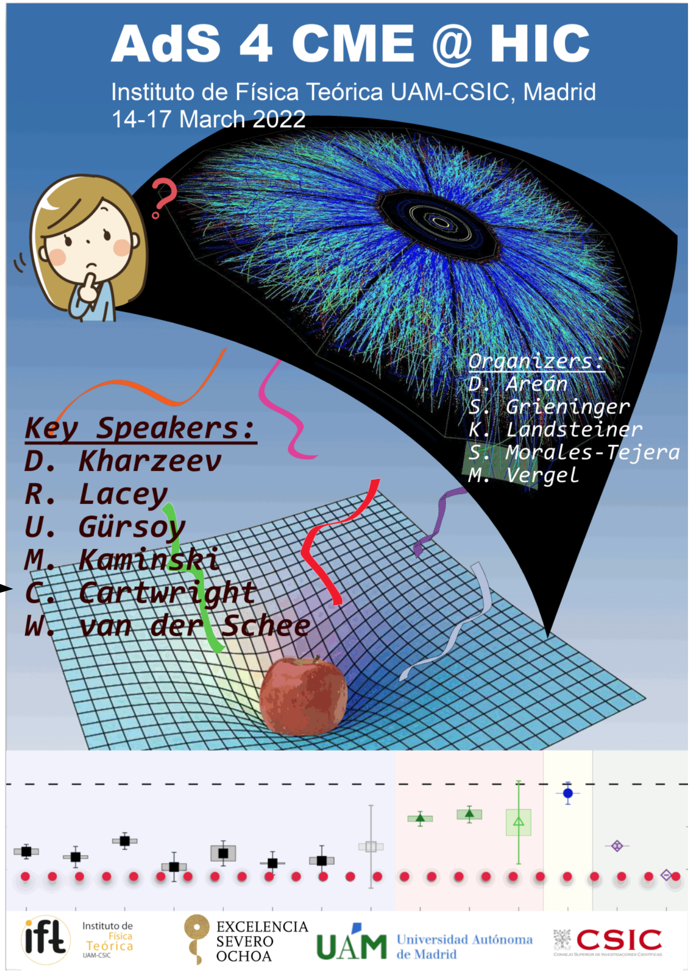
Real time holographic model of the CME

Casey Cartwright

ArXiv:2112.13857 (PRC 2022)



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Outline

1. Introduction and motivation

2. Holographic Model and time dependent AdS solutions

3.Analysis

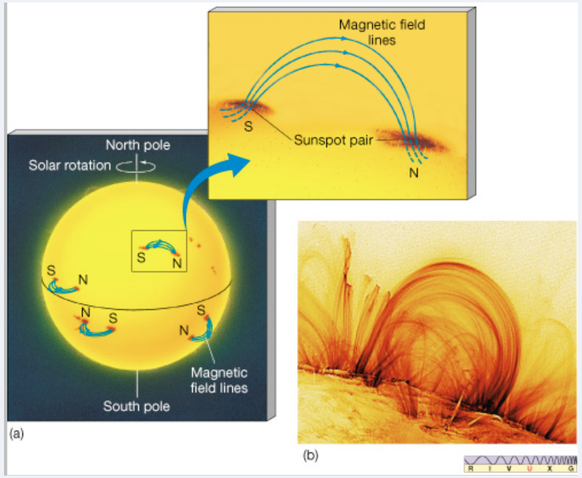
4. Conclusions

Heavy ion collisions and external fields

[Okamoto, Sakurai, 2018]
[Treumann, Baumjohann, Balogh, 2014]
[Skokov, Illarionov, Toneev, 2009]

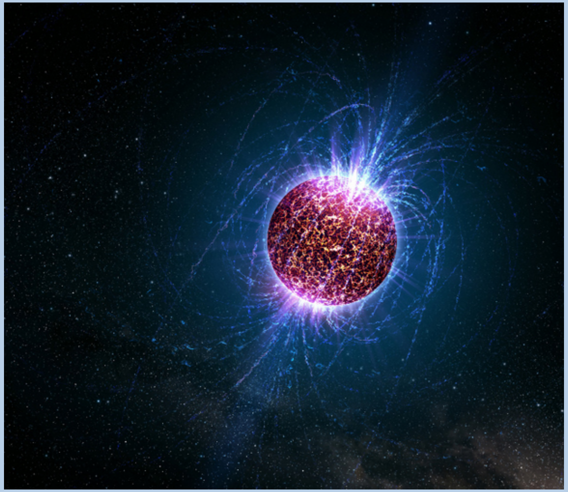
The magnetic fields generated in the collisions of heavy ions are the largest ever created

Estimates of peak values



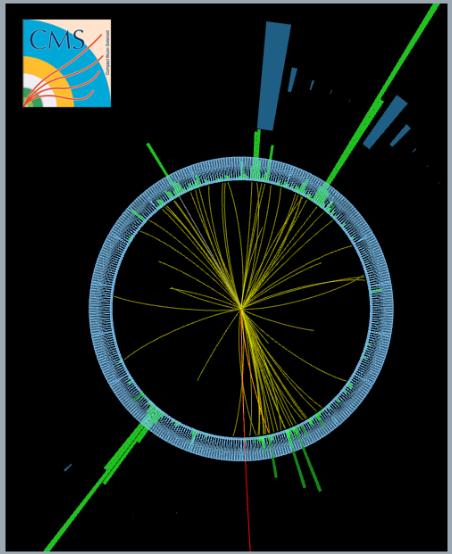
Sunspots $B \approx 10^3 G$

[Chaisson, McMillan, 2014] accessed at here



Magnetars $B \approx 10^{15} G$

[NASA, Casey Reed], accessed here



RHIC $B \approx 10^{19} G$

LHC $B \approx 10^{20} G$

[CERN 2021]

Earth $B \approx 0.65 G$

[NOAA]

Introduction Chiral Magnetic Effect in Heavy Ion Collisions

(CME) — the phenomenon of electric charge separation along the external magnetic field induced by the chirality imbalance. [Kharzeev, 2014]

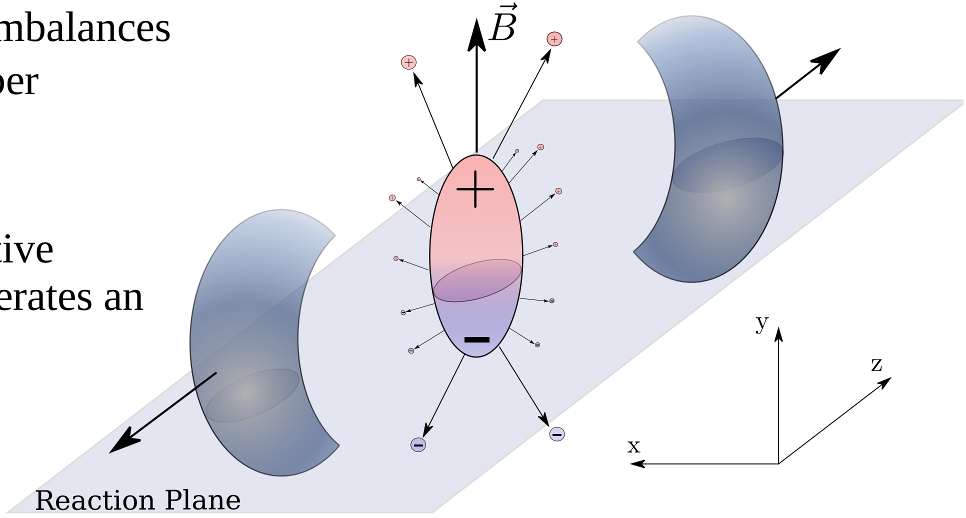
$$\partial_\mu j_5^\mu = 2im_q\bar{q}\gamma_5 q - \frac{g^2 N_f}{32\pi^2}\epsilon^{\alpha\beta\mu\nu}F^c_{\alpha\beta}F^c_{\mu\nu}, \quad j_5^\mu = \bar{q}\gamma^\mu\gamma^5 q$$

1. Incoming nuclei generate enormous magnetic fields

2. Topological transitions in the QCD vacuum generate chiral imbalances

   $N_L - N_R = 2Q_w N_f$ $\quad Q_w$ -Winding number

3. The orientation of the magnetic field aligns the spins of positive (negative) quarks parallel (anti-parallel) to $\vec{B}$ and hence generates an electric current

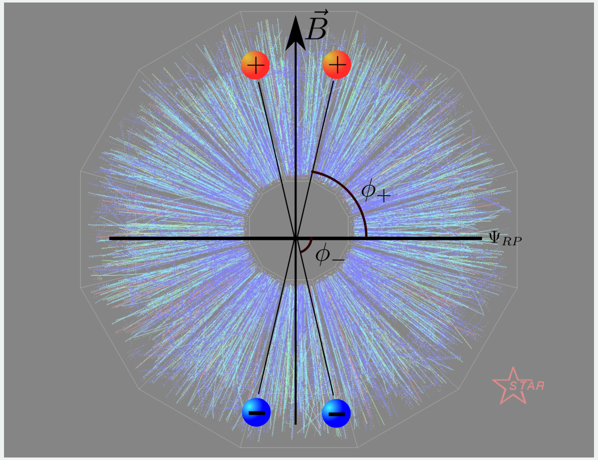


Introduction

Correlator $\gamma = \langle\cos(\phi_1 + \phi_2 - 2\Psi_{RP})\rangle$

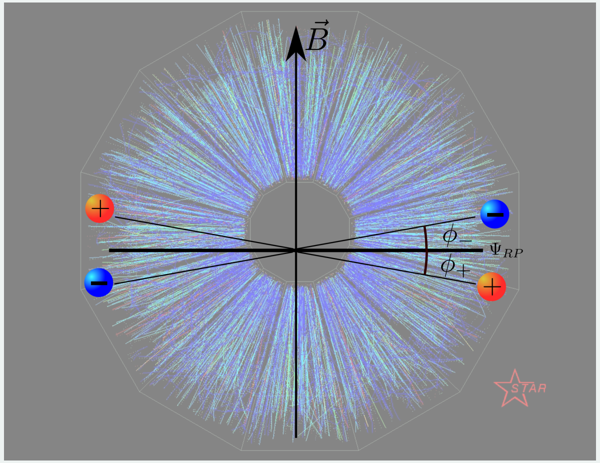
[Voloshin, 2004]
[Tribedy, 2022]

CME Contribution



$$\gamma^{OS} = \cos(\pi/2 - \pi/2) = 1\,,$$
$$\gamma^{SS} = \cos(\pi/2 + \pi/2) = -1\,,$$
$$\Delta\gamma = \gamma^{OS} - \gamma^{SS} > 0\,.$$

Background – Resonance decays

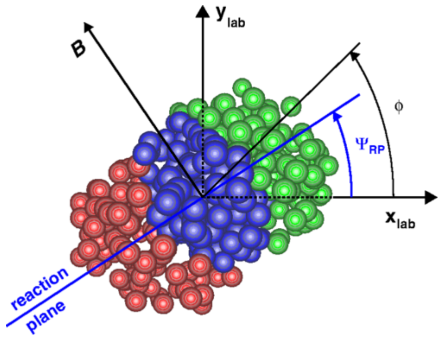


$$\gamma^{OS} = \cos(0 + 0) = 1\,,$$
$$\gamma^{SS} = \cos(0 + \pi) = -1\,,$$
$$\Delta\gamma = \gamma^{OS} - \gamma^{SS} > 0\,.$$

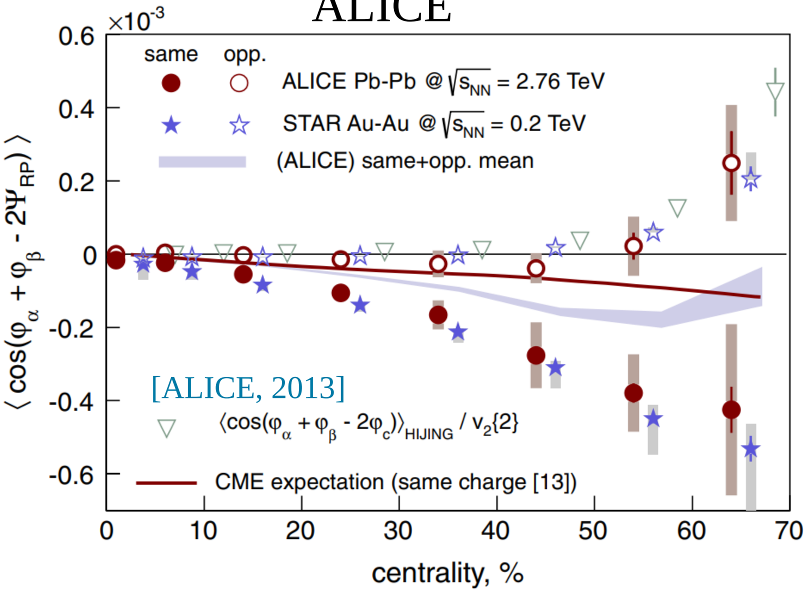
Introduction

Observations

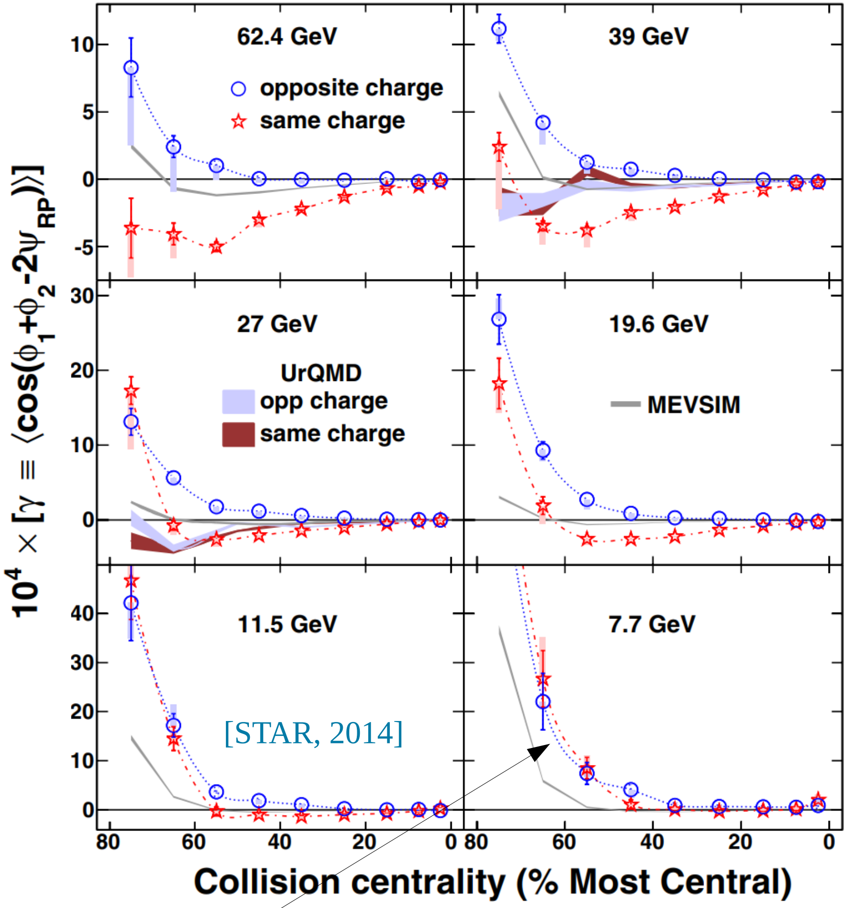
[Cartwright,Kaminski,Schenke, 2021]



ALICE



RHIC-BES



The signal is nearly gone by 7.7 GeV

Image of geometry from [STAR, 2014]

Introduction

Background

CME Signal

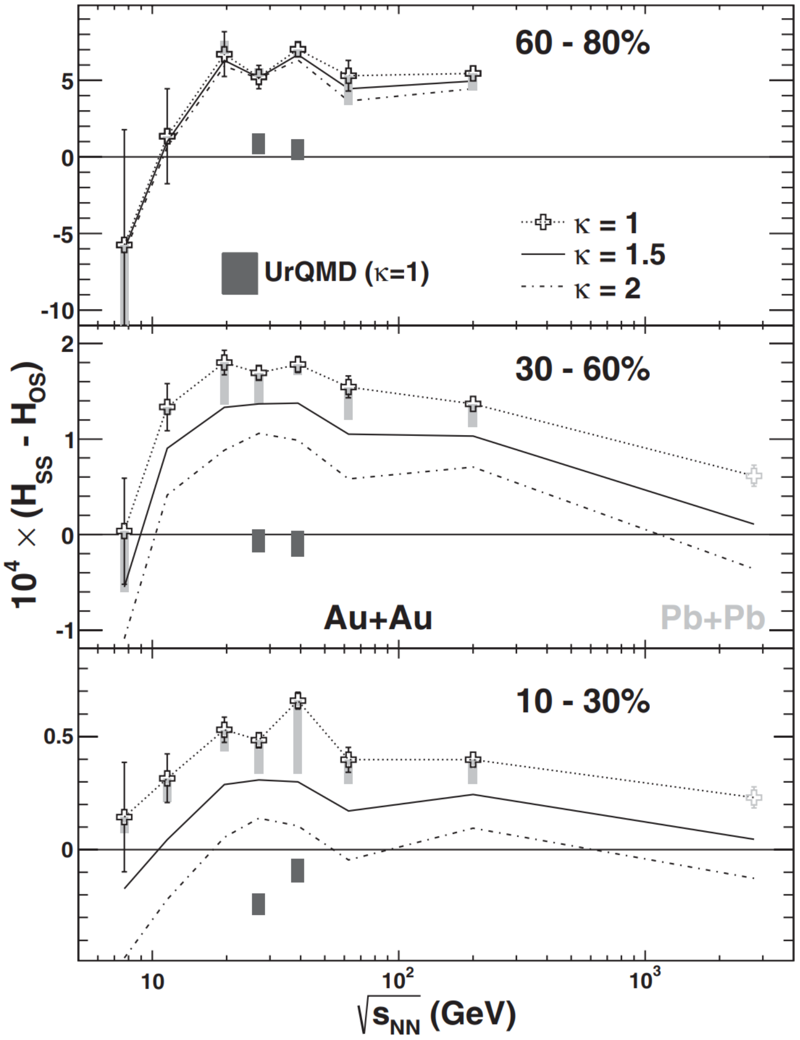
$$\gamma = \chi v_2 F - H$$

$$\delta = \langle \cos(\phi_1 - \phi_2) \rangle = F + H$$

What we want is

$$H^\chi = \frac{\chi v_2 \delta - \gamma}{1 + \chi v_2}$$

$$\Delta H^\chi = H^\chi_{SS} - H^\chi_{OS} > 0$$

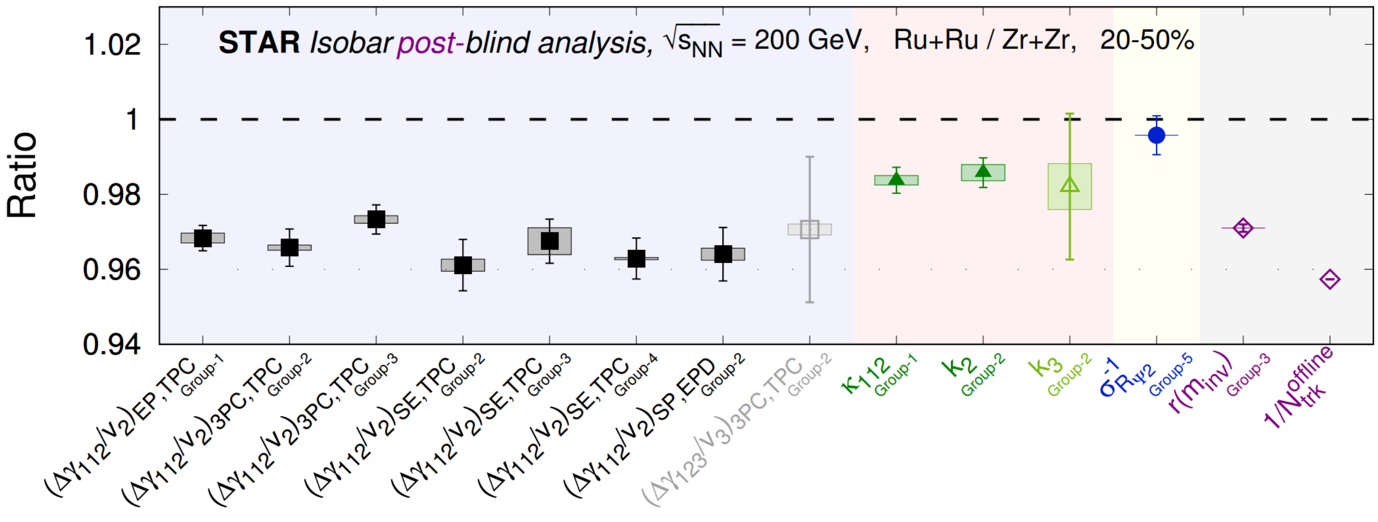


Introduction

Isobar Run

[Cartwright,Kaminski,Schenke, 2021]

[STAR,2021, arXiv:2109.00131]



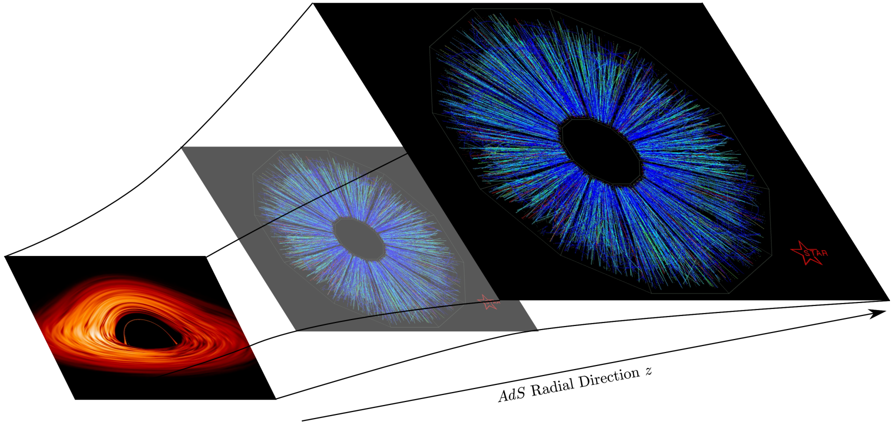
No signal satisfying **pre-blind criteria**

Necessitates a deeper understanding of the baseline and background

- Difference in collision geometry between Ru+Ru and Zr+Zr
- Effects definition of centrality and hence differences in multiplicities

BES-II (2019-2020): should we expect a pleasant surprise in the data for the CME?

AdS/CFT Correspondence or Holography



Equilibrium

Solutions to the Einstein equations correspond to states, ex: a static black hole spacetime to a finite temperature state of the field theory

Out of equilibrium

Black hole formation corresponds to the out of equilibrium evolution of states in the field theory

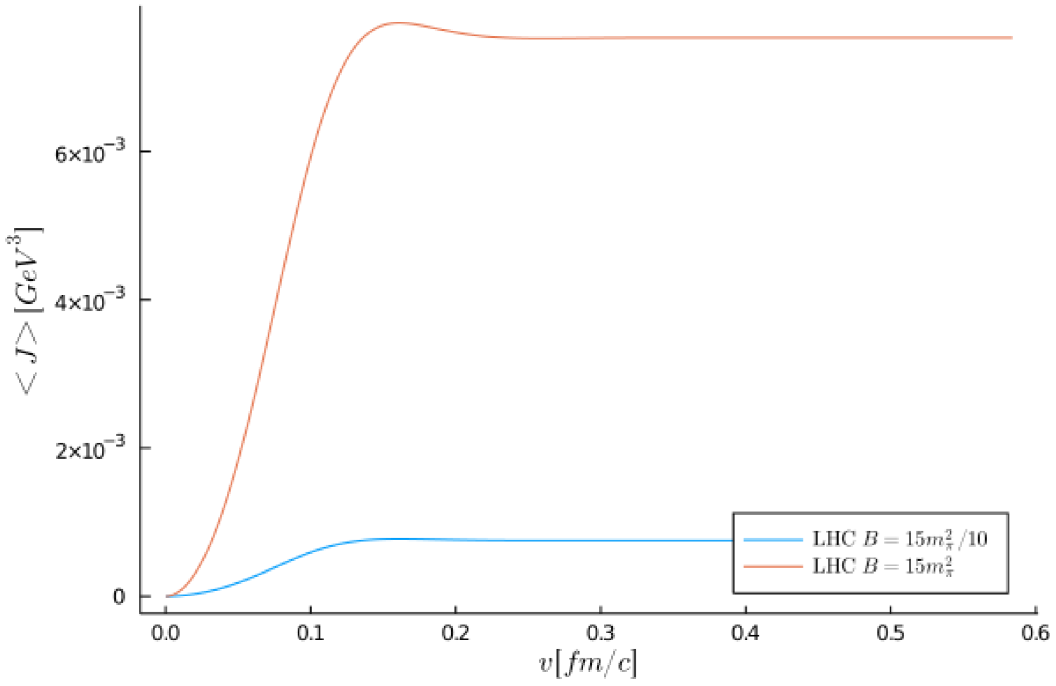
STAR Experiment: https://www.star.bnl.gov/public/tpc/photos/GeneralMisc/event.png
NASA's Goddard Space Flight Center: https://svs.gsfc.nasa.gov/13326

Holographic Model

[Cartwright,Kaminski,Schenke, 2021]

Einstein gravity coupled to $U(1)_V \times U(1)_A$ [Gosh, Grieninger, Landsteiner, Morales-Tejera, 2021]

$$S = \frac{1}{16\pi G_N} \int \mathrm{d}^5 x \sqrt{-g} \left(R - 2\Lambda - \frac{L^2}{4} F_{\mu\nu} F^{\mu\nu} - \frac{L^2}{4} F^{(5)}_{\mu\nu} F^{\mu\nu}_{(5)} + \frac{\alpha}{3} \epsilon^{\mu\nu\rho\sigma\tau} A_\mu \left(3 F_{\nu\rho} F_{\sigma\tau} + F^{(5)}_{\nu\rho} F^{(5)}_{\sigma\tau} \right) \right) + S_{ct}$$



–Isotropization as a model of thermalization

–Time dependent axial charge density $n_5(\tau)$

–Static magnetic field

–Significant build of CME current is possible within the lifetime of the magnetic field

Holographic Model

[Cartwright,Kaminski,Schenke, 2021]

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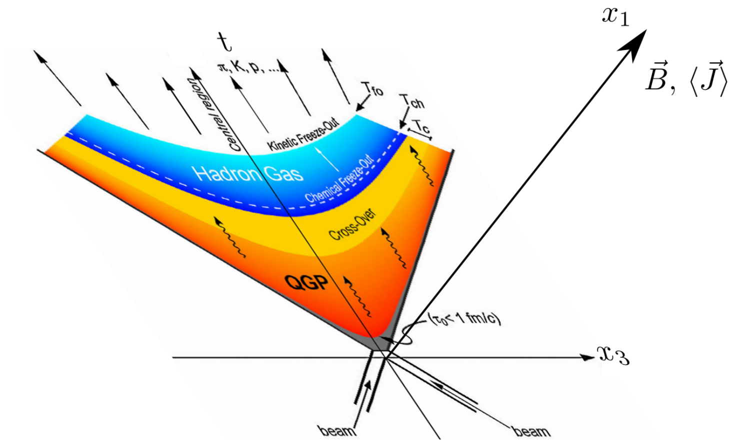


–Boost invariant expanding SYM plasma

–Time dependent axial charge density $n_5(\tau)$

–Time dependent magnetic field perpendicular to expansion

[Braun-Munzinger, Dönigus, 2019]

Holographic Model

[Cartwright,Kaminski,Schenke, 2021]

Einstein gravity coupled to $U(1)_V \times U(1)_A$ [Gosh, Grieninger, Landsteiner, Morales-Tejera, 2021]

$$S = \frac{1}{16\pi G_N}\int \mathrm{d}^5x\sqrt{-g}\left(R - 2\Lambda - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu} - \frac{L^2}{4}F^{(5)}_{\mu\nu}F^{\mu\nu}_{(5)} + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu\left(3F_{\nu\rho}F_{\sigma\tau} + F^{(5)}_{\nu\rho}F^{(5)}_{\sigma\tau}\right)\right) + S_{ct}$$

The AdS/CFT Dictionary:

Metric ansatz ⟷ Dual Energy Momentum tensor

$$\begin{aligned}\mathrm{d}s^2 &= 2\mathrm{d}v(\mathrm{d}r - \frac{1}{2}A(v,r)\mathrm{d}v)\\ &+ S(v,r)^2e^{H_1(v,r)}\mathrm{d}x_1^2\\ &+ S(v,r)^2e^{H_2(v,r)}\mathrm{d}x_2^2\\ &+ L^2S(v,r)^2e^{-H_1(v,r)-H_2(v,r)}\mathrm{d}\xi^2\end{aligned}$$

ξ - Spacetime rapidity

$$\langle T^i_j\rangle \sim \mathrm{diag}(\epsilon(\tau), P_1(\tau), P_2(\tau), P_3(\tau))$$

$$\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, \tau^2)$$

Axial gauge field ansatz ⟷ Dual axial current

$$A_\mu = (-\phi(v,r)/L, 0, 0, 0, 0)$$

$$\langle J^i_{(5)}\rangle \sim (n_5(\tau), 0, 0, 0)$$

Vector gauge field ansatz ⟷ Dual vector current

$$V_\mu = (0, -V_x(v,r)/L, \xi B/L, 0, 0)$$

$$\langle J^i\rangle \sim (0, \tilde{V}_x(\tau), 0, 0)$$

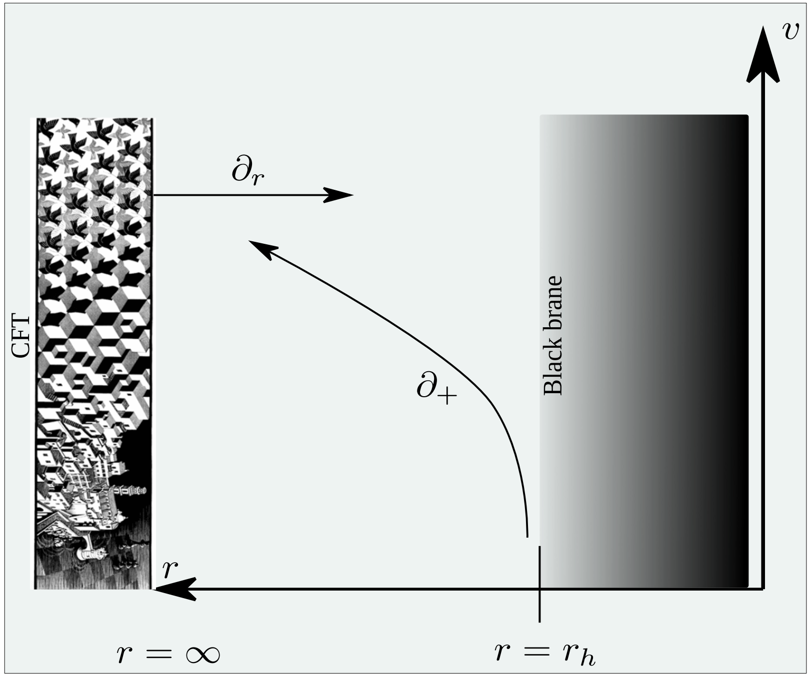
Holographic Model

Numerical Relativity

r Holographic coordinate

$\mathbf{x}$ Field theory coordinates

Einstein Equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$



Escher, M. C., Metamorphosis II, 1940, accessed here

To simplify we use Bondi-Sachs formalism [Chesler,Yaffe,2014]

- Use diffeomorphism invariance to fix ansatz

$$ds^2 = 2\mathrm{d}v\mathrm{d}r + r^2 \tilde{g}_{ij}(r, \mathbf{x})\mathrm{d}x^i \mathrm{d}x^j$$

- Write Einstein equations in terms of directional derivatives along null infalling/out-going geodesics

In-falling $f'(r, \mathbf{x}) = \partial_r f(r, \mathbf{x})$

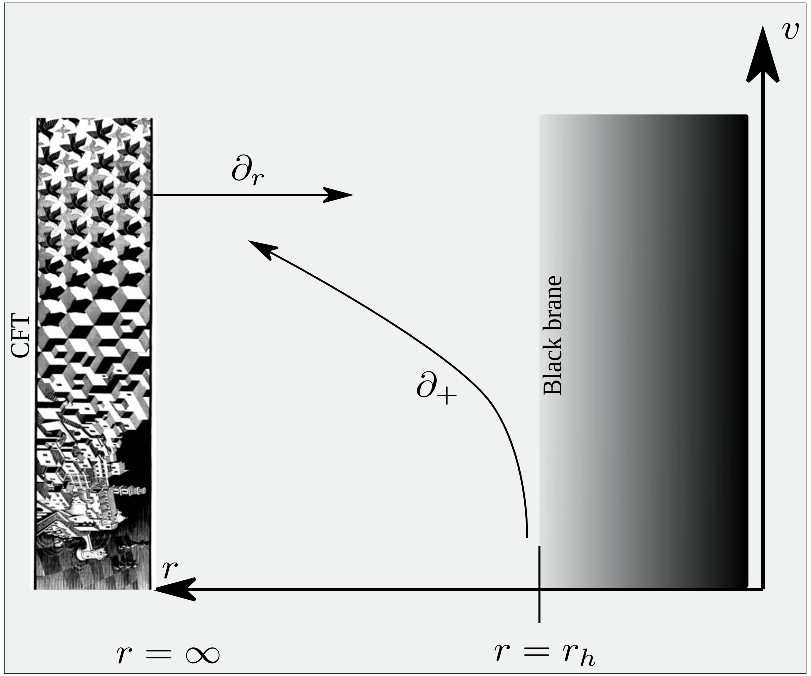
Out-going $\dot{f}(r, \mathbf{x}) \equiv \partial_t f(r, \mathbf{x}) + \frac{1}{2} \partial_r f(r, \mathbf{x}) g_{tt}(r, \mathbf{x})$

Holographic Model

Numerical Relativity

r	Holographic coordinate
$\mathbf{x}$	Field theory coordinates

Einstein Equations $\quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$



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Leads to hierarchical/nested structure for

- A set of hypersurface variables A
- A set of evolution variables B

$$\partial_r A = H_A(A, B)$$
$$\partial_r \dot{B} = H_B(A, B, \dot{B})$$

Characteristic evolution is a predictive scheme in AdS (non-globally hyperbolic) spacetime provided appropriate boundary conditions are applied at the conformal boundary

[Wald, 1980]
[Winicour,2005] [Wald, Ishibashi, 2003]
[Chesler,Yaffe,2014] [Wald, Ishibashi, 2005]

Holographic Model

Numerical routine

[Cartwright,Kaminski,Schenke, 2021]

1. Provide initial data $H_i(v_0, r), V(v_0, r), \epsilon(v_0), \xi(v_0)$

2. Solve line by line

$$
\begin{aligned}
0 &= zS(v,z)^2\left(H_1'(v,z)H_2'(v,z)+H_1'(v,z)^2+H_2'(v,z)^2\right)+ze^{-H_1(v,z)}V'(v,z)^2\\
&\quad+6\left(2S'(v,z)+zS''(v,z)\right)S(v,z)\,,\\
0 &= L^6b^2e^{H_1(v,z)}S(v,z)^2+\left(L^3q_5-8\alpha bV(v,z)\right)^2-24L^6z^2S(v,z)^4S'(v,z)\dot{S}(v,z)\\
&\quad-12L^6z^2S(v,z)^5\dot{S}'(v,z)-24L^6S(v,z)^6\,,\\
0 &= -64\alpha^2b^2e^{H_1(v,z)}V(v,z)+8\alpha bL^3q_5e^{H_1(v,z)}-L^6z^2S(v,z)^3\left(S'(v,z)\dot{V}(v,z)+\dot{S}(v,z)V'(v,z)\right)\\
&\quad+L^6z^2S(v,z)^4\left(H_1'(v,z)\dot{V}(v,z)+\dot{H}_1(v,z)V'(v,z)-2\dot{V}'(v,z)\right)\,,\\
0 &= -9z^2S(v,z)^3\left(H_1'(v,z)\dot{S}(v,z)+\dot{H}_1(v,z)S'(v,z)\right)-4z^2e^{-H_1(v,z)}S(v,z)^2V'(v,z)\dot{V}(v,z)\\
&\quad-6z^2\dot{H}_1{}'(v,z)S(v,z)^4-2b^2e^{H_1(v,z)}\,,\\
0 &= -6z^2\dot{H}_2{}'(v,z)S(v,z)^4+b^2e^{H_1(v,z)}+2z^2e^{-H_1(v,z)}S(v,z)^2V'(v,z)\dot{V}(v,z)\\
&\quad-9z^2S(v,z)^3\left(H_2'(v,z)\dot{S}(v,z)+\dot{H}_2(v,z)S'(v,z)\right)\,,\\
0 &= 3L^4S(v,z)^6\Big(2L^2z^4A''(v,z)+4z^3A'(v,z)-L^2z^2\dot{H}_1(v,z)\left(2H_1'(v,z)+H_2'(v,z)\right)\\
&\quad-L^2z^2H_1'(v,z)\dot{H}_2(v,z)-2L^2z^2H_2'(v,z)\dot{H}_2(v,z)+8L^2\Big)-5b^2L^6e^{H_1(v,z)}S(v,z)^2\\
&\quad+2L^6z^2e^{-H_1(v,z)}S(v,z)^4\left(36e^{H_1(v,z)}S'(v,z)\dot{S}(v,z)-V'(v,z)\dot{V}(v,z)\right)\\
&\quad-7\left(L^3q_5-8\alpha bV(v,z)\right)^2\,,\\
0 &= 3z^2A'(v,z)S(v,z)\dot{S}(v,z)+L^2e^{-H_1(v,z)}\dot{V}(v,z)^2+L^2\dot{H}_1(v,z)\dot{H}_2(v,z)S(v,z)^2\\
&\quad+L^2\dot{H}_1(v,z)^2S(v,z)^2+L^2\dot{H}_2(v,z)^2S(v,z)^2+6L^2S(v,z)\ddot{S}(v,z)\,.
\end{aligned}
$$

3. Obtain time derivative

$$\partial_v H_i(r,v) = \dot{H}_i - \frac{1}{2}A(r,v)\partial_r H_i(r,v)$$

$$\partial_v V(r,v) = \dot{V} - \frac{1}{2}A(r,v)\partial_r V(r,v)$$

4. Step forward in time

5. Provide new initial condition

$H_i(v_0+n\Delta v, r), V(v_0+n\Delta v, r), \epsilon(v_0+n\Delta v), \xi(v_0+n\Delta v)$

6. Repeat steps 2-5 until final time is reached

Initial Data and Solutions

[Cartwright,Kaminski,Schenke, 2021]

Field theory:

Initial time: τ_0

Initial axial charge density: $\langle J^0_{(5)} \rangle = n_5(\tau_0)$

Initial vector current density: $\langle J^1 \rangle (\tau_0) = 0$

Initial energy density: $\epsilon(\tau_0)$

Initial Magnetic field: $B(\tau_0) = B_0/\tau_0$

AdS bulk:

Spacetime ansiotropy: $H_1(v_0, r),\ H_2(v_0, r)$

Deviation from AdS $H_i = H_i^{(d)} - \frac{2}{3}\log(v+z)$

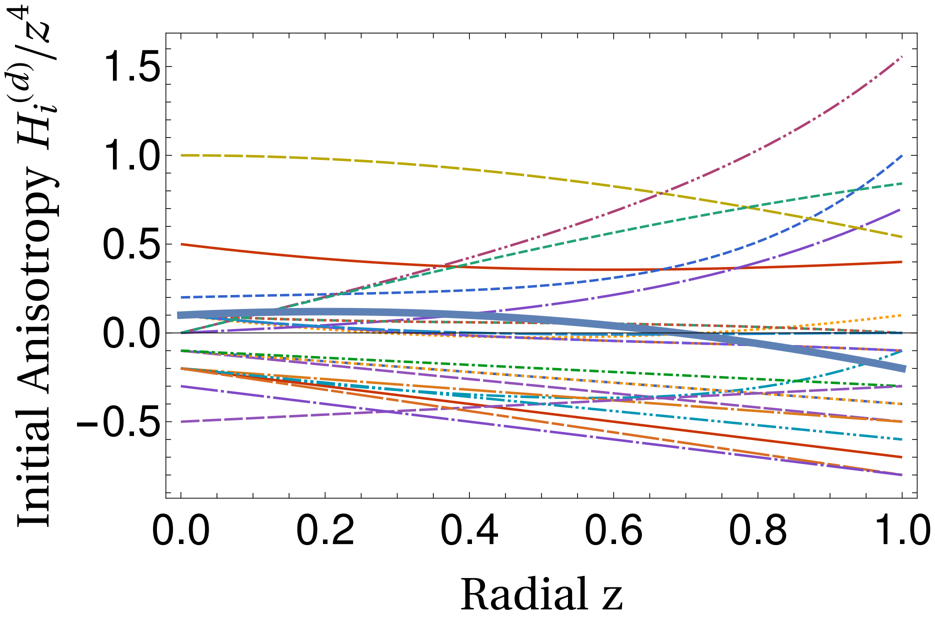
$$H_i^{(d)}(z)/z^4 = \Omega_1 \cos(\gamma_1 z) + \Omega_2 \tan(\gamma_2 z) + \Omega_3 \sin(\gamma_3 z) + \sum_{j=0}^{4} \beta_j z^j$$

[Rougemont et. al.,2021]

Parameters:

Anomaly coefficient: $\alpha = \alpha_{SUSY} = 1/\sqrt{3}$

Highlighted curve will serve as a representative curve in coming slides

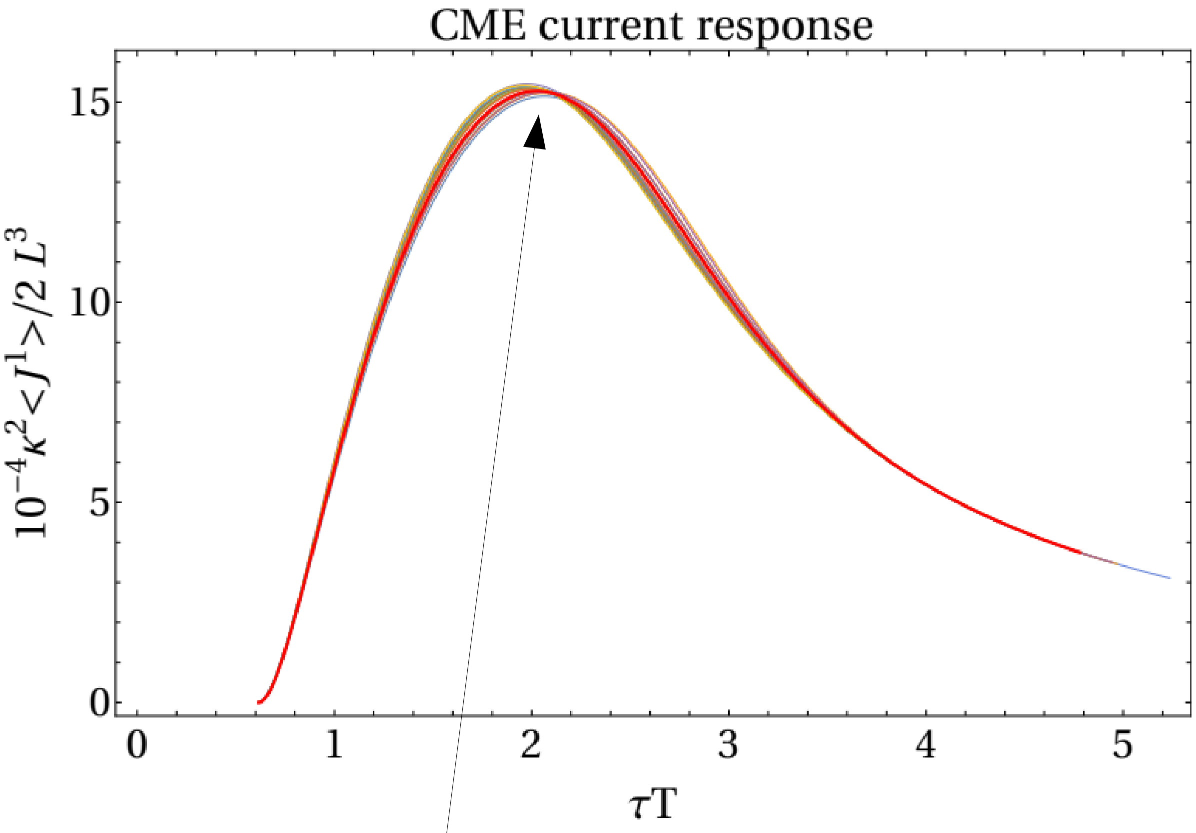


Holographic Model

Energy Momentum and Vector Current

[Cartwright,Kaminski,Schenke, 2021]

22 initial conditions are displayed at fixed $\epsilon(\tau_0),\ n_5(\tau_0),\ B(\tau_0)$



CME response is roughly independent of choices of spacetime anisotropy

Holographic Model

Matching to QCD

[Cartwright,Kaminski,Schenke, 2021]

1. Set AdS radius $L = 1 fm$ and Chern-Simons coupling to $\alpha = \alpha_{QCD} = 6/19$

2. Fix gravitational coupling $\kappa^2 = 8\pi G = \frac{4\pi^2 L^3}{N_c^2}$

3. Late time temperature obeys $T(\tau) = \Lambda^{8/3}\left(\frac{1}{\tau}\right)^{4/3} - \frac{2\Lambda^2}{(3\pi)\tau^2} + O\left(\tau^{-7/3}\right)$

–Extract Λ from late time data

–Use scaling symmetry to set overall scale $T(7\, fm/\lambda)\lambda = 90\, MeV$ $\lambda = 0.768$

$$\tilde{x}^i = \frac{x^i}{\lambda}, \quad \tilde{B} = \lambda^2 B, \quad \tilde{\mu}_{(5)} = \lambda \mu_{(5)}\,,$$

$$\langle \tilde{T}_{\mu\nu} \rangle = \lambda^4 \langle T_{\mu\nu} \rangle\,, \; \langle \tilde{J}^i_{(5)} \rangle = \lambda^3 \langle J^i_{(5)} \rangle\,, \; \langle \tilde{J}^i \rangle = \lambda^3 \langle J^i \rangle \;,$$

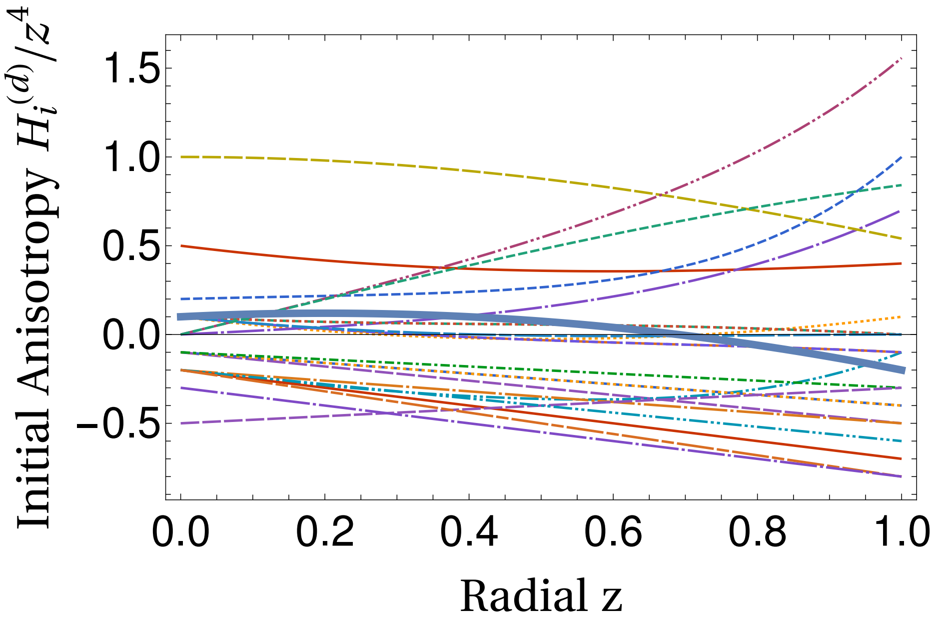
Holographic Model

Program:

1. Fix spacetime anisotropy function to characteristic form

2. Perform initial data run and obtain scaling

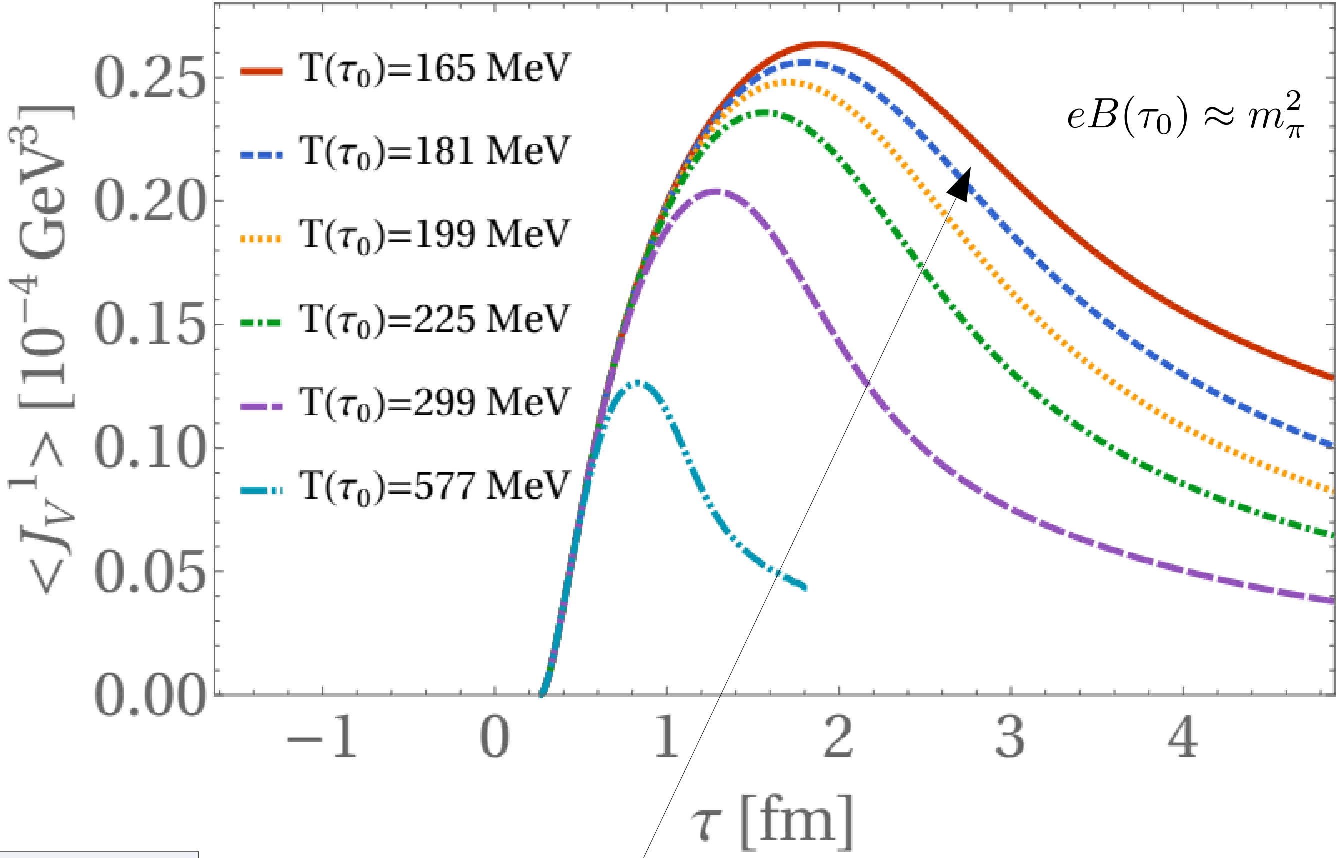
3. Scan over different initial energy densities while changing the energy dependence of initial parameters



Holographic Model

CME – I

[Cartwright,Kaminski,Schenke, 2021]



– Scan over different initial energy densities with $n_5(\tau_0), B(\tau_0), \alpha_{QCD}$

–Estimate for Ru+Ru at $200GeV$

$n_5(\tau_0) = 0.0014GeV^3$

$\sqrt{s}\ [GeV]$	19	27	39	64	200	2750
$T\ [MeV]$	165	181	199	225	299	577

CME signal is enhanced for decreasing initial energy

Holographic Model

CME – II

Axial charge density depends on energy of the collision [Sun, Ko, 2018]

$$\sqrt{N_5} \approx \frac{2\tau_0 \pi \rho_{\mathrm{tube}} Q_s^4 \sqrt{N_{\mathrm{coll}}}}{16\pi^2}$$

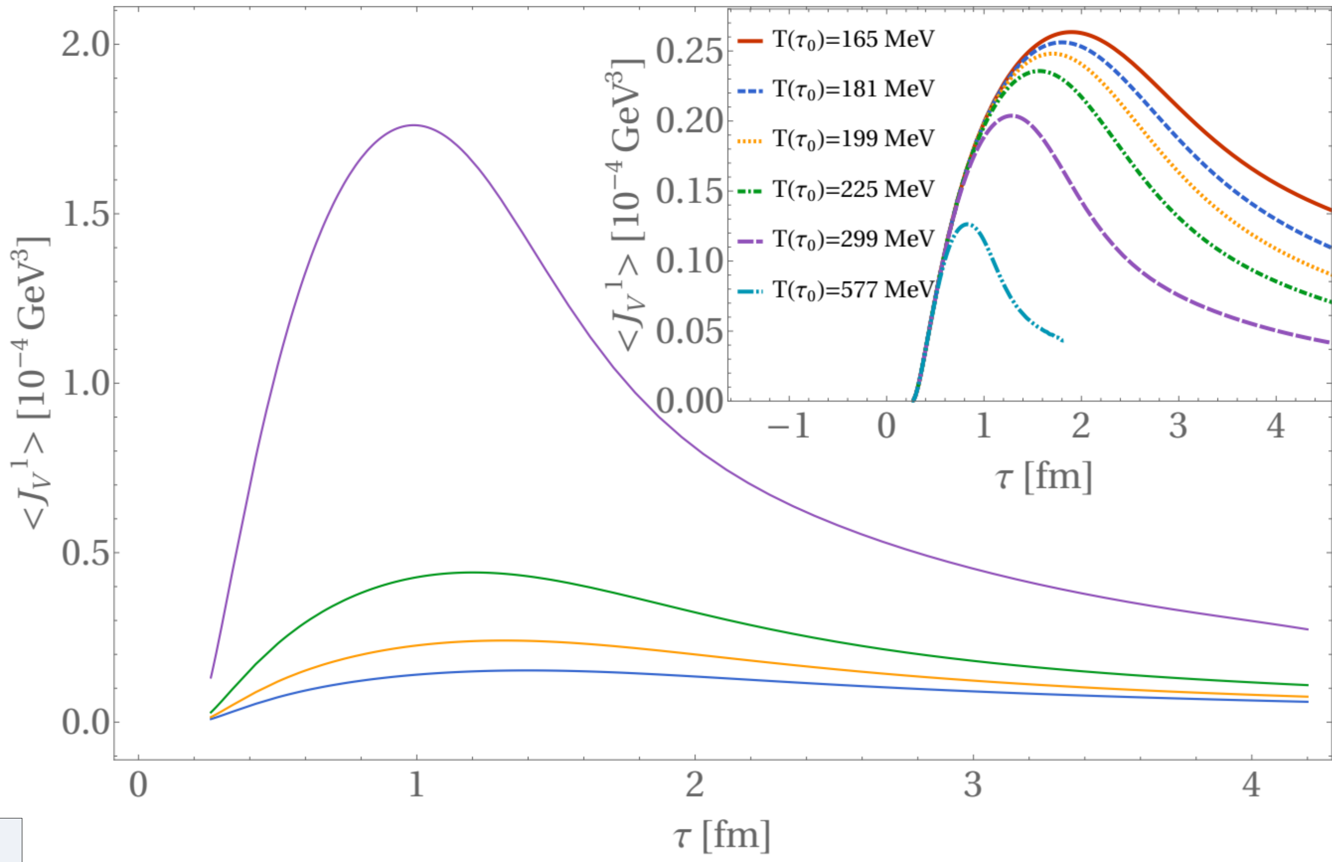
$\sqrt{s}\ [GeV]$	19	27	39	64	200	2750
$\langle J^0_{(5)} \rangle\ [10^{-3} GeV^3]$	1.2	1.4	1.6	1.9	2.7	6.6

Magnetic field strength also depends on the collision energy as well as on the particular nuclei [McLerran, Skokov, 2014]

$$\frac{eB_{RHIC}}{eB_{LHC}} = \frac{1}{2} \frac{\gamma_{LHC}}{\gamma_{RHIC}} \left(\frac{Q_s^{RHIC}}{Q_s^{LHC}} \right)^2$$

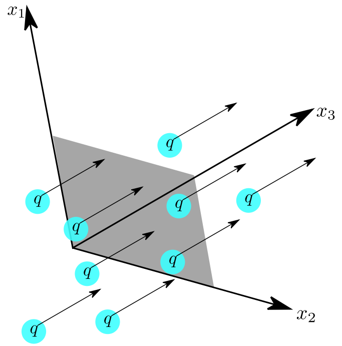
$\sqrt{s}[GeV]$	19	27	39	64	200
eB	$0.095 m_\pi^2$	$0.135 m_\pi^2$	$0.195 m_\pi^2$	$0.32 m_\pi^2$	$1 m_\pi^2$

[Cartwright,Kaminski,Schenke, 2021]



Far from equilibrium

How much signal is generated?

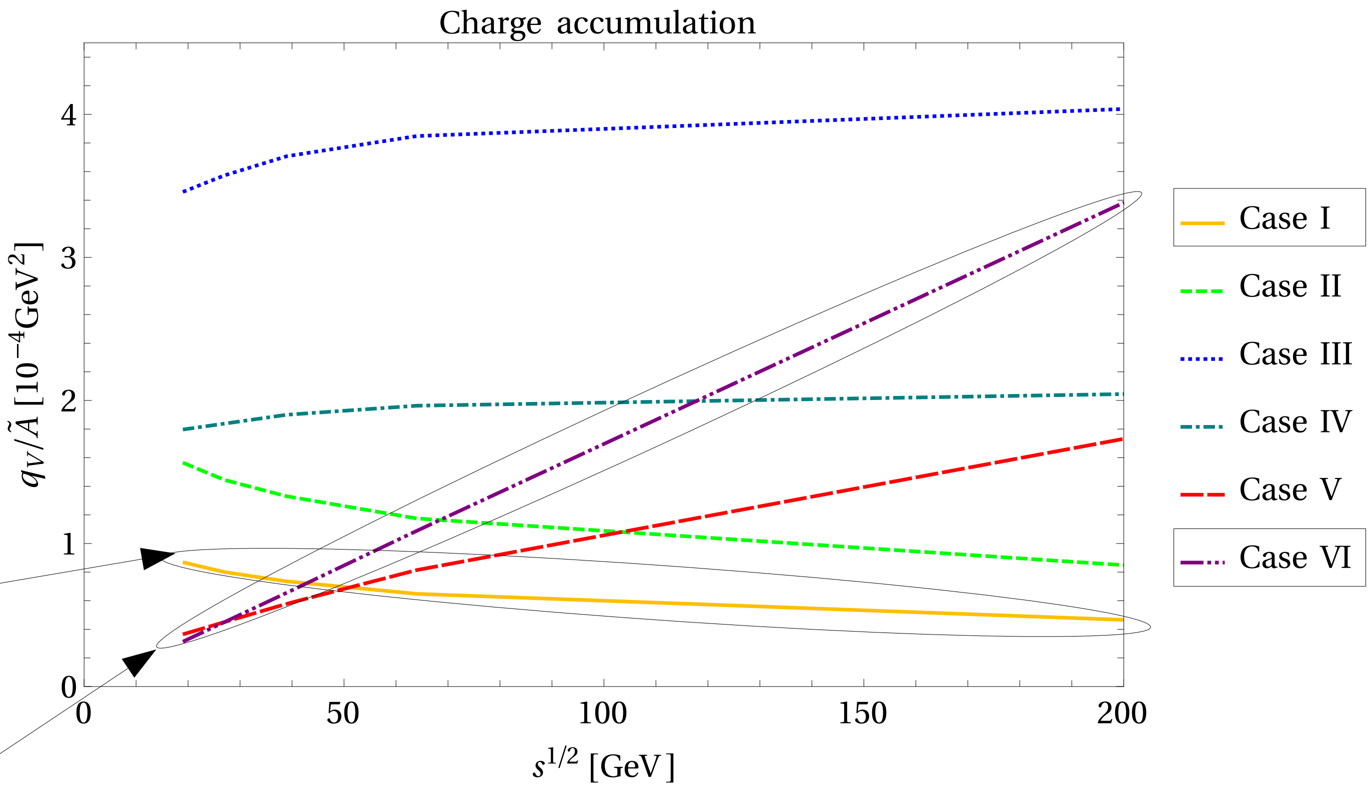


Charge passing through a surface

$$q_V = \int \mathrm{d}t \int \langle \vec{J} \rangle \cdot \mathrm{d}\vec{A}$$

Case I
Initial parameters do not depend on initial energy
i.e. $n_5 \neq n_5(\epsilon_0)$ or $B \neq B(\epsilon_0)$

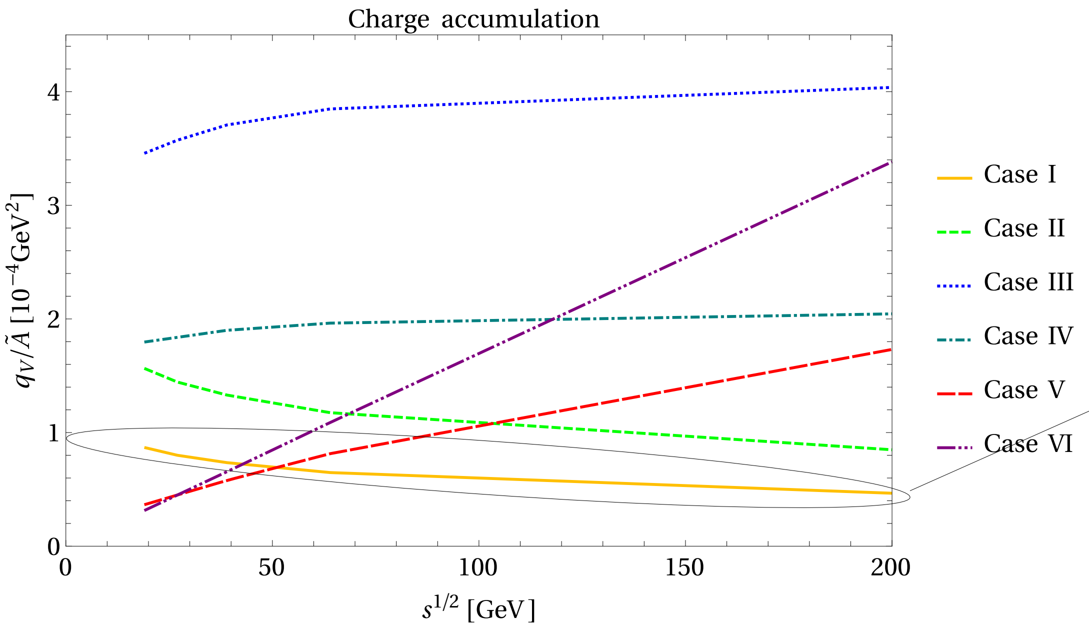
Case VI
Initial parameters do depend on initial energy

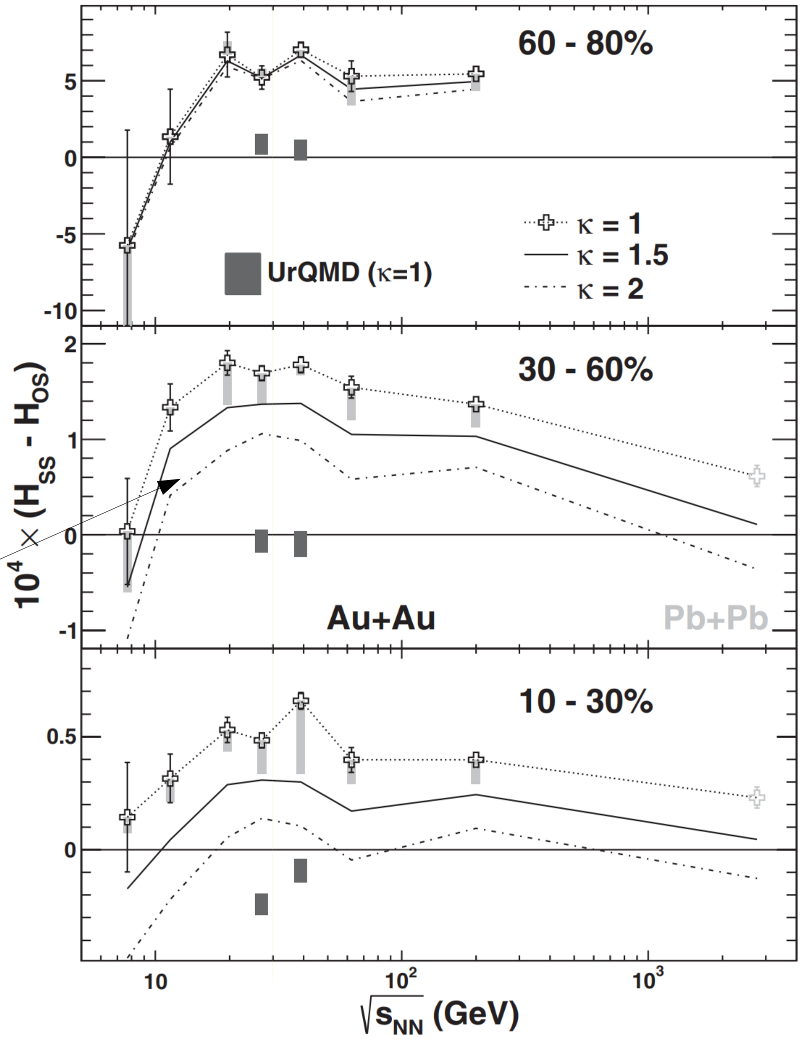


More charge passes through a unit surface for all other cases as we increase the energy

Far from equilibrium

How does it compare to observations?

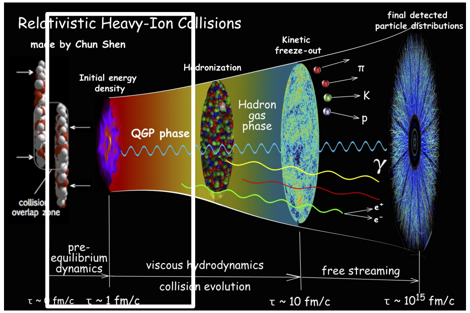




Case I is the best match to current observation

Far from equilibrium

Summary



How does the energy dependence of initial parameters alter the time evolution of the CME current?

Generated a time dependent model of the CME which includes:

- Longitudinal expansion
- Initial far from equilibrium evolution
- Asymptotic to Bjorken flow
- Time dependent magnetic field
- Time dependent axial charge density

Varying $\epsilon(\tau_0)$	CME is larger for smaller energy
Varying $\epsilon(\tau_0),\, n_5(\tau_0,\epsilon(\tau_0))$	CME is larger for larger energy
Varying $\epsilon(\tau_0),\, n_5(\tau_0,\epsilon(\tau_0))\,, B(\tau_0,\epsilon(\tau_0))$	CME is larger for larger energy

- Clearly we need to understand better what the actual energy dependence of the parameters is
- Radial/directed flow in the transverse plane, higher harmonics
- Non homogeneous axial charge density

[Cartwright,Kaminski,Schenke, 2021]

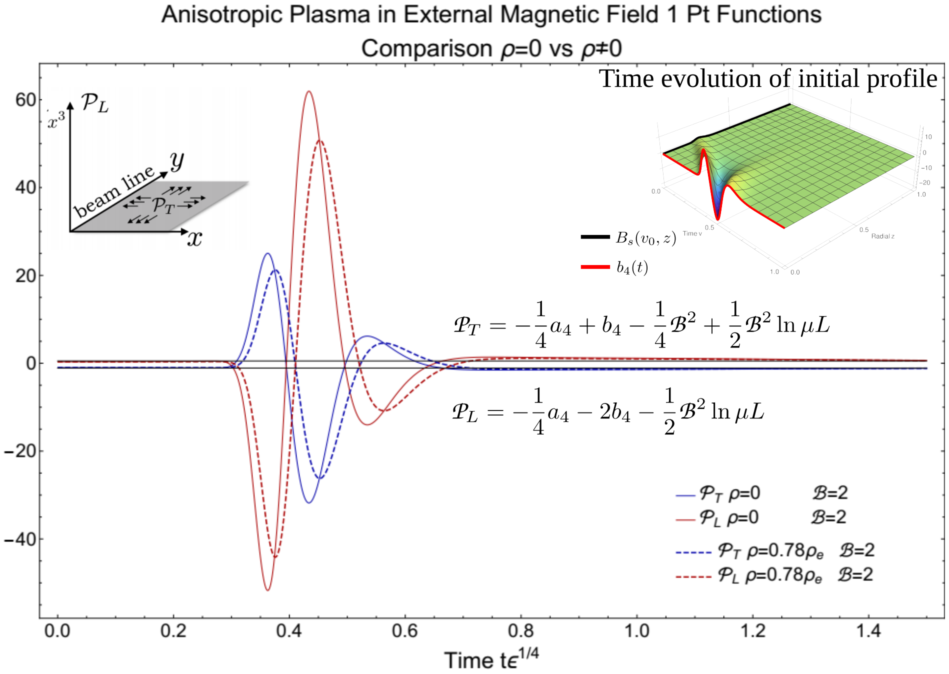
Thank you!

Questions?

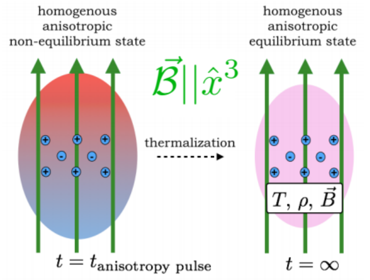
Far from equilibrium II

[Cartwright, Kaminski 2019]

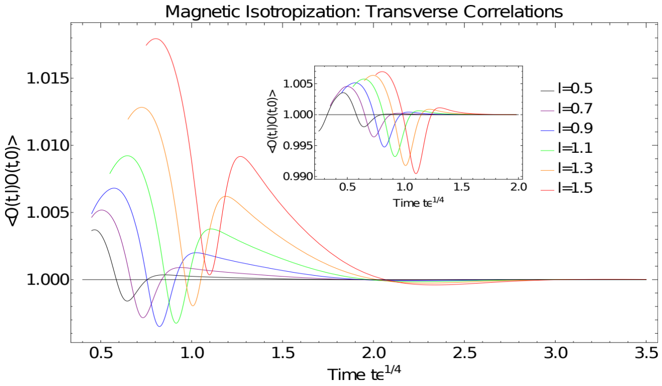
Physics: A homogeneous 3+1 dimension strongly coupled plasma at finite temperature undergoes process of isotropization of its initially anisotropic energy momentum tensor subjected to strong magnetic fields

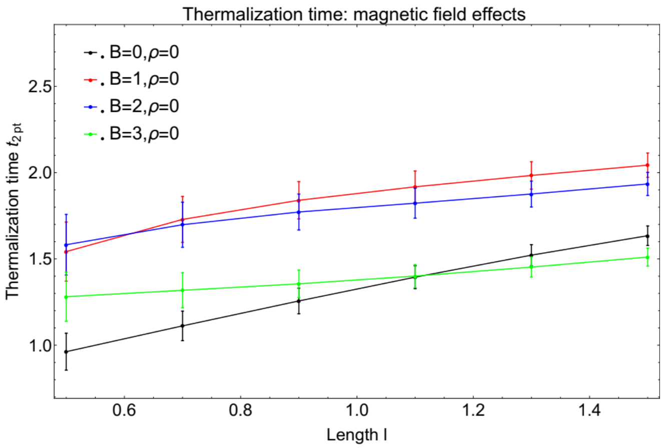


$$\langle O(t,\vec{x}_1)O(t,\vec{x}_2)\rangle = \int \mathcal{D}\mathcal{P} e^{i\Delta\mathcal{L}(\mathcal{P})} \approx \sum_{\text{geodesics}} e^{-\Delta L} \approx e^{-\Delta L}$$



(d) charged fluid in magnetic field





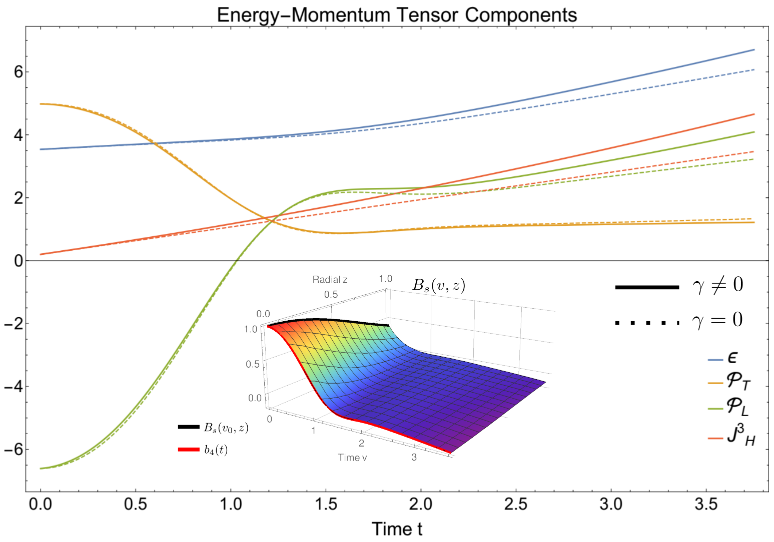
Summary

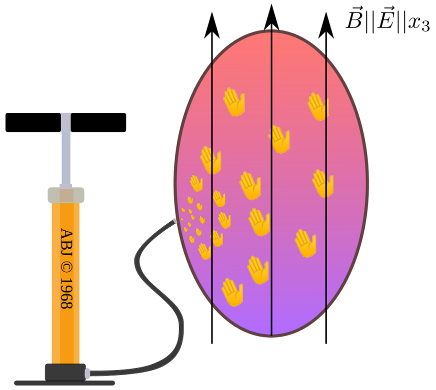
- Computed thermalization times of 1- & 2- op.'s
- 2- correalations take approximately 3 times as long to thermalize
- Increasing magnetic fields lead shallow dependence on time
  Interpreted as a approaching a universal thermalization time

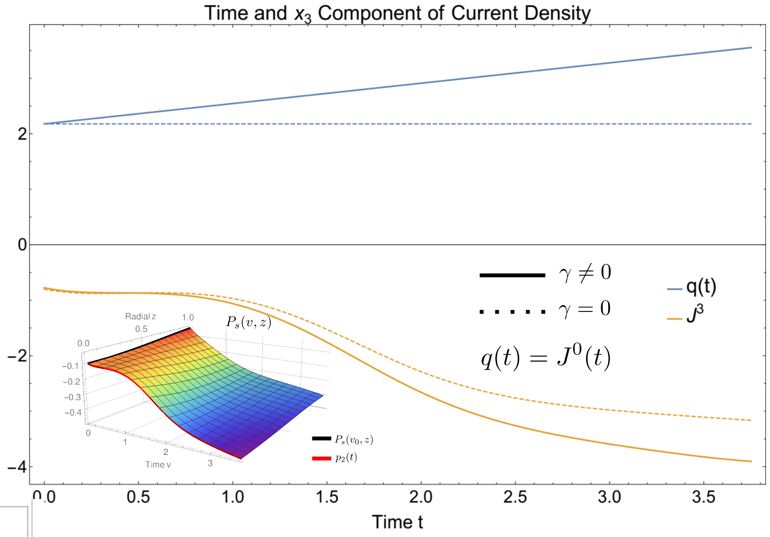
Far from equilibrium III

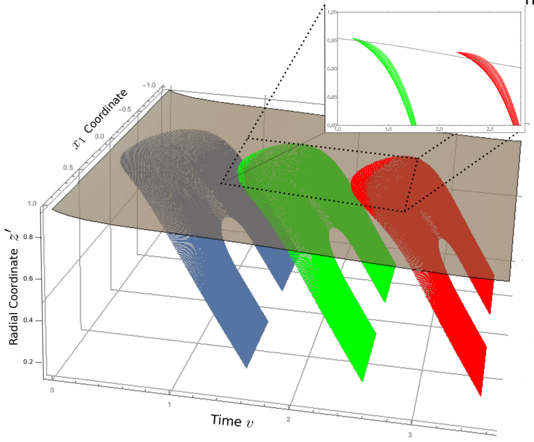
[Cartwright, 2020]

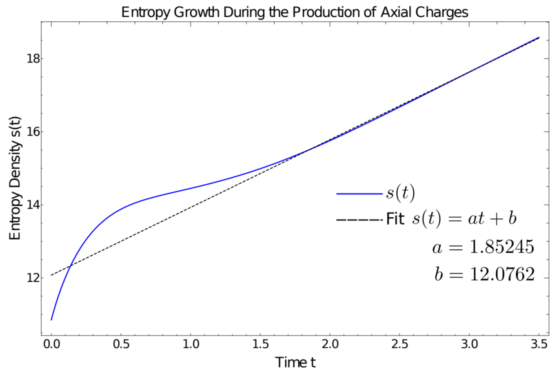
Physics: A homogeneous 3+1 dimension strongly coupled **chiral** plasma at finite temperature undergoes process of isotropization of its initially anisotropic energy momentum tensor











Summary

- Evolution of anomalous chiral plasma experiences Joule heating as a result of accelerated axial axial charges
- Entropy and entanglement entropy grow linearly in time

Introduction

Why can’t I just use Lattice field theory?

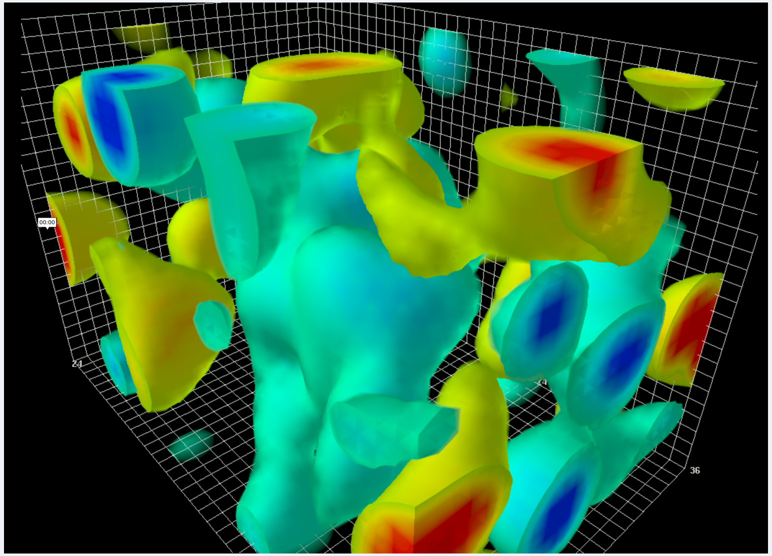
In strongly coupled regime QCD can be in principle modeled directly via the lattice

$$\mathcal{L}_{QCD} = \sum_q \left(\bar{\psi}_{qi} i\gamma^\mu D_{\mu ij}\psi_{qj} - m_q \bar{\psi_q} i\psi_{qi}\right) - \frac{1}{4}G_{a\mu\nu}G^{a\mu\nu}$$

Drawbacks: [Ratti, 2018]

- Not available for large baryon chemical potential $\mu_B/T \lessapprox 2$

- Not available for real time processes. Ill-posed inverse from imaginary time to real time



[Leinweber, 2003] accessed here

Even basic questions are of general interest, e.g.

-How does QCD matter thermalize?

- How does the presence of external fields alter the course to equilibrium?

Far from equilibrium

CME - III

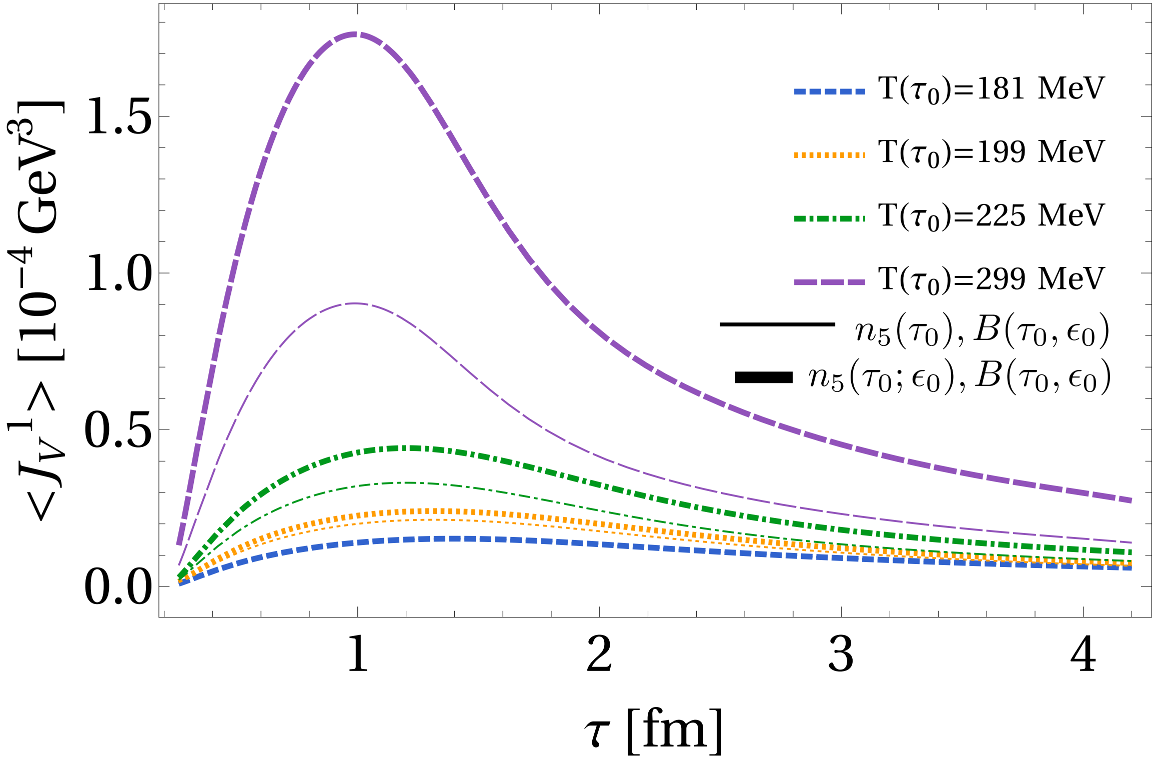
[Cartwright,Kaminski,Schenke, 2021]

Magnetic field strength also depends on the collision energy as well as on the particular nuclei

$$\frac{eB_{RHIC}}{eB_{LHC}} = \frac{1}{2}\frac{\gamma_{LHC}}{\gamma_{RHIC}}\left(\frac{Q_s^{RHIC}}{Q_s^{LHC}}\right)^2$$

– Case V Scan over different initial energy densities with $n_5(\tau_0), B(\tau_0, \epsilon_0)$ Thin lines

– Case VI Scan over different initial energy densities with $n_5(\tau_0;\epsilon_0), B(\tau_0, \epsilon_0)$ Thick lines



Note: Naive estimate of characteristic time scale of the magnetic field [Guo et. al., 2019]

$$t_B \sim \frac{115 GeV fm}{\sqrt{s}}$$

AdS/CFT Correspondence or Holography

[Maldacena, 1999]

N=4 SYM theory with gauge group $SU(N)$ and coupling g_{YM}

dynamically equivalent

$g_{\mathrm{YM}}^2 = 2\pi g_s \quad 2g_{\mathrm{YM}}^2 N = L^4/l_s^4$

Type IIB superstring theory on $AdS_5 \times S^5$ with string length l_s and coupling g_s

In practice the correspondence is a relation between generating functionals

$$\langle e^{\int_{\partial AdS} \phi_0(x) O(x)} \rangle_{CFT} = \mathcal{Z}_s[\phi(z=\epsilon, x) = \phi_0(x)]$$
$$= e^{-W[\phi_0]}$$

Correlation functions can be computed by standard variations

$$\langle O_1(x_1) O_2(x_2) \cdots O_n(x_n) \rangle_{CFT} = \frac{\delta^n W}{\delta\phi_0^1(x_1)\delta\phi_0^2(x_2)\cdots\delta\phi_0^n(x_n)}$$

AdS		CFT	
Metric	$g_{\mu\nu}$	EM-Tensor	$\langle T_{ij} \rangle$
Gauge Field	A_μ	Current	$\langle J_i \rangle$
Scalar Field	ϕ	Scalar Op.	$\langle O \rangle$
Black hole		Finite T state	

[Erdmenger, Ammon, 2015]