New insights on quantum black holes from braneworld gravity

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Based on:

Emparan, Pedraza, Svesko, Tomašević & Visser [arXiv:2207.03302]
Frassino, Pedraza, Svesko & Visser [arXiv:2212.14055]
Frassino, Pedraza, Svesko & Visser [arXiv:2310.12220]
Hosseini Mansoori, Pedraza, Rafiee [arXiv:2403.13063]
Panella, Pedraza & Svesko [arXiv:2407.03410] (Review on quantum BHs)
Cartwright, Gürsoy, Pedraza & Planella Planas [2408.08010]
Cartwright, Gürsoy, Pedraza & Svesko [2501.17231]

Outline

Part I:

- Motivation: backreaction in semi-classical gravity
- Braneworld gravity and braneworld black holes: exact solutions

Part II:

- Thermodynamics: equilibrium
- QNMs: out-of-equilibrium

Semi-classical gravity & backreaction

- Quantum effects on BHs are crucial in our quest for a theory of QG!
- Consider a theory of gravity coupled to matter, e.g.,

$$S = \int d^{d+1}x\sqrt{-g}\left(\frac{(R-2\Lambda)}{16\pi G_N} + \mathcal{L}_{\text{matter}}\right)$$

- Often, we are interested in studying leading quantum effects
- Focus on semi-classical regime where

$$\ell_P \ll \ell \ll \ell_{\text{macro}}, \qquad \ell_P \sim (\hbar G_N)^{1/(d-1)}$$

- Treat geometry classically; quantize matter fields
- At zeroth order QFT in a fixed background. More generally,

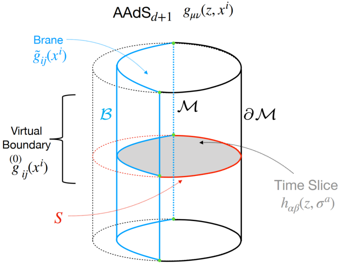
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu}\rangle$$

Technical problem: iterative calculation with increasing complexity

Braneworld gravity: basics

- Start with pure AdS/CFT. Bulk AdS is dual to a CFT living on the boundary
- Introduce a pure tensional brane [Randall-Sundrum '99, Karch-Randall '00]:

$$S = S_{\text{Bulk}}[\mathcal{M}] + S_{\text{GHY}}[\partial\mathcal{M}] + S_{\text{Brane}}[\mathcal{B}]\,, \qquad S_{\text{Brane}} = -\tau \int_{\mathcal{B}} d^d x \sqrt{-h}$$



- 'Integrate out' the UV [de Haro, Skenderis, Solodukhin '00]
- This pulls the CFT to the brane, coupled to dynamical gravity

Braneworld gravity: basics

- This process yields:

$$\tilde{S}_{\text{Brane}} = S_{\text{Bgrav}}[\mathcal{B}] + S_{\text{CFT}}[\mathcal{B}]\,,$$

$$S_{\text{Bgrav}} = \frac{1}{16\pi G_d}\int_{\mathcal{B}} d^d x\sqrt{-h}\left[R - 2\Lambda_d + \ell^2(R^2\text{-terms}) + \cdots\right],$$

- $\ell \propto 1/\tau$ is a scale generated by integration
- The '$\cdots$' in S_{Bgrav} arise from modes above the cutoff
- S_{CFT} arises from modes below the cutoff

Important: Classical bulk solutions induce semi-classical solutions on the brane

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + \cdots = 8\pi G_N \langle T_{\mu\nu}\rangle$$

exactly to all orders in backreaction!

- 'Double holographic' interpretation (3 descriptions: Bulk/Brane/CFT)

Braneworld gravity: basics

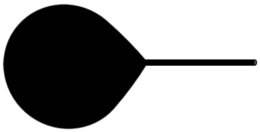
- Very useful to derive exact semi-classical BHs: [Emparan, Frassino, Way '20; Emparan, Pedraza, Svesko, Tomašević, Visser '22; Panella, Svesko '23; Feng, Ma, Mann, Xue, Zhang '24; Climent, Emparan, Hennigar '24; Climent, Hennigar, Panella, Svesko '24]
- And understand their semi-classical properties!
  - Semi-classical BH thermodynamics [Emparan, Frassino, Way '20; Frassino, Pedraza, Svesko, Visser '22; Frassino, Pedraza, Svesko, Visser '23; Johnson, Nazario '23; Mansoori, Pedraza, Rafiee '24]
  - Entanglement islands and entropy of Hawking radiation [Almheiri, Mahajan, Maldacena, Zhao '19; Chen, Myers, Neuenfeld, Reyes, Sandor '20, Geng, Karch '20].
  - Semi-classical corrections to complexity [Hernandez, Myers, Ruan '20; Emparan, Frassino, Sasieta, Tomašević '21; Chen, Liu, Yu '23]
  - BH evaporation [Emparan, Luna, Suzuki, Tomašević, Way '22]

Braneworld black holes: exact solutions

- **Want:** exact solution of bulk+brane system with brane localized BH
- Difficult! Exploit algebraic properties of special class of metrics in 4d AdS_4 C-metric with Karch-Randall brane (Emparan, Horowitz, Myers, '99)

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + H^{-1}(r)dr^2 + r^2\left(G^{-1}(x)dx^2 + G(x)d\phi^2\right)\right]$$

$$H(r) = \kappa + \frac{r^2}{\ell_3^2} - \frac{\mu\ell}{r}\,, \quad G(x) = 1 - \kappa x^2 - \mu x^3$$

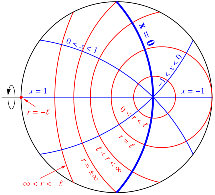


  - $\kappa = \pm 1, 0 \Rightarrow$ slicings on the brane; $\mu \geq 0$ mass parameter; ℓ is (inverse) acceleration; ℓ_3 determines the curvature of 3-slices

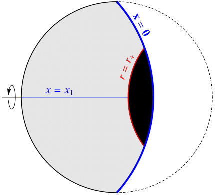
- Key: $x = 0$ surface is 'umbilic.' Brane satisfies Israel conditions

Braneworld black holes: exact solutions

- Cutting & paste:

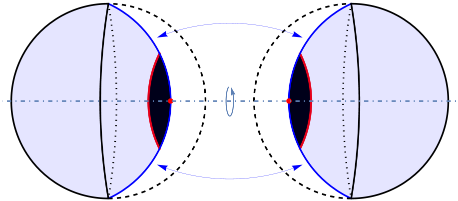


$(\mu = 0)$



$(\mu \neq 0)$

- End up with a $\mathbb{Z}_2$-symmetric brane construction



Braneworld black holes: quantum BTZ (qBTZ)

- Brane geometry is known as the quantum BTZ black hole:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\phi^2 \,, \quad f(r) = \frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \frac{\ell F(M)}{r}$$

$$\mathcal{G}_3 = G_3/\sqrt{1+(\ell/\ell_3)^2} \,, \quad \ell = \frac{1}{2\pi G_4 \tau}$$

$$G_3 = \frac{1}{2L_4} G_4 \,, \quad \frac{1}{L_3^2} = \frac{2}{L_4^2}\left(1 - 2\pi G_4 L_4 \tau\right)$$

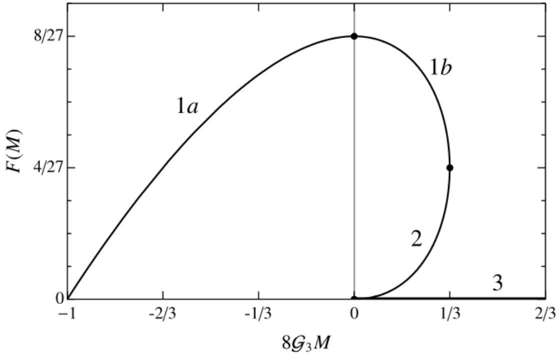
- The CFT stress tensor on the brane yields [Emparan, Frassino, Way, '20]

$$\langle T^{\mu}_{\nu} \rangle = \frac{\ell}{16\pi G_3} \frac{F(M)}{r^3} \mathrm{diag}\{1, 1, -2\} + \ldots$$

  - Central charge $c \sim \ell/G_3$
  - Agrees with perturbative calculation, but valid at all orders
  - Backreaction controlled by $\ell \sim c\ell_P$; corrections are *not* Planck-sized!

Braneworld black holes: quantum BTZ (qBTZ)

- The function $F(M)$ is non-monotonic; it smoothly connects the quantum-corrected BTZ BHs (with $M > 0$) with 'backreacted' conical singularities ($M < 0$), which develop a true singularity and a horizon due to quantum effects:



Thermodynamics of qBTZ

Thermodynamics are inherited from 4d bulk: [Emparan, Frassino, Way, '20]

$$M = \frac{\sqrt{1+\nu^2}}{2G_3}\frac{z^2(1-\nu z^3)(1+\nu z)}{(1+3z^2+2\nu z^3)^2}$$

$$T = \frac{\kappa}{2\pi} = \frac{1}{2\pi\ell_3}\frac{z(2+3\nu z+\nu z^3)}{1+3z^2+2\nu z^3}$$

$$S = \frac{A_4}{4G_4} = \frac{\pi\ell_3\sqrt{1+\nu^2}}{G_3}\frac{z}{1+3z^2+2\nu z^3}$$

- Here $\nu = \ell/\ell_3$ and $z = \ell_3/(r_+ x_1) \geq 0$
- Follows from on-shell partition function [Kudoh, Kurita, '04]
- *Classical* first law of thermodynamics

$$\boxed{dM = TdS}$$

Induced thermodynamics on the brane

Bulk BH thermodynamics *induces* thermodynamics of qBTZ:

$$T = T_{\text{qBTZ}}$$

$$S = \frac{A_3}{4G_3} + S_{\text{Wald}} + S_{\text{CFT}} \equiv S_{\text{gen}}$$

First law of quantum black holes [Emparan, Frassino, Way, '20]

$$\boxed{dM = TdS_{\text{gen}}}$$

- Consistent with 'semi-classical' intuition in 2d [Pedraza, Svesko, Sybesma, Visser, '21]
- In $\ell \to 0$ limit, recover thermodynamics of BTZ

Extended thermodynamics [Frassino, Pedraza, Svesko, Visser '22]

- Treat tension as variable, like fluid surface tension
- Brane performs work on the bulk BH system

Bulk first law:

$$\boxed{dM = TdS + A_\tau d\tau}$$

- $A_\tau \equiv (\partial M/\partial\tau)_S$ – "regularized brane area"
- M plays role as *enthalpy*

Bulk Smarr law:

$$\boxed{M = 2TS - 2P_4 V_4 - \tau A_\tau \ , \quad P_4 = -\frac{\Lambda_4}{8\pi G_4}}$$

- Working in *fixed* pressure P_4 ensemble

Extended thermodynamics [Frassino, Pedraza, Svesko, Visser '22]

Variable τ **induces** *extended thermodynamics!*

$$\boxed{\delta\tau = \frac{\delta\Lambda_3}{8\pi G_3} = -\delta P_3}$$

Extended first law of qBTZ:

$$\boxed{dM = TdS_{\text{gen}} + V_3 dP_3}$$

More generally, allowing variations of Λ_4:

$$\boxed{dM = TdS_{\text{gen}} + V_3 dP_3 + \mu dc}$$

Smarr law for qBTZ:

$$0 = TS_{\text{gen}} - 2P_3 V_3 + \mu_3 c_3$$

- $G_3 M$ has vanishing scaling dimension

Entropy inequalities [Frassino, Hennigar, Pedraza, Svesko '24]

Treating backreaction small $\nu \ll 1$:

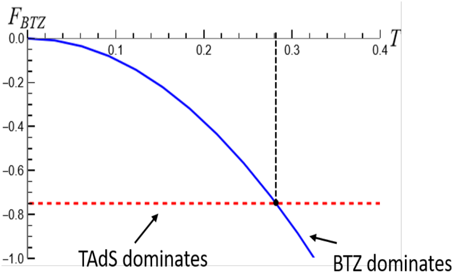
$$V_3 = V_{\text{BTZ}} + 8\pi\ell_3^2 \frac{z^3(1+z^2)}{(1+3z^2)^3}\nu + \dots$$

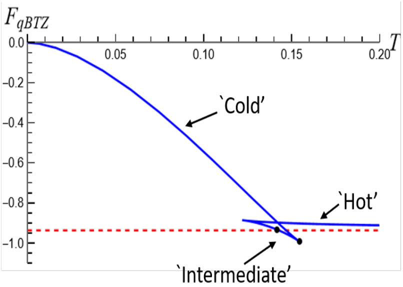
- *Classically* $V_{\text{BTZ}} \sim S_{\text{BTZ}}^2$
- Backreaction modifies volume from *geometric volume*
- 'Reverse isoperimetric inequality' obeyed [Cvetic, Gibbons, Kubiznak, Pope '11]

$$\mathcal{R} \equiv \left(\frac{V_3}{\pi}\right)^{1/2} \left(\frac{\pi}{2S_{\text{gen}}}\right) = 1 + z\nu + \mathcal{O}(\nu^2) \geq 1$$

- qBTZ is *sub-entropic*: a BH with V_3 and entropy *less* than BTZ
- Thermodynamically *stable* for small backreaction [Johnson '19]
- Naively violated for $\nu \sim 1$. Casimir effects dominate over thermal ones
  - New semi-classical entropy inequalities (RII and Penrose)! [FHPS '24]

Thermal phase transitions [Frassino, Pedraza, Svesko, Visser '23]





- In canonical ensemble (fixed T, c and P_3): $F_{\text{qBTZ}} = M - TS_{\text{gen}}$
- Large backreaction $\Rightarrow$ 'reentrant' phase transitions

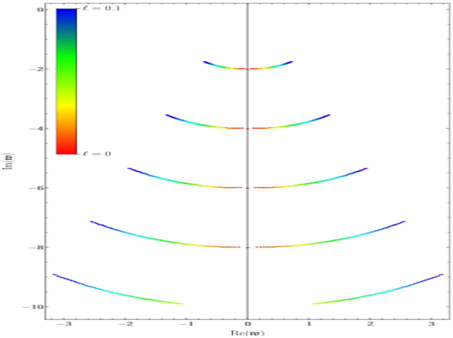
$$\text{As } \; T \; \text{ increases,} \quad \text{TAdS} \overset{\text{1st}}{\to} \text{qBTZ} \overset{\text{0th}}{\to} \text{TAdS}$$

- Other ensembles equally interesting [Mansoori, Pedraza, Rafiee '24]: e.g., a novel critical point with non-mean-field exponents (not Van der Waals -like) at fixed T, c and V

Black hole perturbations [Cartwright, Gürsoy, Pedraza & Planella Planas '24]

- We consider brane localized matter (scalar and fermions) and solve the corresponding wave equations with Dirichlet BC at the AdS boundary (no-source) and ongoing boundary conditions at the horizon
- Solving the eigenvalue problem we obtain the black hole QNMs, which give the poles of the retarded correlator [Horowitz & Hubeny '99]

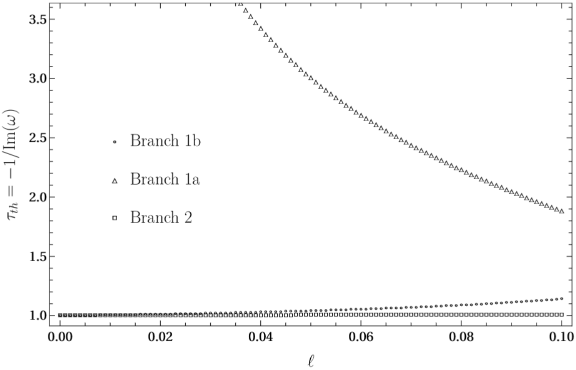
$$G_R(\omega, k) = \frac{A(\omega, k)}{B(\omega, k)}$$



Black hole perturbations [Cartwright, Gürsoy, Pedraza & Planella Planas '24]

- The leading QNM $\omega = \omega_R - i\omega_I$ controls the late-time decay of perturbations. This defines the thermalization time $\tau_{th} = -1/\text{Im}(\omega)$:

$$G_R(t,x) \sim e^{-\omega_I t}$$



- For the BTZ branches τ_{th} increases with the strength of the backreaction. For conical defects: prediction for DCFT+BCFT system

Pole skipping points

- Pole skipping are locations in the complex momentum plane where the Green's function becomes multivalued
- Zeroes at particular points where there are multiple ingoing solutions:

$$G_R(\omega, k) = \frac{A(\omega, k)}{B(\omega, k)}$$

Motivation is twofold:

- Analytic constrains on boundary correlators from near-horizon physics [Blake, Davison & Vegh '19]
- Connections with quantum chaos [Grozdanov, Schalm & Scopelliti '17]
  - e.g.: the lowest pole skipping point for energy-energy correlators satisfy:

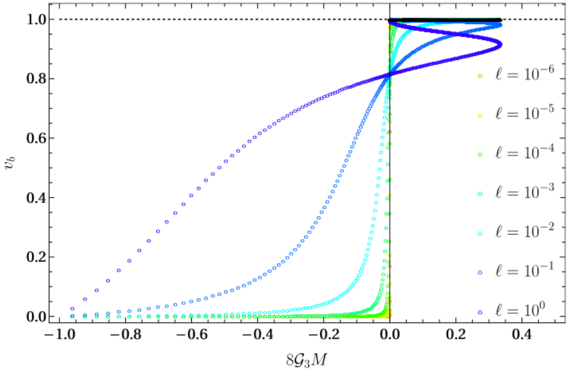
    $$\omega = i\lambda_L \,, \qquad k = \lambda_L / v_B$$

    where λ_L and v_B are the Lyapunov exponent and butterfly velocity that appear bounding generic OTOCs:

    $$C(t, x) \sim e^{\lambda_L (t - x/v_B)}$$

Pole skipping points

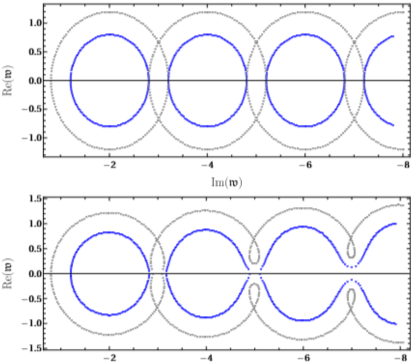
- At finite quantum backreaction, frequency is constant but momentum has a non-trivial dependence with temperature. This implies a novel dependence of the butterfly velocity:



- These results are for a massless scalar field; A honest-to-god calculation would require gravitational perturbations (work in progress!).

Pole collisions in the complex plane

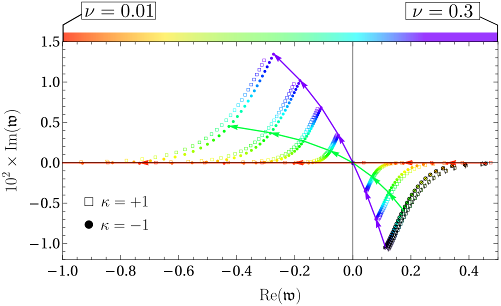
- Pole collisions bound the radius of convergence for hydrodynamic expansion [Grozdanov, Kovtun, Starinets & Tadic '19]
- In BTZ the poles of the Green's functions are analytic yielding level-touching events. In qBTZ, these become level crossing events.



- Such non-analyticities are clear semi-classical imprints, which has implications for the spectral reconstruction program [Grozdanov & Lemut '22]

Superradiance in rotating qBTZ [Cartwright, Gürsoy, Pedraza & Svesko '25]

- Set of superradiant modes close to extremality (quantum induced)!



- For qBTZ, all eigenvalues of the Hessian are negative, while for the qCone at least one is positive: thermodynamic instability $\rightarrow$ dynamical instability.
- This suggest a semi-classical version of Gubser-Mitra conjecture for black hole stability [Gubser, Mitra '00].

Summary and future work

Summary:

- Braneworlds are useful to construct and study exact semi-classical BHs
- Novel thermal phenomena induced by quantum backreaction effects
- Relaxation and thermalization in CFT beyond the strict large-N limit

Future directions:

- Gravitational perturbations
- Adding rotation & charge
- dS and flat space BHs
- Higher dimensions
- Top-down models from string theory

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