Heterotic Cosmic String Vacua and Generalized Half-flat Compactifications

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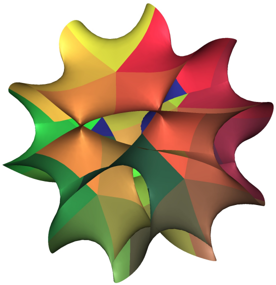
Why the heterotic string?

Some reasons to study heterotic string theory:

- String theory: consistent theory of quantum gravity!
- Comes equipped with an $E_8 \times E_8$ (or SO(32)) gauge group $\Rightarrow$ good framework for grand unified models.
- Many candidate Standard Model compactifications known.
- Calabi–Yau compactification gives $\mathcal{N} = 1$ SUSY in $d = 4$ $\rightarrow$ suitable for MSSM-style models.
- Appealing mathematical framework, reasonably well-studied.

Calabi–Yau compactification

- Superstring theory is self-consistent only in 10 spacetime dimensions.
- Assume the extra 6 spatial dimensions are compactified.
- Lots of supersymmetry in $d = 10$ $\rightarrow$ want to break most of it.
- Amount of broken SUSY $\Rightarrow$ holonomy group of compactification manifold.



- Maximum holonomy is SO(6) $\cong$ SU(4) $\Rightarrow$ no SUSY preserved.
- Calabi–Yau manifold: SU(3) holonomy $\Rightarrow$ 1/4 SUSY preserved
  - e.g. heterotic Calabi-Yau: 4 of 16 supercharges unbroken.

Moduli stabilization: Type IIB example

Compactification gives rise to moduli: flat directions in the potential $\rightarrow$ need to stabilize.

- Example: moduli stabilization in type IIB string theory.
- Theory contains R-R 3-form flux F_3 and NS-NS 3-form flux H_3.
- Compactify such that on the manifold, F_3 and H_3 are non-zero $\rightarrow$ flux compactification.
- In the right combination, dilaton and all complex structure moduli can be stabilized by these fluxes
- Kähler moduli remain unstabilized, can fix with eg.
  - non-perturbative effects (KKLT),
  - non-perturbative effects and perturbative α' corrections (LVS).
- All moduli stabilized!

Problems with heterotic moduli stabilization

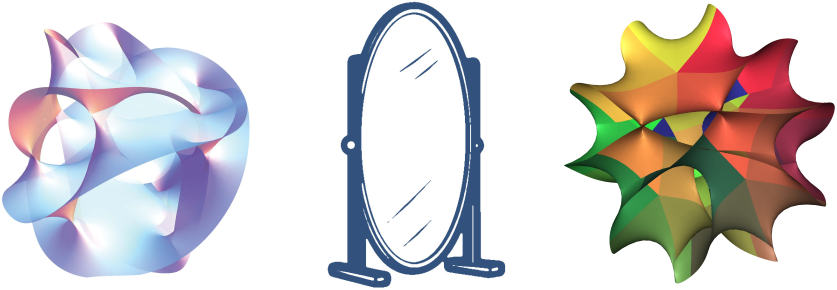
- In heterotic string theory, only have NS-NS flux H_3.
- Can stabilize complex structure moduli... then what?
- Dilaton can be stabilized by gaugino condensation.
- No other non-perturbative effects, no other options for flux quantisation.
- In fact, problem is even worse:

Strominger, 1986

If a heterotic compactification on a manifold Y has a maximally symmetric (e.g. Poincaré) vacuum and non-vanishing H_3, Y is non-Calabi–Yau.

- Hence for a Calabi–Yau compactification, $H_3 = 0$!

Mirror symmetry



- A related issue is mirror symmetry.
- Type IIA compactified on $Y \leftrightarrow$ type IIB compactified on $\widetilde{Y}$.
- Flux compactifications: R-R flux $F_3 \leftrightarrow F_0, F_2, F_4, F_6$.
- Problem: no obvious mirror dual for NS-NS flux H_3!

What is an SU(3) structure manifold?

- Mirror dual: manifold with SU(3) structure, but not Calabi-Yau hep-th/0008142 (Vafa), hep-th/0211102 (Gurrieri, Louis, Micu, Waldram).
- SU(3) structure: there is a globally-defined spinor ζ that leaves 1/4 of the SUSY unbroken.
- Calabi–Yau case: ζ is covariantly constant with respect to the Levi-Civita connection ∇.
- Non-CY case: $\nabla\zeta \sim T^0\zeta$ (note: Γ matrices/indices suppressed).
- T^0 is the intrinsic torsion of the manifold.
- SU(3) decomposition: torsion splits into 5 torsion classes,

$$T^0 \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4 \oplus \mathcal{W}_5 \ .$$

Half-flat manifolds

Two (not mutually exclusive) ways to satisfy Strominger's theorem:

Option 1:

Study compactifications on SU(3) structure manifolds with torsion.

- Has been studied in eg. hep-th/0408121 (Gurrieri, Lukas, Micu), hep-th/0507173 (de Carlos, Gurrieri, Lukas, Micu).
- Torsion quantization understood for half-flat manifolds.
- Expanding the SU(3) invariant forms on appropriate bases, the only non-closed basis forms in the half-flat case satisfy

$$d\omega_i = e_i \beta^0 \ , \quad d\alpha_0 = e_i \tilde{\omega}^i \ .$$

- For half-flat manifolds, torsion falls into the SU(3) classes

$$T^0 \in \mathcal{W}_1^+ \oplus \mathcal{W}_2^+ \oplus \mathcal{W}_3 \ ,$$

where $+$ denotes the real part of $\mathcal{W}$.

Domain wall vacuum

Option 2:

Break maximal symmetry of $d = 4$ spacetime.

- Compactification with H-flux on a half-flat or Calabi-Yau manifold.
- There exist 1/2-BPS domain wall solutions 1305.0594 (Klaput, Lukas, Svanes).
- 1/2-BPS: 2 of the 4 SUSY generators in $d = 4, \mathcal{N} = 1$ unbroken.
- $d = (2 + 1)$ Poincaré symmetry preserved; DW breaks symmetry in transverse y direction.
- Moduli satisfy flow equations in the y coordinate.
- 10d perspective: SU(3) fibred over $y \rightarrow$ G_2 structure arXiv:1005.5302 (Lukas, Matti).

Extension: cosmic strings

The following is based on research in progress, in collaboration with Cyril Matti (City University, London) and Eirik Svanes (LPTHE, Paris).

- The 1/2-BPS domain wall breaks maximal symmetry along one noncompact direction.

Proposition:

Fibre the SU(3) structure over an interval that is not a Cartesian coordinate direction.

- Cylindrical polar coordinates (ρ, ϕ, z): fibre along ρ $\Rightarrow$ cosmic string.
- 4d metric:

$$ds_4^2 = e^{-2B(\rho)} \left(\eta_{\alpha\beta} dx^\alpha dx^\beta + d\rho^2 + \rho^2 d\phi^2 \right) .$$

Cosmic string flow equations

The Killing spinor equations reduce to

$$\begin{aligned}
\partial_\rho A^I &= -ie^{-B} e^{K/2} K^{IJ^*} D_{J^*} W^* \ , \\
\partial_\rho B &= ie^{-B} e^{K/2} W \ , \\
0 &= \mathrm{Im}(K_I \partial_\rho A^I) \ , \\
2\partial_\rho \zeta &= -\partial_\rho B \zeta \ ,
\end{aligned}$$

with the 1/2-BPS constraint $\overline{\zeta} = \sigma^{\underline{r}} \zeta$, where

$$\sigma^{\underline{\rho}} = \frac{\widehat{x}}{\rho}\sigma^{\underline{1}} + \frac{\widehat{y}}{\rho}\sigma^{\underline{2}} = \begin{pmatrix} 0 & e^{-i\phi} \\ e^{i\phi} & 0 \end{pmatrix} .$$

- Note: same structure as in the domain wall case (with $\rho \leftrightarrow y$).
- Related to DW by conformal transformation: $\rho = e^y$; $x = -\phi$.

Spin(7): two transverse coordinates

- The cosmic string solution is a special case of the metric

$$ds_4^2 = e^{-2B(x_a)}\left(\eta_{\alpha\beta}d\tilde{x}^\alpha d\tilde{x}^\beta + g_{ab}dx^a dx^b\right) \ .$$

- We can consider more general codimension-2 topological defects.
- 10d perspective: looks like an 8-dimensional Spin(7) structure.
- For the corresponding 6d compact SU(3) structure manifold, consider a generalized half-flat manifold, which satisfies

$$d\omega_i = p_{Ai}\beta^A - q_i^A\alpha_A \ , \quad d\alpha_A = p_{Ai}\tilde{\omega}^i \ , \quad d\beta^A = q_i^A\tilde{\omega}^i \ , \quad d\tilde{\omega}^i = 0 \ ,$$

  where ω_i and (α_A, β^B) are basis 2- and 3-forms, respectively.
- Relevant SU(3) torsion classes are now

$$T^0 \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \ .$$

Simple example

- 1/4-BPS ansatz: $\overline{\zeta} = \sigma^1 \zeta = -i\sigma^2 \zeta$ gives Killing spinor equations

$$(\partial_x - i\partial_y)\, A^I = -ie^{-B} e^{K/2} K^{IJ^*} D_{J*} W^* \ ,$$
$$(\partial_x + i\partial_y)\, B = ie^{-B} e^{K/2} W \ ,$$
$$0 = \mathrm{Im}(K_I \partial_a A^I) \ ,$$
$$2\partial_a \zeta = -\partial_a B \zeta \ .$$

- Simple case: $p_{0i} = q_i^A = 0$, $H = 0$, vanishing axions.
- In this simple example, 4d and 10d equations match when x-dependence vanishes
  $\Rightarrow$ alternative 1/2-BPS DW solution, G_2 sublocus of Spin(7).
- KSEs satisfy

$$\frac{\mathcal{K}'}{\mathcal{K}} = \frac{\tilde{\mathcal{K}}'}{\tilde{\mathcal{K}}} = -2\phi' = \frac{3}{\sqrt{\mathcal{K}\tilde{\mathcal{K}}}} p_{ai} v^i w^a \ ,$$

where $(v^i, w^a) = \mathrm{Im}(T^i, Z^a)$, $\mathcal{K} = \mathcal{K}_{ijk} v^i v^j v^k$, $\tilde{\mathcal{K}} = \tilde{\mathcal{K}}_{abc} w^a w^b w^c$.

Summary

- String compactifications generate moduli, which must be stabilized. This can be done using fluxes.
- For the heterotic string, only H_3 present. One can compactify on SU(3) structure manifolds which are not Calabi–Yau, and/or sacrifice maximal symmetry in $d = 4$. Domain wall solutions have been considered.
- We found a cosmic string solution, which is a 1/2-BPS G_2 fibration. It is related to the domain wall case by a conformal transformation.
- In the more general codimension-2 case, for vanishing H-flux we found a new type of domain wall solution at a G_2 locus of Spin(7).
- Goal: include flux; 1/4-BPS solutions (e.g. intersecting domain walls $\rightarrow$ extra warp factor?); more general Spin(7) structure ...work in progress!