Disordered Charged Horizons

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Based on work to appear
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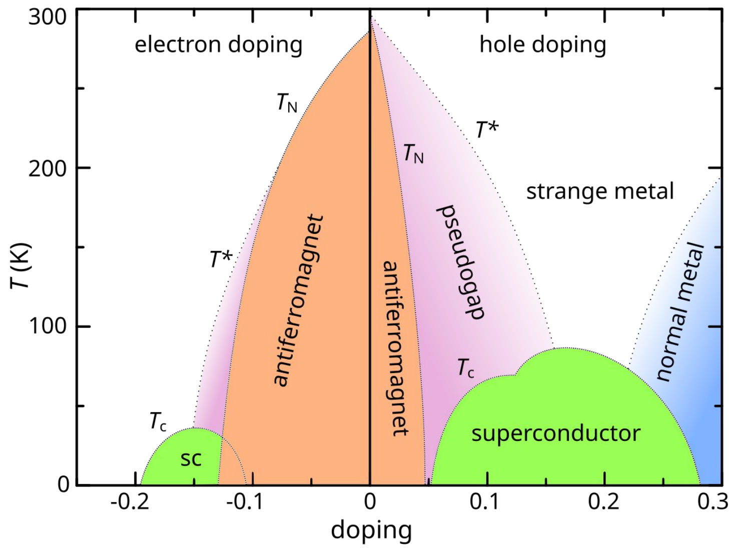
Disordered systems

Cool Condensed Matter Phenomenology with no clean counterpart, specially when combined with **strong interactions**:

- Finite transport
- Griffiths phase transitions [Vojta '10]
- Anderson Localisation [Anderson '58]
- Disordered fixed points
- High Tc superconductivity



Disordered systems

Disorder is always present in nature and systems that we want to study.

- Doping to get cool phenomenology
- Impurities in samples
- Defects in crystals

We need to understand how disorder affects our predictions:

- Does disorder change qualitatively our observables (conductivities, entropy, 2-point functions…) ?
- Are IR fixed points stable under disorder?

Outline

- Harris Criterion
- Holographic setup + disorder
- Numerical implementation
- AdS_4 Results
- AdS_3 Results
- Summary + Outlook

Harris Criterion

[Harris '74]

In (perturbative) QFT, we describe disorder by deforming a theory with an operator with a **space-dependant coupling**

$$S = S_0 + \int d^d x \; \gamma(x) \mathcal{O}_\Delta(x)$$

Observables are **disorder-averaged quantities** calculated by summing over different choices of $\gamma(x)$, using a measure that fixes disorder correlations and strength.

$$\langle ... \rangle_D = \int D\gamma \; P[\gamma] \, (...)$$

[Vojta '13 ;
Aharony, Komargodski, Yankielowicz '15]

We will focus on white Gaussian disorder along n spatial directions.

$$\langle \gamma(x) \rangle_D = 0 \qquad \langle \gamma(x)\gamma(y) \rangle_D = V^2 \delta^{(n)}(x-y)$$

Harris Criterion

For Gaussian disorder we can do the disorder-averaging path integral using the replica trick. We end up with m copies of the clean theory deformed by an operator with an homogeneous coupling, the disorder strength.

$$\tilde{S} = \sum^{m} S_0 + \int d^dx \; V^2 \; \tilde{\mathcal{O}}$$

$$[\ \tilde{\mathcal{O}}\ ] = 2\Delta - (d - n)$$

$$\mathrm{UV} \Big\downarrow \mathrm{IR}^{(\mathrm{Clean})}$$

[Harris ’74]

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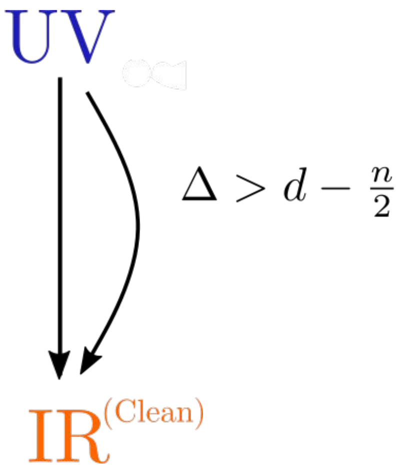

[Harris '74]

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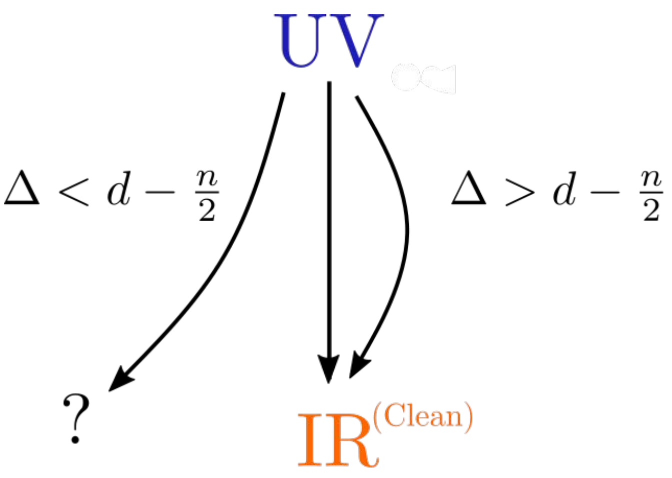

Holographic setup with disorder

Our setup is the minimal theory with a (dual) $U(1)$ current

$$S = \int d^{d+1}x \, \sqrt{-g} \left(\mathcal{R} + \Lambda - \frac{1}{4} F^2 \right)$$

Disorder is introduced via an spatially modulated chemical potential

[Areán, Farahi, Pando Zayas, Salazar Landea, Scardicchio '15]

$$A|_{\partial AdS_3} = \mu(x) \log(z) dt$$

$$A|_{\partial AdS_4} = \mu(x) dt$$

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$$A|_{\partial AdS_4} = \mu(x) dt \qquad \langle \mu(x) \mu(y) \rangle - \mu_0^2 = V^2 \, \text{“}\delta\text{”} (x - y)$$

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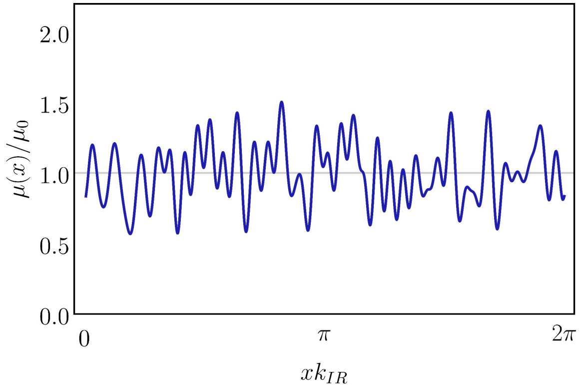

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Holographic setup with disorder

To change the IR we must turn on a Harris relevant disorder $\Delta < d - \frac{n}{2}$

We source disorder via the chemical potential $\mu(x)$ associated with a conserved current

$$\Delta_\mu = d - 1$$

$$\Delta_\mu < d - \frac{n}{2} \rightarrow n < 2$$

We only add disorder along one direction ($n = 1$), this ensures that it is Harris-relevant $\forall d$.

Holographic setup with disorder (Recap)

$$\begin{array}{c} AdS_{d+1} \\ \Big\downarrow \mu = \mu_0 \\ AdS_2 \times \mathbb{R}^{d-1} \end{array}$$

Holographic setup with disorder (Recap)

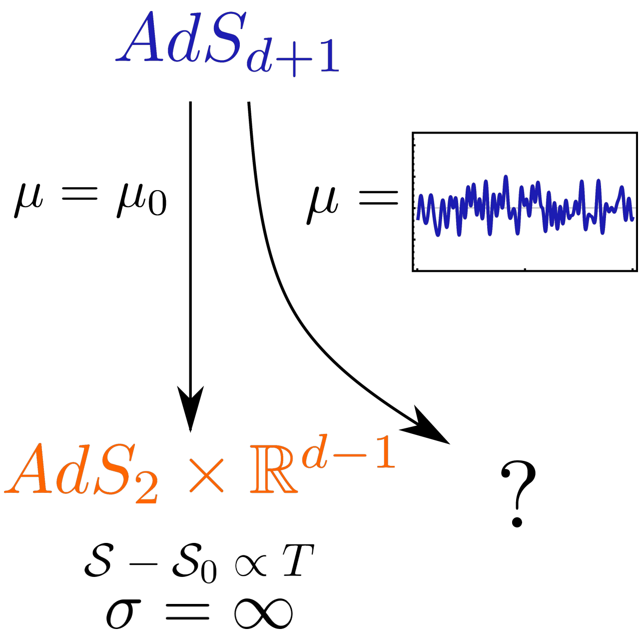
$$AdS_{d+1}$$

$$\Big\downarrow \mu = \mu_0$$

$$AdS_2 \times \mathbb{R}^{d-1}$$

$$\mathcal{S} - \mathcal{S}_0 \propto T$$
$$\sigma = \infty$$

Holographic setup with disorder (Recap)



Solutions of Einstein-Maxwell eq. with a horizon and a random source for the gauge field.

Non-linear 2D PDE.

Solve numerically with a Newton Raphson algorithm.

Numerical Implementation

$$ds^2 = \frac{L^2}{z^2}\left(-H_1 f(z)dt^2 + \frac{H_2}{f(z)}dz^2 + H_3(dx + H_4 dz)^2 + ds_\perp^2\right)$$

The ansatz doesn't fully fix the gauge. We use DeTurck trick to have a well defined boundary problem

$$\mathcal{R}_{\mu\nu} \;\rightarrow\; \mathcal{R}_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} \text{ with } \xi^\nu = g^{\alpha\beta}(\Gamma(g)^\nu_{\alpha\beta} - \Gamma(\bar{g})^\nu_{\alpha\beta})$$

[Headrick, Kitchen, Wiseman ‘09]

Numerical Implementation

$$\begin{cases} 0 \\ H_5 dy^2 \end{cases}$$

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Clean metric

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[Headrick, Kitchen, Wiseman ‘09]

Disordered Regime

$$\mu(x) = \mu_0 + V\sqrt{\frac{k_{UV}}{N}}\sum_j^N \cos\left(\frac{j}{N}\ k_{UV}\ x + \delta_j\right)$$

Only reproduces (white) Gaussian disorder in the regime $[k_{IR}, k_{UV}]$.

For our horizon to feel the disorder we must consider Black Holes with

$$\frac{k_{IR}}{\mu_0} < \frac{T}{\mu_0} < \frac{k_{UV}}{\mu_0}$$

$$k_{UV} \to \infty$$

$$k_{IR} \to 0$$

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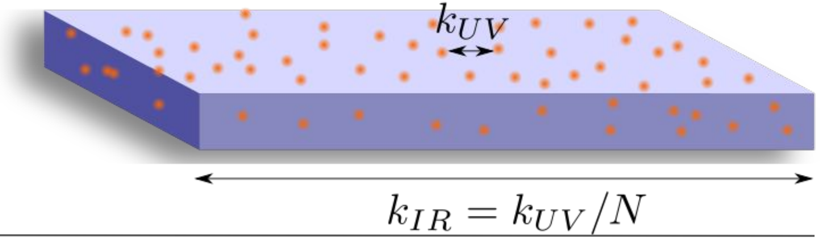

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$$\cancel{k_{UV} \to \infty}$$
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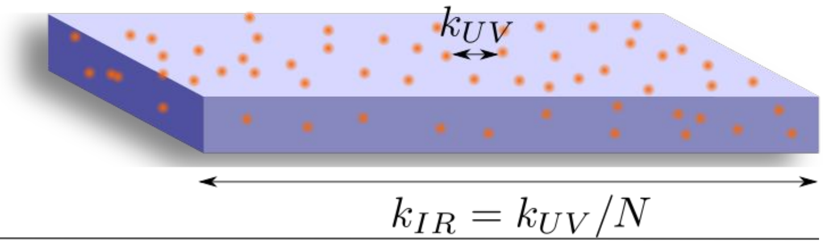
$$k_{UV} = \text{const.}$$
$$k_{IR} \to 0 \ (N \to \infty)$$

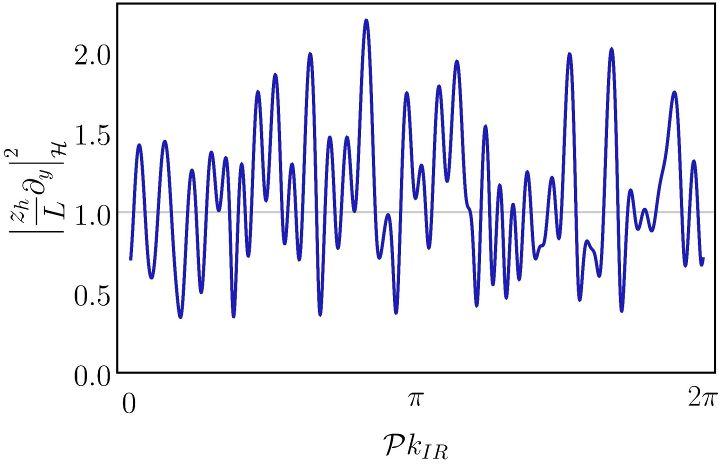


AdS_4 results

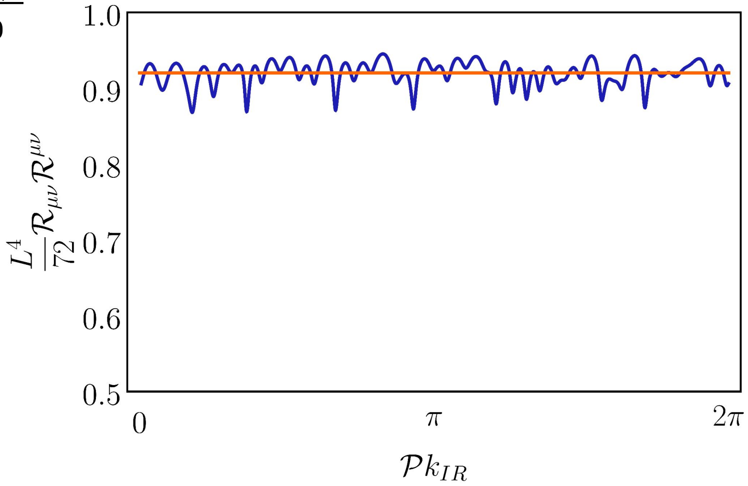
Strongly coupled 2+1 dimensional system with a random chemical potential.

AdS_4 Disordered Charged Horizons

$$\frac{T}{\mu_0} \sim \frac{k_{UV}}{\mu_0}$$



How distances contract/expand in the direction perpendicular to the disorder as a function of proper distance along the horizon.

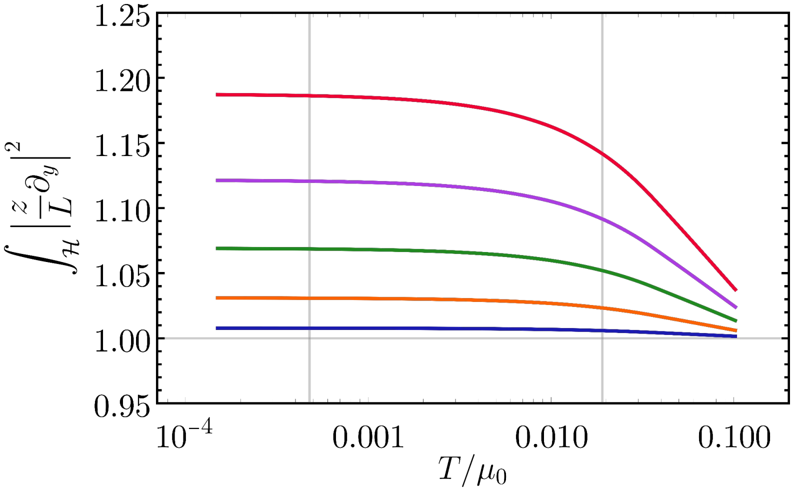


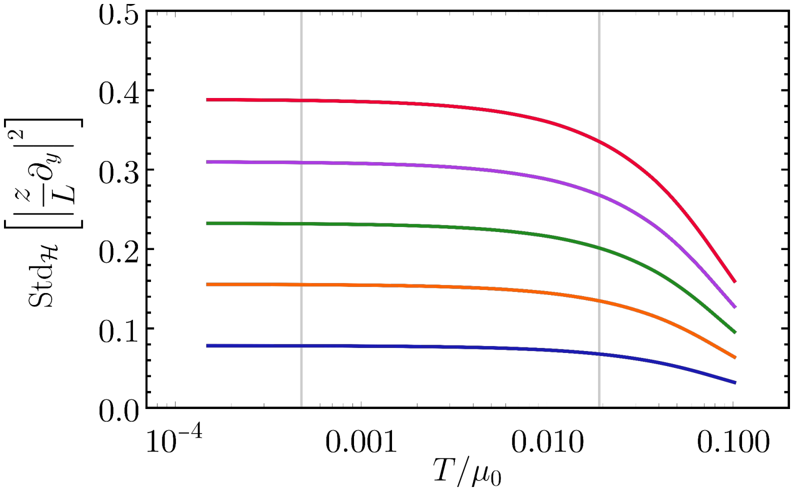
Norm of the Ricci tensor in the horizon

AdS_4 Fate of the IR fixed point

$V/\sqrt{\mu} = 1.25$
$V/\sqrt{\mu} = 1$
$V/\sqrt{\mu} = 0.75$
$V/\sqrt{\mu} = 0.5$
$V/\sqrt{\mu} = 0.25$

$$\int_{\mathcal{H}} X = \frac{1}{\int dx \sqrt{\gamma_{\mathcal{H}}}} \int dx \sqrt{\gamma_{\mathcal{H}}} \left. X \right|_{z=1} \qquad \mathrm{Std}_{\mathcal{H}}[X] = \int_{\mathcal{H}} X^2 - \left(\int_{\mathcal{H}} X\right)^2$$



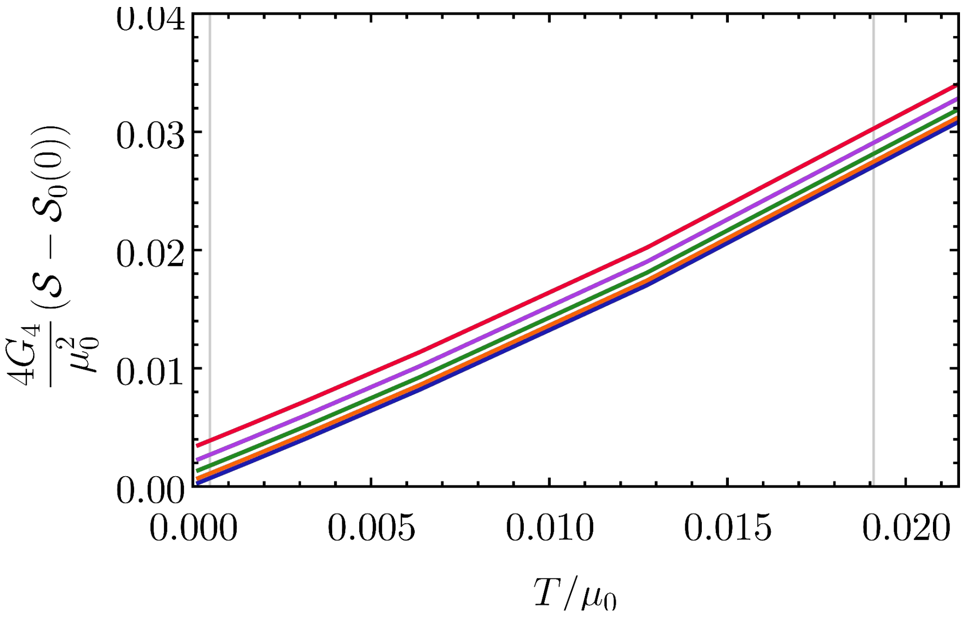


AdS_4 Fate of the IR fixed point (Entropy)

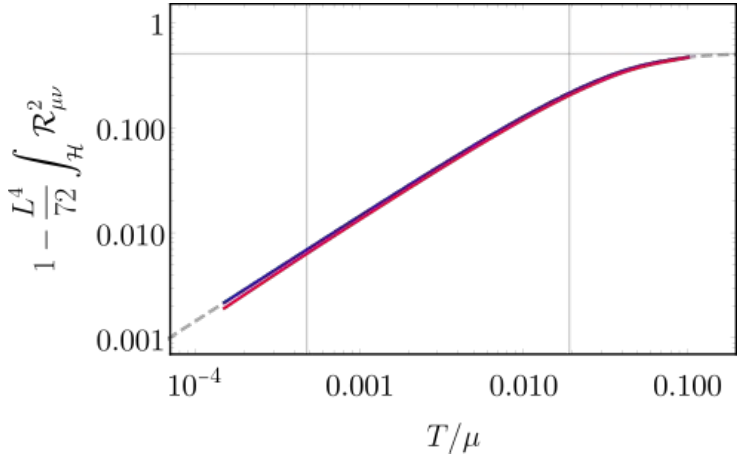
Disorder just shifts the value of the entropy a zero temperature depending on disorder strength.

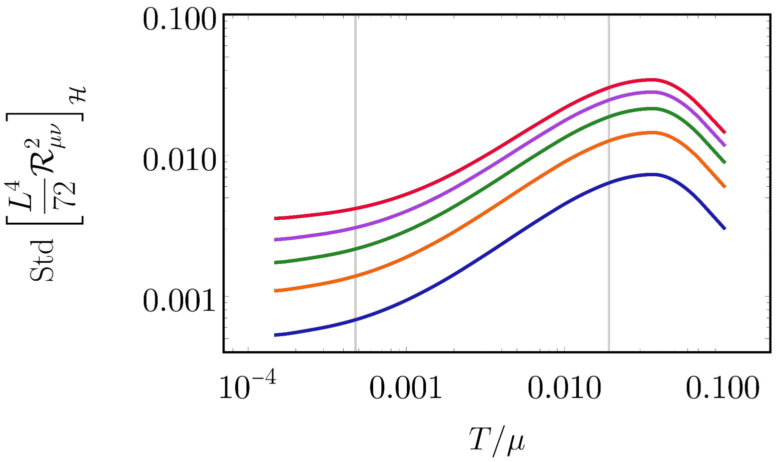
The linear-in T scaling remains unchanged.

$$\mathcal{S} - \mathcal{S}_V \propto T$$



AdS_4 Fate of the IR fixed point (Ricci Tensor)

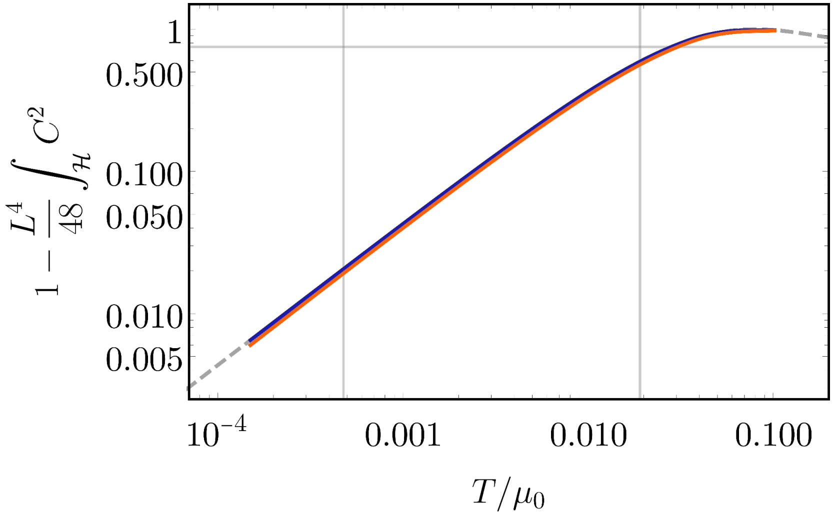


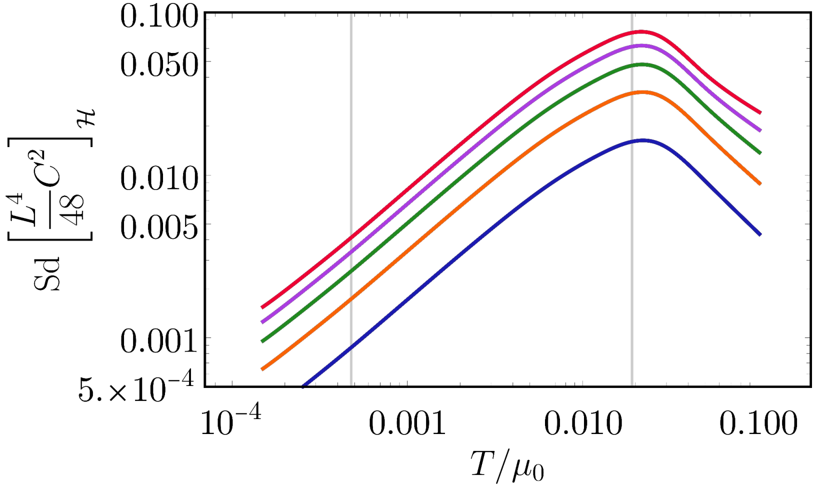


The average geometry goes like the clean case.

The inhomogeneity in the Ricci tensor dies off slower than power-law.

AdS_4 Fate of the IR fixed point (Weyl tensor)





The average value also scales like the clean case.

The inhomogeneity in the Weyl tensor decreases like a power-law.

AdS_4 DC conductivities

[Donos, Gauntlett 14']

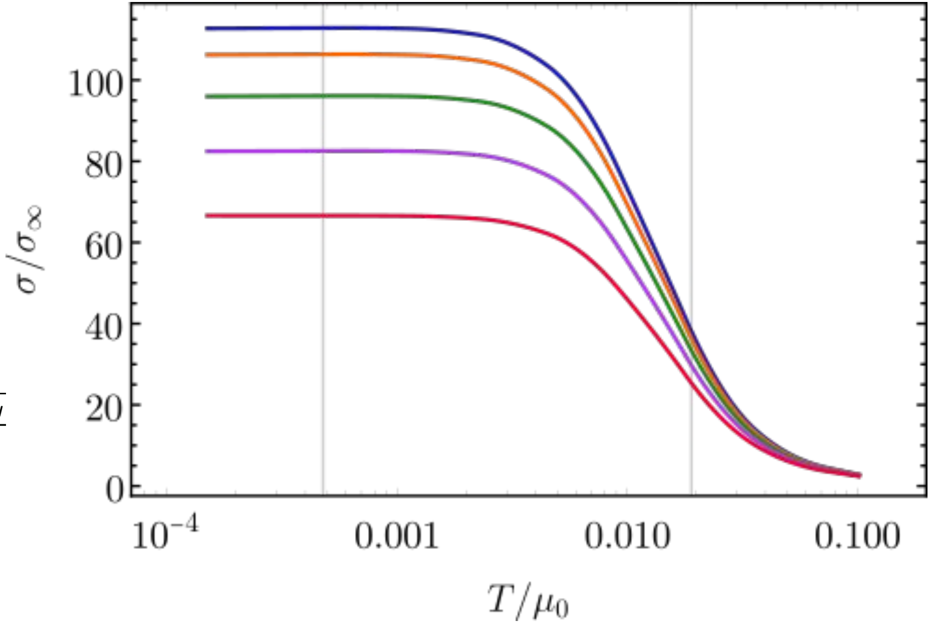
We can obtain DC transport coefficients from integrals over the horizon.

$$\sigma = \frac{1}{Z}\left(1 + \frac{\langle\rho\rangle^2}{\langle\rho^2\rangle - \langle\rho\rangle^2 + \langle\Upsilon^2\rangle}\right)$$

$$\langle\ldots\rangle = \frac{1}{Z}\int dx \frac{\sqrt{g_{xx}g_{yy}}}{g_{yy}}(\ldots) \text{ with } Z = \int dx \frac{\sqrt{g_{xx}g_{yy}}}{g_{yy}}$$

$$\rho = \frac{A_t'}{h_{tt}} \text{ and } \Upsilon^2 = \frac{1}{g_{xx}}\left(\frac{\partial_x g_{yy}}{g_{yy}}\right)^2$$

$$\sigma_\infty = 1 + \frac{\pi\mu_0^2}{V^2 k_0}$$



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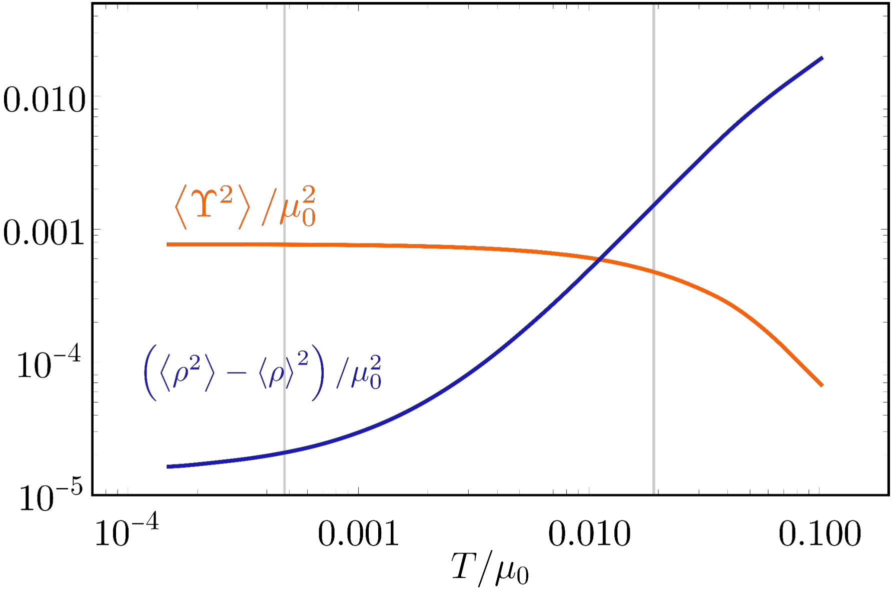
$$\rho = \frac{A_t'}{h_{tt}} \quad \text{and} \quad \Upsilon^2 = \frac{1}{g_{xx}}\left(\frac{\partial_x g_{yy}}{g_{yy}}\right)^2$$

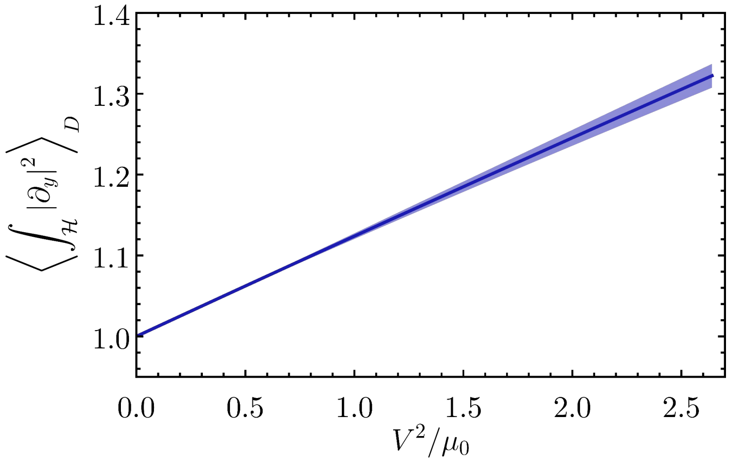
At low temperatures the dominant contribution to the resistivity comes from the inhomogeneous geometry.

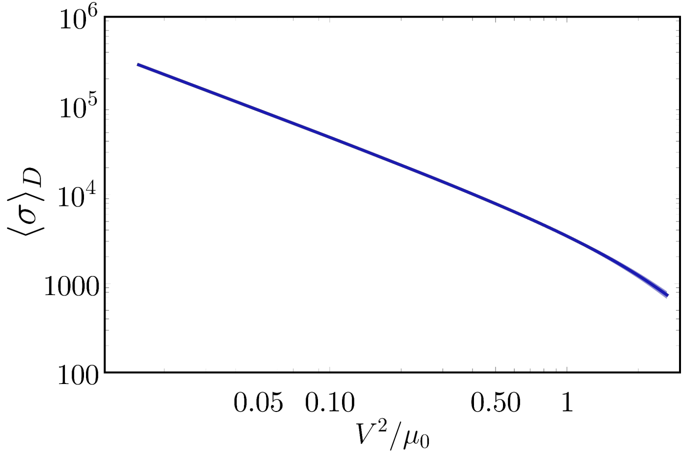


Realisation dependance

Integration along the spatial coordinate kills off the dependance on the random phases. Disorder averaging doesn't change qualitatively our results.

$$\frac{T}{\mu_0} \sim \frac{k_{IR}}{\mu_0}$$





AdS_3 results

Strongly coupled 1+1 dimensional system with a random chemical potential.

AdS_3 Disordered Charged Horizons

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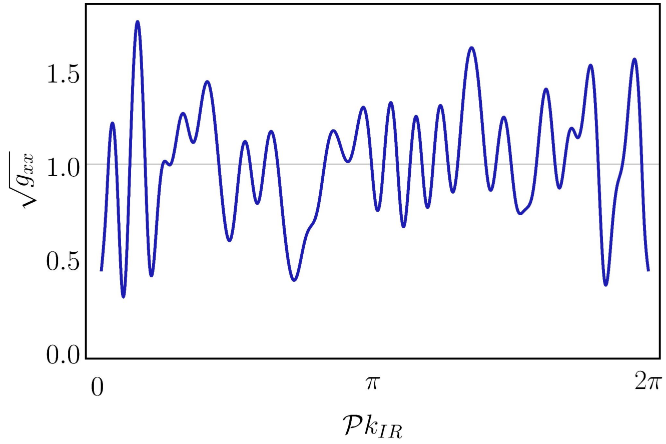
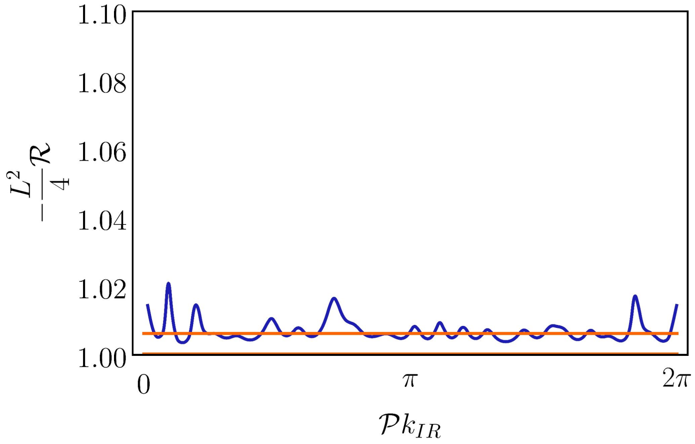
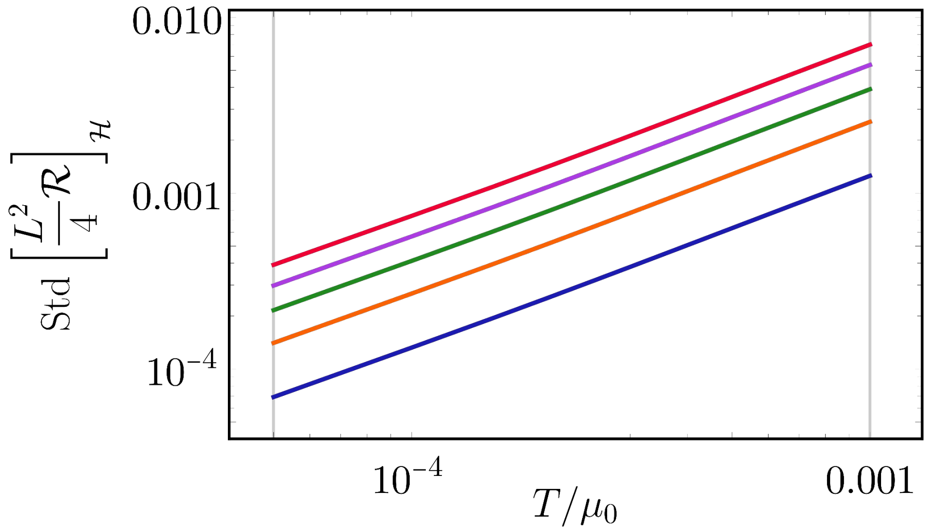


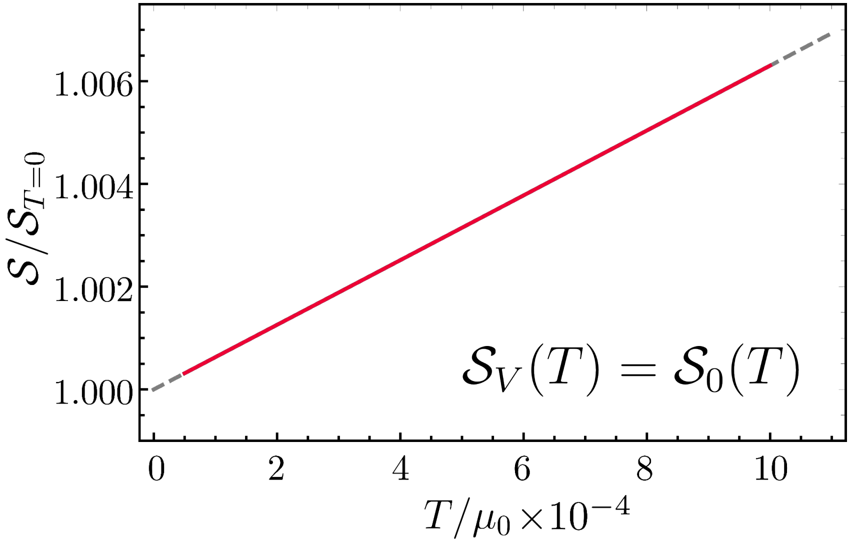


[Schrauth, Portela, Goth ’18; Fayfar, Bretaña, Montfrooji ’21; Topchyan, Nuding, Klümper, Sedrakyan ’24]

AdS_3 Fate of the IR fixed point



Inhomogeneity dies off as we lower the temperature.



Entropy is independent of disorder for all temperatures.

AdS_3 DC conductivities

[Donos, Gauntlett 14']

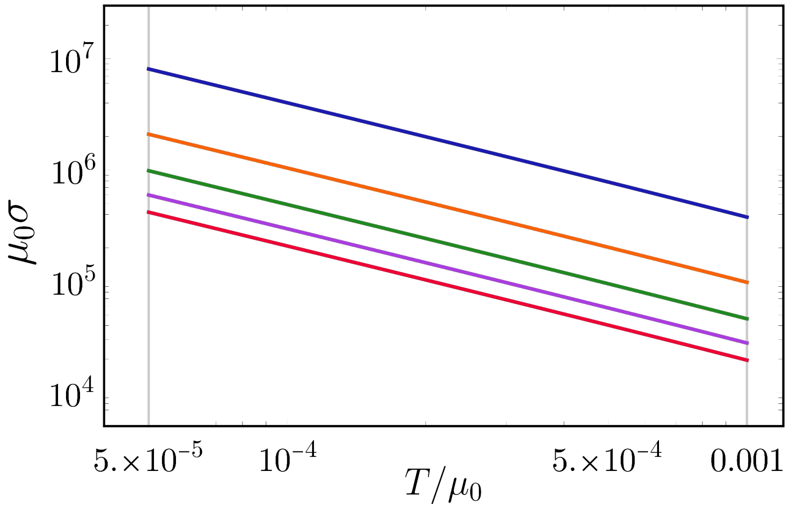
In the DC conductivity we don't have the contribution from the transverse geometry.

As we lower the temperature electrical conductivity diverges.

$$\sigma = \frac{1}{Z}\left(1 + \frac{\langle\rho\rangle^2}{\langle\rho^2\rangle - \langle\rho\rangle^2}\right)$$

$$\langle\ldots\rangle = \frac{1}{Z}\int dx\sqrt{g_{xx}}(\ldots) \text{ with } Z = \int dx\sqrt{g_{xx}}$$

$$\rho = \frac{A_t'}{H_1}$$



Summary

- We obtain disordered horizons with Harris-relevant disorder
- Asymptotically AdS_4
  - Inhomogeneity stabilizes *within* the disordered regime + finite transport coefficients
  - In the IR we find a *continuous family* of disordered fixed points labeled by the disorder strength
  - Still retains characteristics of the clean fixed point: average geometry, entropy scaling with temperature.
- Asymptotically AdS_3
  - IR goes back to the clean case, explicit violation of the Harris criterion.

Outlook

- What happens in AdS_5 ?
- What if we change the disorder (correlations, source, directions…) ?
- Is there any (simple) way to predict how disorder will affect our IR fixed point à la Harris ?
- Can we obtain our disorder transport coefficients directly from the (homogeneous) average geometry (massive gravity) ?

Thank you!