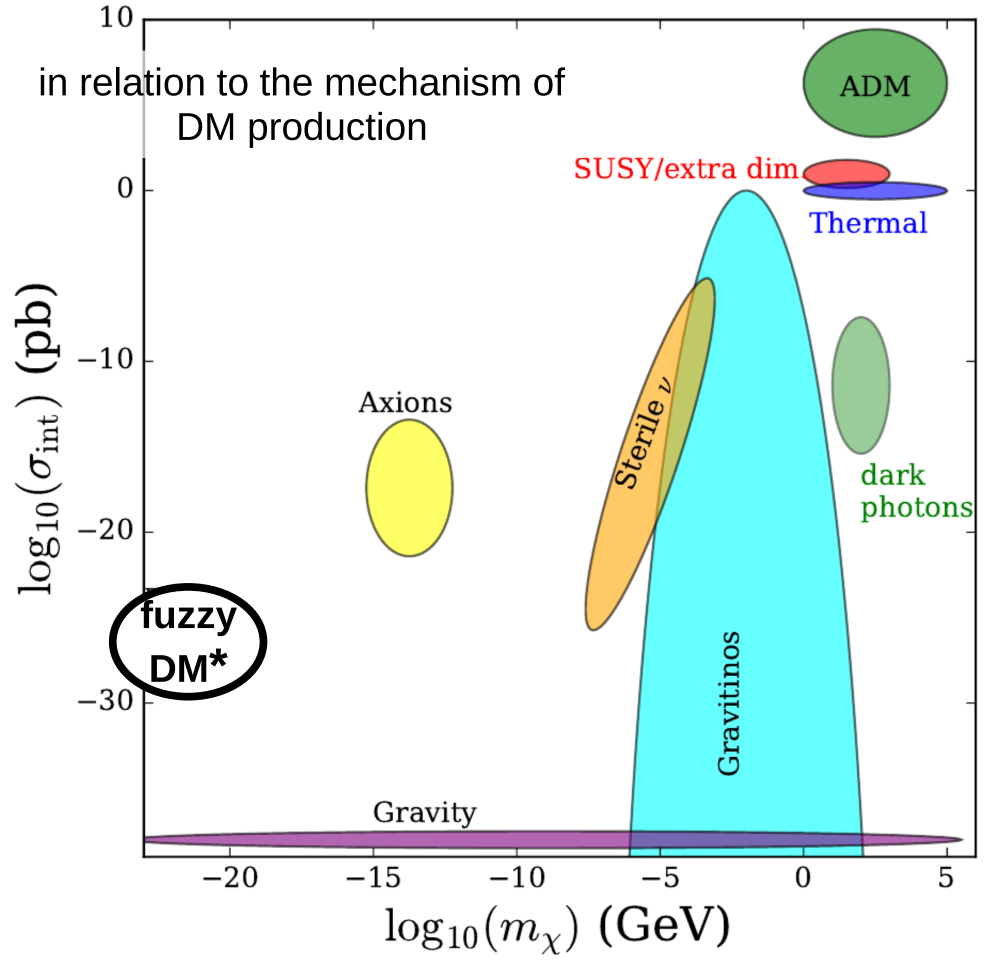
What is the mass(es) of the DM particle(s) and through which forces does it interact?

Is gravity the only dark matter interaction that matters in the physics of galaxies?

Although there is no indisputable evidence that the CDM hypothesis is wrong, there are reasonable physical motivations to consider alternatives

The (incomplete) particle DM landscape

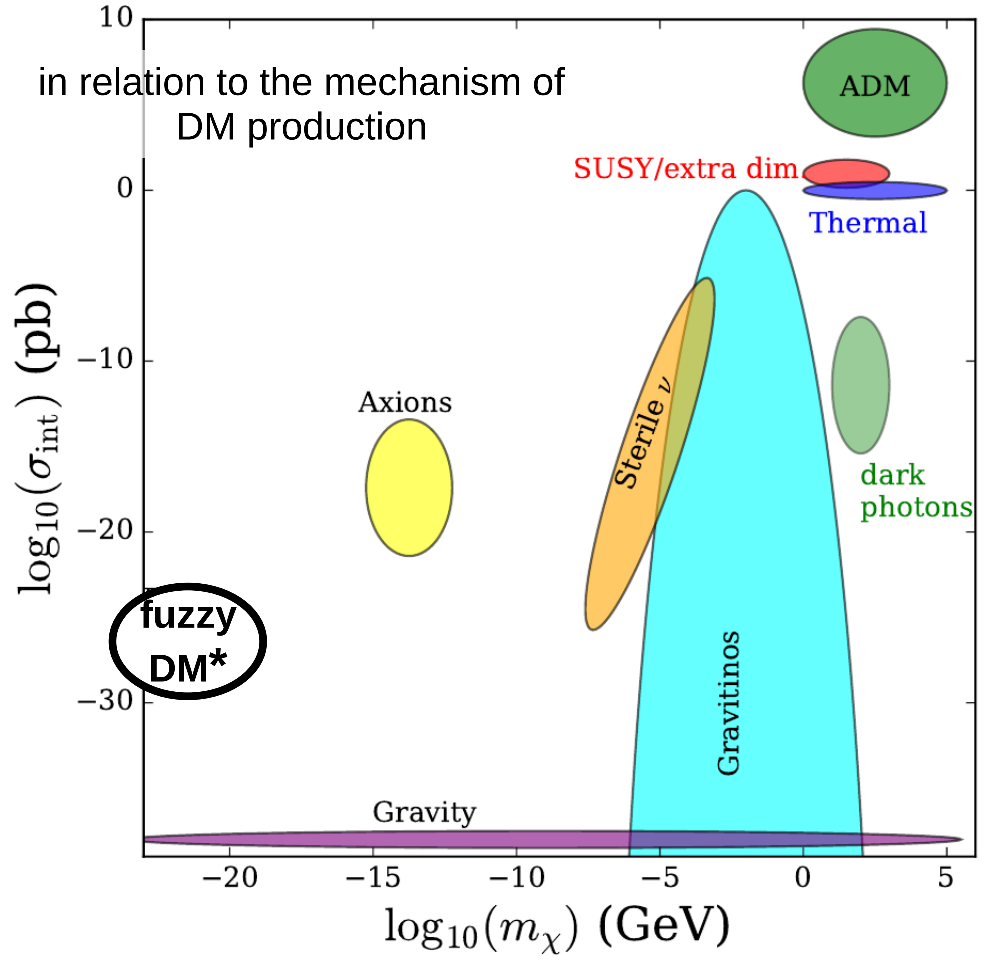
Particle physics parameter space



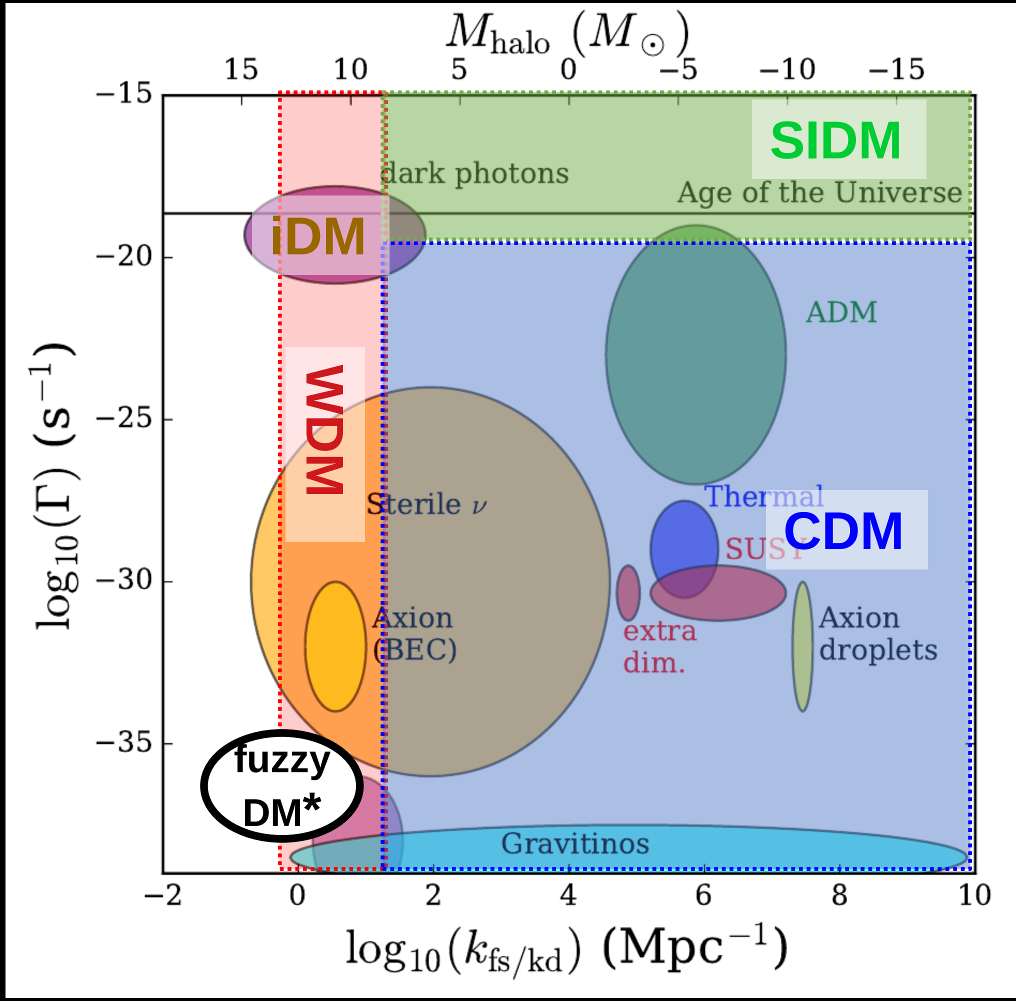
Adapted from: Buckley & Peter 2018

The (incomplete) particle DM landscape

Particle physics parameter space



Astrophysics parameter space



Adapted from: Buckley & Peter 2018

WDM: Warm Dark Matter

SIDM: Self-Interacting Dark Matter

iDM: interacting Dark Matter

two major unresolved questions in structure/galaxy formation theory

What physical mechanisms set the minimum mass scale for galaxy formation?

What physical mechanisms set the (central) dynamics within the visible galaxy?

Is it baryonic physics, is it new DM physics, or is it both?

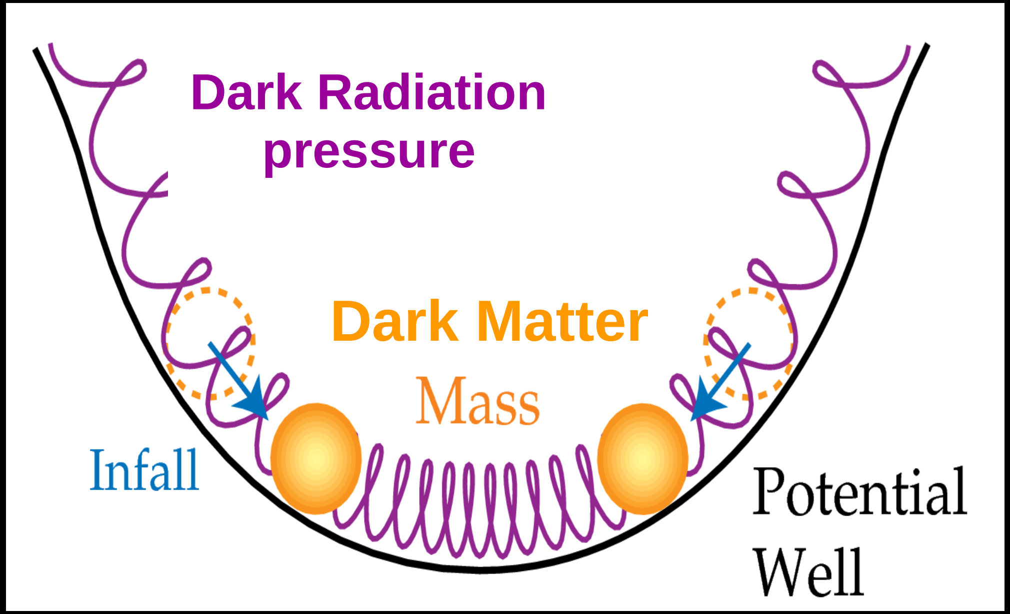
Unknown but simple "dark physics"

can DM physics induce a galactic-scale primordial power spectrum cut-off?

Allowed interactions between DM and relativistic particles (e.g. "dark radiation") in the early Universe introduce pressure effects that impact the growth of DM structures

analogous to the photon-electron-baryon plasma case: BAOs

Dark Acoustic Oscillations (DAOs)



there is also the traditional collisionless (free streaming) damping (e.g. thermal WDM)

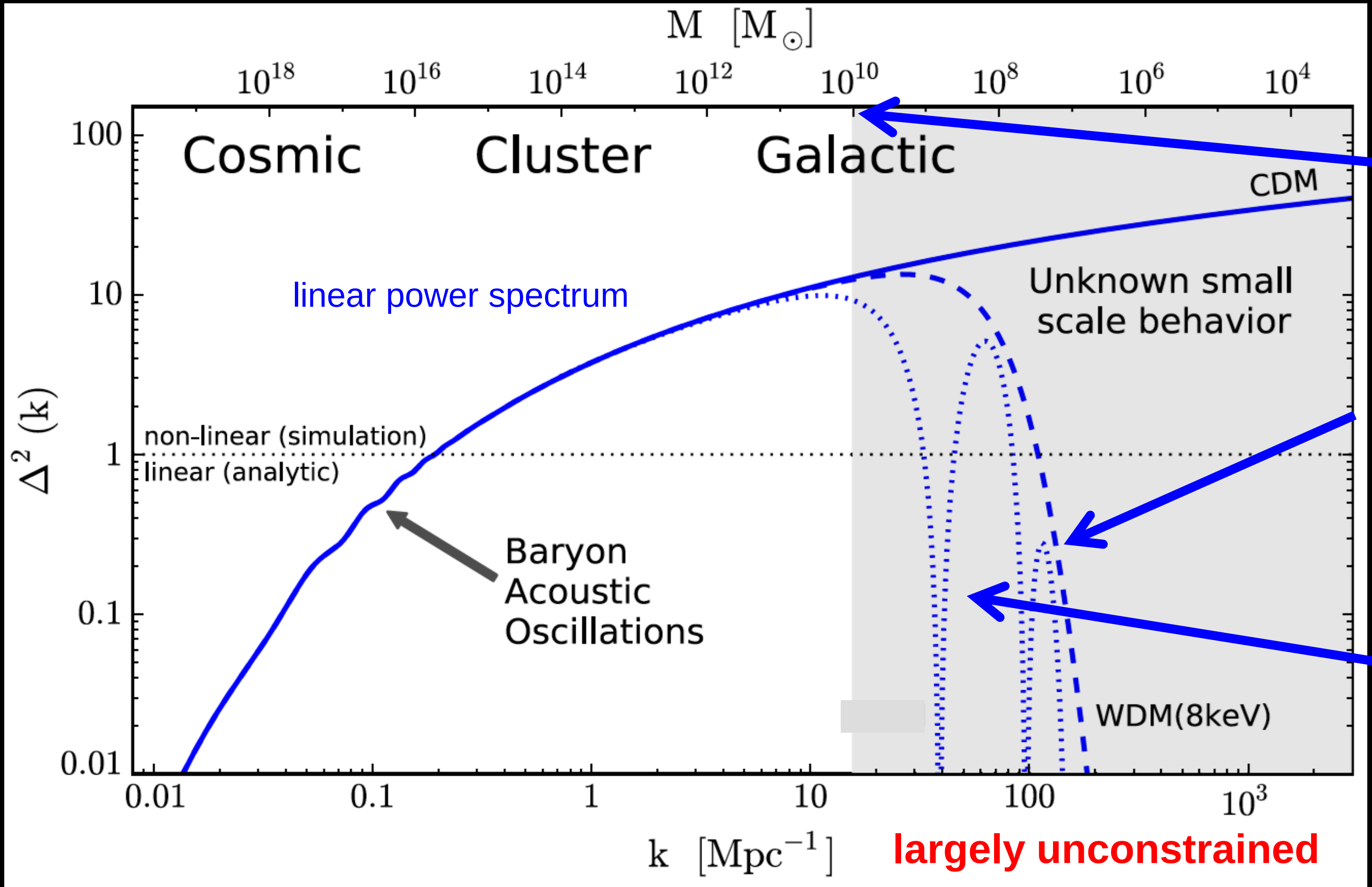
iDM

Unknown but simple “dark physics”

can DM physics induce a galactic-scale primordial power spectrum cut-off?

Observations have yet to measure the clustering of dark matter at the scale of the smallest galaxies

Kuhlen+12



Dwarf galaxies

DM is relativistic at earlier times ‘thermal’ cut-off (WDM free-streaming)

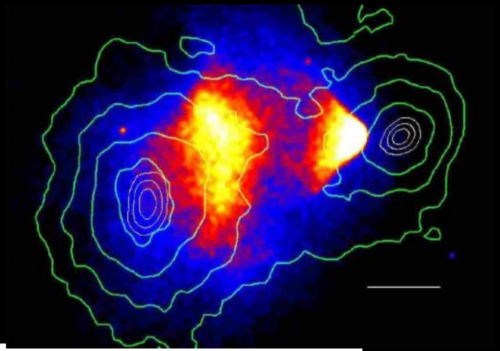
DM interacts with relativistic particles at earlier times:

DM-dark-photons DAOs
and
Silk damping

Unknown but simple "dark physics"

can DM physics change the phase-space structure of DM haloes during their evolution?

constraints allow collisional DM that is astrophysically significant in the center of galaxies

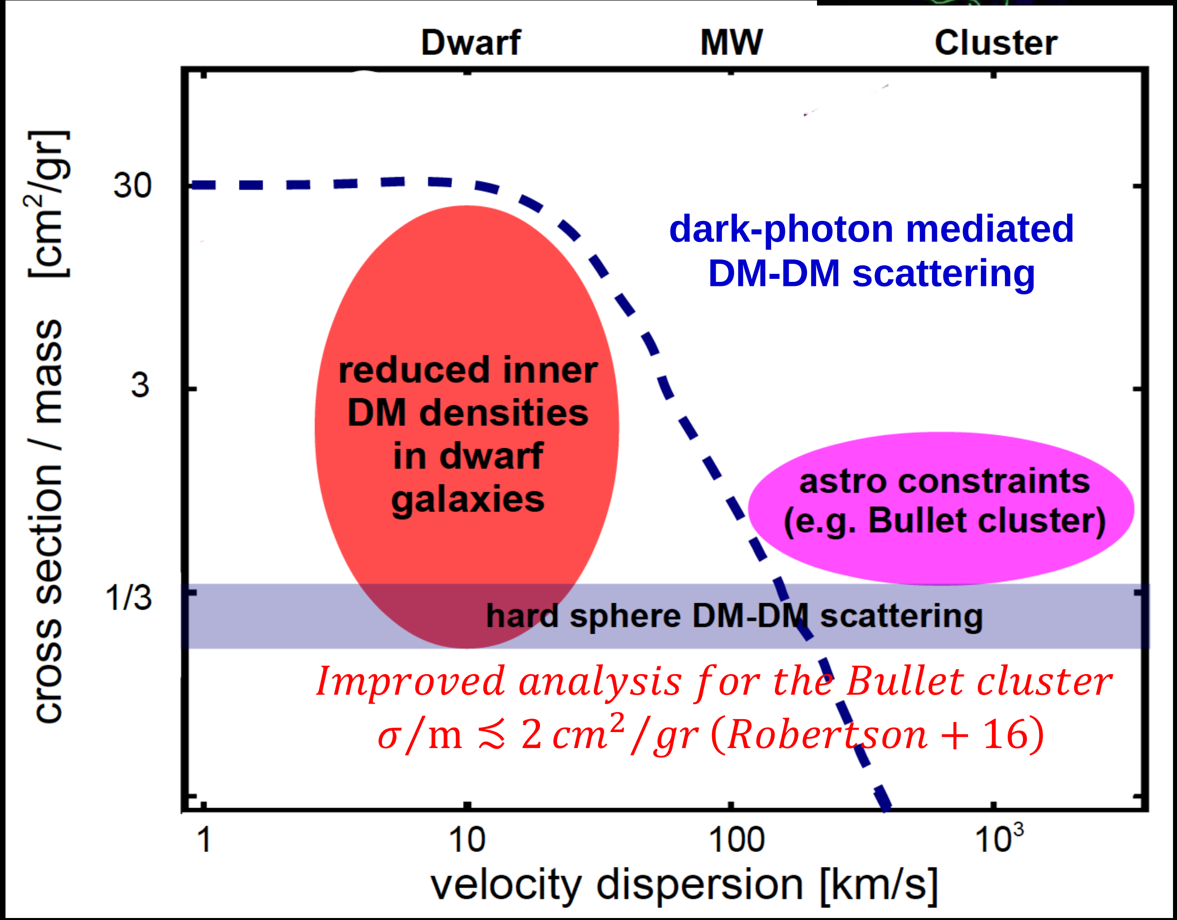


average scattering rate per particle:

$$\frac{\overline{R}_{sc}}{\Delta t} = \left(\frac{\sigma_{\rm sc}}{m_\chi}\right) \overline{\rho}_{\rm dm}\ \overline{v}_{\rm typ}$$

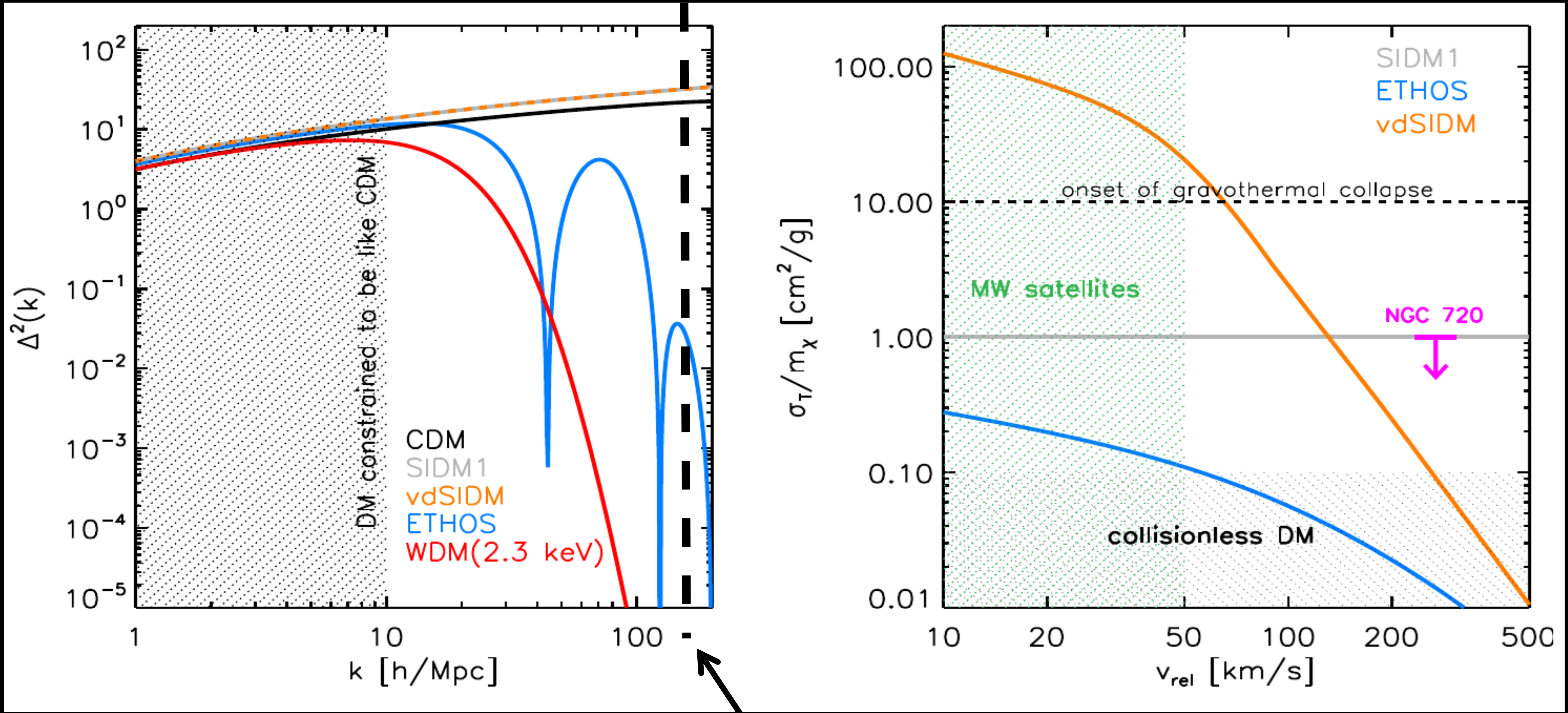
~ 1 scatter / particle / Hubble time

Neither a fluid nor a collisionless system: ~ rarefied gas



SIDM

Additional free DM parameters might play a key role in the physics of galaxies. The window is relatively narrow.

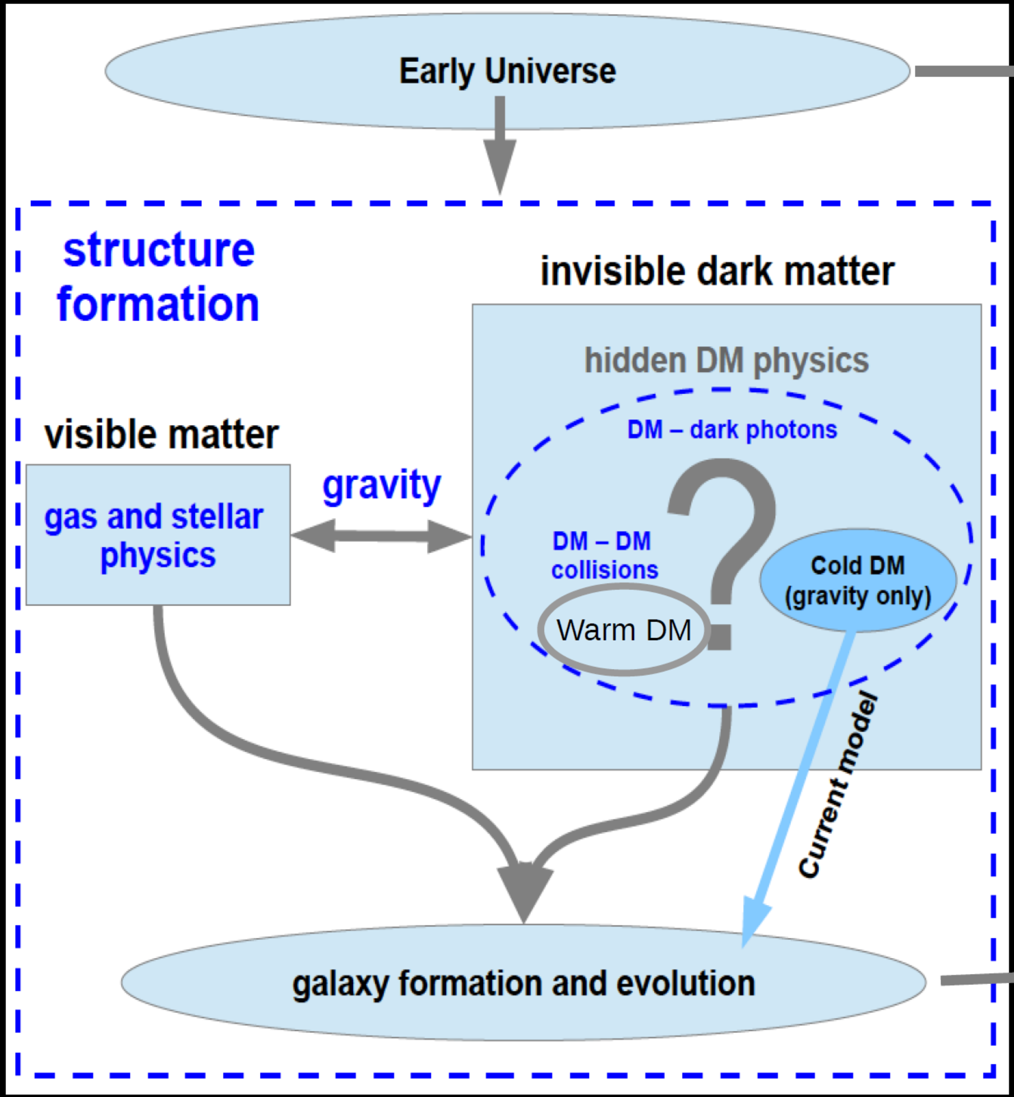


below this value galaxy formation Is highly supressed (reionisation)

$10^{9.5}\mathbf{M}_{Sun}$ *at* $\mathbf{z} = 0$

Zavala et al. 2019

An Effective THeory Of Structure formation (ETHOS)



DM production mechanism (verify consistency with global DM abundance)

Generalize the theory of structure formation (CDM) to include **a broader range of allowed DM phenomenology** coupled with our knowledge of galaxy formation/evolution

Signatures of non-gravitational DM interactions (dynamical, visible byproducts)

Cyr-Racine et al. 2016, Vogelsberger et al. 2016, Lovell et al. 2018, Bohr et al. 2020,…